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LECTURES

ON

SELECT SUBJECTS

IN

MECHANICS,
HYDROSTATICS,
HYDRAULICS,
PNEUMATICS,
OPTICS,
GEOGRAPHY,
ASTRONOMY, AND
DIALLING.

By JAMES FERGUSON, F. R. S.

WITH

NOTES, AND AN ADDITIONAL VOLUME,

CONTAINING

THE MOST RECENT DISCOVERIES IN THE ARTS AND SCIENCES.

By DAVID BREWSTER, LL. D.

F. R. S. LOND. & SEC. R. S. ED. &c. &c.

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IN TWO VOLUMES.

WITH TWENTY-SEVEN PLATES.

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THE PREFACE

OF THE EDITOR.

THE writings of Mr. Ferguson have been long and justly distinguished for their perspicuity and plainness. It seems to have been the chief object of his labours to give a familiar view of the various branches of physical science, and to render them accessible to those who are not accustomed to mathematical investigation; and the favourable reception which his works have everywhere experienced is a satisfactory proof that he did not labour in vain.

The treasures of science had been long concealed in the recesses of algebraical formulæ and geometrical discussion, and men of ordinary capacity were deterred from exploring them by the repulsive form in which they were exhibited. There were some works, indeed, which, from the absence of mathematical reasoning, may be regarded as exceptions to this general observation; but most of them wanted that perspicuity of style, that method of viewing a difficult subject in different aspects, and that happy manner of illustrating the abstrusest facts in mechanical philosophy by new and ingenious experiments, which the author of these lectures so eminently possessed. Mr. Ferguson, therefore, may in some degree be regarded as the first elementary writer on natural philosophy, and to his labours we must attribute that general diffusion of scientific knowledge among the practical mechanics of this country, which has in a great measure banished those antiquated prejudices, and erroneous maxims of construction, that perpetually mislead the unlettered artist.

But it is not merely to the praise of a popular writer that Mr. Ferguson is entitled. While he is illustrating the discoveries of others, and accommodating them to the capacities of his readers, we are frequently introduced to inventions and improvements of his own. Many of these are well known to the public; and while some of them have been of great service to experimental philosophy, they all evince a considerable share of mechanical genius. To a still higher commendation, however, our author may justly lay claim. It has long been fashionable with a certain class of philosophers to keep the Creator totally out of view, when describing the noblest of his works. But Mr. Ferguson had not imbibed those gloomy principles which steel the heart against its earliest and strongest impressions, and prompt us to suppress those feelings of devotion and gratitude which the structure and harmony of the universe are so fitted to inspire. When benevolence and design are particularly exhibited in the works or in the phenomena of nature, he dwells with delight upon the goodness and wisdom of their Author; and never fails to impress upon the reader what is too apt to escape his notice, that the wonders of creation, and the various changes which the material world displays, are the result of that unerring wisdom and boundless goodness which are unceasingly exerted for the comfort and happiness of man.

For these reasons the present work has gone through a great number of editions; and no book whatever upon the same subject has been so generally read, and so widely circulated, among all ranks of the community. We meet with it in the workshop of every mechanic. We find it transfused into many of the Encyclopædias which this country has produced; and we may easily trace it in those popular systems of philosophy which have lately appeared.

Since these Lectures were first published, the boundaries of the arts and sciences have been widely enlarged by many important discoveries and improvements. Some of these discoveries the Editor endeavoured, in the two first editions of the enlarged work to communicate in the notes, and in a very ample Appendix. In this third edition, however, the Appendix has been almost wholly recomposed, and made to embrace a much wider range of practical information It occupies the whole of the second volume, and may be read as a separate work on the practical branches of natural philosophy, without any reference to the lectures in the first volume.

The greatest pains have been taken by the Editor to render this additional volume popular and useful. He has introduced descriptions of most of those valuable machines and instruments which have been recently invented, and he has added two chapters on the double refraction and the polarisation of light, in the expectation that these new and interesting subjects may be rendered intelligible to readers who have little or no knowledge either of algebra or geometry.

EDINBURGH, *Nov.* 25, 1822.

THE PREFACE

OF THE AUTHOR.

Ever since the days of the Lord-chancellor Bacon, natural philosophy hath been more and more cultivated in England. That great genius first set out with taking a general survey of all the natural sciences, dividing them into distinct branches, which he enumerated with great exactness. He inquired scrupulously into the degree of knowledge already attained to in each, and drew up a list of what still remained to be discovered: this was the scope of his first undertaking. Afterwards he carried his views much farther, and shewed the necessity of an experimental philosophy, a thing never before thought of. As he was a professed enemy to systems, he considered philosophy no otherwise than as that part of knowledge which contributes to make men better and happier: he seems to limit it to the knowledge of things useful, recommending above all the study of nature, and shewing that no progress can be made therein, but by collecting facts, and comparing experiments, of which he points out a great number proper to be made.

But notwithstanding the true path to science was thus exactly marked out, the old notions of the schools so strongly possessed people's minds at that time, as not to be eradicated by any opinions, how rationally soever advanced, until the illustrious Mr. Boyle, the first who pursued Lord Bacon's plan, began to put experiments in practice, with an assiduity equal to his great talents. Next, the Royal Society being established, the true philosophy began to be the reigning taste of the age, and continues so to this day.

The immortal Sir Isaac Newton insisted, even in his early years, that it was high time to banish vague conjectures and hypotheses from natural philosophy, and to bring that science under an entire subjection to experiments and geometry. He frequently called it *the experimental philosophy*, so as to express significantly the difference between it and the numberless systems which had arisen merely out of the conceits of inventive brains : the one subsisting no longer than the spirit of novelty lasts ; the other never failing while the nature of things remains unchanged.

The method of teaching and laying the foundation of physics, by public courses of experiments, was first undertaken in this kingdom, I believe, by Dr. John Keill, and since improved and enlarged by Mr. Hauksbee, Dr. Desaguliers, Mr. Whiston, Mr. Cotes, Mr. Whiteside, Dr. Bradley, our late Regius and Savilian Professor of Astronomy, and Dr. Bliss his successor. Nor has the same been neglected by Dr. James, and Dr. David Gregory, Sir Robert Stewart, and after him Mr. Maclaurin.—Dr. Helsham in Ireland, Messieurs Gravesande and Muschenbroek, and the Abbè Nollet in France, have also acquired just applause thereby.

The substance of my own attempts in this way of instrumental instruction, the following sheets (exclusive of the astronomical part) will shew: the satisfaction they have generally given, read as lectures to different audiences, affords me some hope that they may be favourably received in the same form by the public.

I ought to observe, that though the last five lectures cannot properly be said to concern experimental philosophy, I considered, however, that they were not of so different a class, but that they might, without much impropriety, be subjoined to the preceding ones.

My apparatus (part of which is described here, and the rest in a former work*) is rather simple than magnificent, which is owing to a particular point I had in view at first setting out, namely, to avoid all superfluity, and to render every thing as plain and intelligible as I thought the subject would admit of.

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* *Astronomy explained upon Sir Isaac Newton's Principles, and made easy to those who have not studied Mathematics.*

A SHORT ACCOUNT OF THE LIFE OF THE AUTHOR,

WRITTEN BY HIMSELF.

As this is probably the last book I shall ever publish,* I beg leave to prefix to it a short account of myself, and of the manner I first began, and have since prosecuted, my studies. For, as my setting out in life from a very low station, and in a remote part of the island, has occasioned some false, and indeed very improbable, particulars to be related of me, I therefore think it the better way, instead of contradicting them one by one, to give a faithful and circumstantial detail of my whole proceedings, from my first obscure beginning to the present time: wherein, if I should insert some particulars of little moment, I hope the good-natured reader will kindly excuse me.

I was born in the year 1710, a few miles from Keith, a little village in Banffshire, in the north of Scotland; and can with pleasure say, that my parents, though poor, were religious and honest; lived in good repute with all who knew them, and died with good characters.

As my father had nothing to support a large family but his daily labour, and the profits arising from a few acres of land which he rented, it was not to be expected that he could bestow much on the education of his children: yet they were not neglected; for, at his leisure hours, he taught them to read and write. And it was while he was teaching my elder brother to read the Scotch Catechism that I acquired my reading. Ashamed to ask my father to instruct me, I used, when he and my brother were abroad, to take the catechism and study the lesson which he had been teaching my brother: and when any difficulty occurred, I went to a neighbouring old woman, who gave me such help as enabled me to read tolerably well before my father had thought of teaching me.

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* This account of Mr. Ferguson's life was originally prefixed to his Select Mechanical Exercises. He lived to publish another work, entitled, "The Art of Drawing in Perspective made easy to those who have no knowledge of the Mathematics."—*Ed.*

Some time after, he was agreeably surprised to find me reading by myself: he thereupon gave me further instruction, and also taught me to write; which, with about three months I afterwards had at the grammar-school at Keith, was all the education I ever received.

My taste for mechanics arose from an odd accident.—When about seven or eight years of age, a part of the roof of the house being decayed, my father, desirous of mending it, applied a prop and lever to an upright spar to raise it to its former situation; and, to my great astonishment, I saw him, without considering the reason, lift up the ponderous roof, as if it had been a small weight. I attributed this at first to a degree of strength that excited my terror as well as wonder: but thinking farther of the matter, I recollected that he had applied his strength to that end of the lever which was farthest from the prop; and finding, on inquiry, that this was the means whereby the seeming wonder was effected, I began making levers (which I then called bars); and by applying weights to them different ways, I found the power gained by my bar was just in proportion to the lengths of the different parts of the bar on either side of the prop. I then thought it was a great pity that, by means of this bar, a weight could be raised but a very little way. On this, I soon imagined, that, by pulling round a wheel, the weight might be raised to any height, by tying a rope to the weight, and winding the rope round the axle of the wheel; and that the power gained must be just as great as the wheel was broader than the axle was thick; and found it to be exactly so, by hanging one weight to a rope put round the wheel, and another to the rope that coiled round the axle. So that, in these two machines, it appeared very plain, that their advantage was as great as the space gone through by the working power exceeded the space gone through by the weight: and this property I also thought must take place in a wedge for cleaving wood; but then I happened not to think of the screw.—By means of a turning lathe which my father had, and sometimes used, and a little knife, I was enabled to make wheels and other things necessary for my purpose.

I then wrote a short account of these machines, and sketched out figures of them with a pen, imagining it to be the first treatise of the kind that ever was written: but found my mistake when I afterwards shewed it to a gentleman, who told me that these things were known long before, and shewed me a printed book in which they were treated of: and I was much pleased when I found, that my account (so far as I had carried it) agreed with the principles of mechanics in the book he shewed me. And from that time my mind preserved a constant tendency to improve in that science.

But, as my father could not afford to maintain me while I was in pursuit only of these matters, and I was rather too young and weak for hard labour, he put me out to a neighbour to keep sheep, which I continued to do for some years;* and in that time I began to study the stars in the night. In the day-time I amused myself by making models of mills, spinning-wheels, and such other things as I happened to see.

I then went to serve a considerable farmer in the neighbourhood, whose name was James Glashan. I found him very kind and indulgent; but he soon observed that, when my work was over, I went into a field with a blanket about me: lay down on my back, and stretched a thread with small beads upon it, at arms' length, between my eye and the stars; sliding the beads upon it till they hid such and such stars from my eye, in order to take their apparent distances from one another; and then laying the thread down on a paper, I marked the stars thereon by the beads, according to their respective positions, having a candle by me. My master at first laughed at me; but, when I explained my meaning to him, he encouraged me to go on: and that I might make fair copies in the day-time of what I had done in the night, he often worked for me himself. I shall always have a respect for the memory of that man.

One day he happened to send me with a message to the Reverend Mr. John Gilchrist, minister at Keith, to whom I had been known from my childhood. I carried my star-papers to shew them to him, and found him looking over a large parcel of maps, which I surveyed with great pleasure, as they were the first I had ever seen. He then told me that the earth is round like a ball, and explained the map of it to me. I requested him to lend me *that* map, to take a copy of it in the evenings. He cheerfully consented to this, giving me at the same time a pair of compasses, a ruler, pens, ink, and paper; and dismissed me with an injunction not to neglect my master's business by copying the map, which I might keep as long as I pleased.

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* From a misconception of this part of Mr. Ferguson's history, the celebrated La Lande, in his account of famous astronomers, very ludicrously observes, "Ferguson qui nous avons citè, etoit *Berger au Roi d'Angleterre en Ecosse.*" "Ferguson whom we have quoted, was *Shepherd to the King of England for Scotland.*" *Astronomie de La Lande*, tom. i, p. 163.—*Ed.*

For this pleasant employment, my master gave me more time than I could reasonably expect; and often took the threshing-flail out of my hands, and worked himself, while I sat by him in the barn, busy with my compasses, ruler, and pen.

When I had finished the copy, I asked leave to carry home the map: he told me I was at liberty to do so, and might stay two hours to converse with the minister. In my way thither, I happened to pass by the school at which I had been before, and saw a genteel-looking man (whose name I afterwards learnt was Cantley) painting a sun-dial on the wall. I stopt a while to observe him, and the schoolmaster came out, and asked me what parcel it was that I had under my arm. I shewed him the map, and the copy I had made of it, wherewith he appeared to be very well pleased, and asked me whether I should not like to learn of Mr. Cantley to make sun-dials. Mr. Cantley looked at the copy of the map, and commended it much; telling the schoolmaster (Mr. John Skinner) that it was a pity I did not meet with notice and encouragement. I had a good deal of conversation with him, and found him to be quite affable and communicative, which made me think I should be extremely happy if I could be further acquainted with him.

I then proceeded with the map to the minister, and shewed him the copy of it. While we were conversing together, a neighbouring gentleman, Thomas Grant, Esq. of Achoynaney, happened to come in; and the minister immediately introduced me to him, shewing him what I had done. He expressed great satisfaction, asked me some questions about the construction of maps, and told me, that if I would go and live at his house, he would order his butler, Alexander Cantley, to give me a great deal of instruction. Finding that this Cantley was the man whom I had seen painting the sun-dial, and of whom I had already conceived a very high opinion, I told 'Squire Grant that I should rejoice to be at his house as soon as the time was expired for which I was engaged with my present master. He very politely offered to put one in my place, but this I declined.

When the term of my servitude was out, I left my good master, and went to the gentleman's house (1730), where I quickly found myself with a most humane good family. Mr. Cantley the butler soon became my friend, and continued so till his death. He was the most extraordinary man that I ever was acquainted with, or perhaps ever shall see; for he was a complete master of arithmetic, a good mathematician, a master of music on every known instrument except the harp, understood Latin, French, and Greek, let blood extremely well, and could even prescribe as a physician upon any urgent occasion. He was what is generally called *self-taught;* but, I think, he might, with much greater propriety, have been termed GOD ALMIGHTY's scholar.

He immediately began to teach me decimal arithmetic and algebra; for I had already learnt vulgar arithmetic, at my leisure hours, from books. He then proceeded to teach me the elements of geometry; but, to my inexpressible grief, just as I was beginning that branch of science, he left Mr. Grant, and went to the late Earl of Fife's, at several miles' distance. The good family I was then with could not prevail with me to stay after he was gone: so I left them, and went to my father's.

He had made me a present of Gordon's Geographical Grammar, which, at that time, was to me a great treasure. There is no figure of a globe in it, although it contains a tolerable description of the globes, and their use. From this description I made a globe in three weeks at my father's, having turned the ball thereof out of a piece of wood; which ball I covered with paper, and delineated a map of the world upon it; made the meridian-ring and horizon of wood, covered them with paper, and graduated them; and was happy to find, that by my globe (which was the first I ever saw) I could solve the problems.

But this was not likely to afford me bread, and I could not think of staying with my father, who I knew full well could not maintain me in that way, as it would be of no service to him; and he had, without my assistance, hands sufficient for all his work.

I then went to a miller, thinking it would be a very easy business to attend the mill, and that I should have a great deal of leisure time to study decimal arithmetic and geometry. But my master, being too fond of tippling at an alehouse, left the whole care of the mill to me, and almost starved me for want of victuals; so that I was glad when I could have a little oatmeal mixed with cold water to eat. I was engaged for a year in this man's service, at the end of which I left him, and returned in a very weak state to my father's.

Soon after I had recovered my former strength, a neighbouring farmer, who practised as a physician in that part of the country, came to my father's, wanting to have me as a labouring servant. My father advised me to go to Doctor Young, telling me that the Doctor would instruct me in that part of his business. This he promised to do, which was a temptation to me. But instead of performing his promise, he kept me constantly to very hard labour, and never once shewed me one of his books. All his servants complained that he was the hardest master they had ever lived with; and it was my misfortune to be engaged with him for half a year. But, at the end of three months, I was so much overwrought, that I was almost disabled, which obliged me to leave him: and he was so unjust as to give me nothing at all for the time I had been with him, because I did not complete my half-year's service; though he knew that I was not able, and had seen me working for the last fortnight, as much as possible, with one hand and arm, when I could not lift the other from my side. And what I thought was particularly hard, he never once tried to give me the least relief, further than once bleeding me, which rather did me hurt than good, as I was very weak, and much emaciated. I then went to my father's, where I was confined for two months on account of my hurt, and despaired of ever recovering the use of my left arm. And during all that time, the doctor never once came to see me, although the distance was not quite two miles. But my friend Mr. Cantley, hearing of my misfortune, at twelve miles distance, sent me proper medicines and applications, by means of which I recovered the use of my arm; but found myself too weak to think of going into service again, and had entirely lost my appetite, so that I could take nothing but a draught of milk once a-day for many weeks.

In order to amuse myself in this low state, I made a wooden clock, the frame of which was also of wood; and it kept time pretty well. The bell, on which the hammer struck the hours, was the neck of a broken bottle.

Having then no idea how any time-keeper could go but by a weight and a line, I wondered how a watch could go in all positions; and was sorry that I had never thought of asking Mr. Cantley, who could very easily have informed me. But happening one day to see a gentleman ride by my father's house (which was close by a public road), I asked him what o'clock it then was: he looked at his watch, and told me. As he did that with so much good-nature, I begged of him to shew me the inside of his watch: and though he was an entire stranger, he immediately opened the watch, and put into my hands. I saw the spring-box with part of the chain round it, and asked him what it was that made the box turn round: he told me that it was turned round by a steel spring within it. Having then never seen any other spring than that of my father's gun lock, I asked how a spring within a box could turn the box so often round as to wind all the chain upon it. He answered, that the spring was long and thin; that one end of it was fastened to the axis of the box, and the other end to the inside of the box; that the axis was fixed, and the box was loose upon it. I told him I did not yet thoroughly understand the matter: well, my lad, says he, take a long thin piece of whalebone, hold one end of it fast between your finger and thumb, and wind it round your finger: it will then endeavour to unwind itself; and if you fix the other end of it to the inside of a small hoop, and leave it to itself, it will turn the hoop round and round, and wind up a thread tied to the outside of the hoop. I thanked the gentleman, and told him that I understood the thing very well. I then tried to make a watch with wooden wheels, and made the spring of whalebone; but found that I could not make the watch go when the balance was put on, because the teeth of the wheels were rather too weak to bear the force of a spring sufficient to move the balance; although the wheels would run fast enough when the balance was taken off. I inclosed the whole in a wooden case, very little bigger than a breakfast tea-cup: but a clumsy neighbour one day looking at my watch, happened to let it fall; and turning hastily about to pick it up, set his foot upon it, and crushed it all to pieces; which so provoked my father, that he was almost ready to beat the man; and discouraged me so much, that I never attempted to make such another machine again, especially as I was thoroughly convinced I could never make one that would be of any real use.

As soon as I was able to go abroad, I carried my globe, clock, and copies of some other maps besides that of the world, to the late Sir James Dunbar of Durn (about seven miles from where my father lived), as I had heard that Sir James was a very good-natured, friendly, inquisitive gentleman. He received me in a very kind manner, was pleased with what I shewed him, and desired that I would clean his clocks. This, for the first time, I attempted; and then begun to pick up some money in that way about the country, making Sir James's house my home, at his desire.

Two large globular stones stood on the top of his gate: on one of them I painted (with oil colours) a map of the terrestrial globe, and on the other a map of the celestial, from a planisphere of the stars which I copied on paper from a celestial globe belonging to a neighbouring gentleman. The poles of the painted globes stood toward the poles of the heavens; on each, the twenty-four hours were placed around the equinoctial, so as to shew the time of the day when the sun shone out, by the boundary where the half of the globe at any time enlightened by the sun was parted from the other half in the shade; the enlightened parts of the terrestrial globe answering to the like enlightened parts of the earth at all times. So that, whenever the sun shone on the globe, one might see to what places the sun was then rising, to what places it was setting, and all the places where it was then day or night, throughout the earth.

During the time I was at Sir James's hospitable house, his sister, the honourable the Lady Dipple, came there on a visit, and Sir James introduced me to her. She asked me whether I could draw patterns for needle work on aprons and gowns. On shewing me some, I undertook the work, and drew several for her; some of which were copied from her patterns, and the rest I did according to my own fancy. On this, I was sent for by other ladies in the country, and began to think myself growing very rich by the money I got for such drawings; out of which I had the pleasure of occasionally supplying the wants of my poor father.

Yet all this while I could not leave off star-gazing in the nights, and taking the places of the planets among the stars by my above mentioned thread. By this I could observe how the planets changed their places among the stars, and delineated their paths on the celestial map, which I had copied from the above mentioned celestial globe.

By observing what constellations the ecliptic passed through in that map, and comparing these with the starry heaven, I was so impressed as sometimes to imagine that I saw the ecliptic in the heaven, among the stars, like a broad circular road for the sun's apparent course; and fancied the paths of the planets to resemble the narrow ruts made by cart wheels, sometimes on one side of a plain road and sometimes on the other, crossing the road at small angles, but never going far from either side of it.

Sir James's house was full of pictures and prints, several of which I copied with pen and ink: this made him think I might become a painter.

Lady Dipple had been but a few weeks there, when William Baird, Esq. of Auchmeddan, came on a visit: he was the husband of one of that lady's daughters, and I found him to be very ingenious and communicative: he invited me to go to his house, and stay some time with him, telling me that I should have free access to his library, which was a very large one; and that he would furnish me with all sorts of implements for drawing. I went thither, and staid about eight months; but was much disappointed in finding no books of astronomy in his library, except what was in the two volumes of Harris's Lexicon Technicum, although there were many books on geography and other sciences: several of these indeed were in Latin, and more in French; which being languages that I did not understand, I had recourse to him for what I wanted to know of these subjects, which he cheerfully read to me; and it was as easy for him, at sight, to read English from a Greek, Latin, or French book, as from an English one. He furnished me with pencils and Indian ink, shewing me how to draw with them: and although he had but an indifferent hand at that work, yet he was a very acute judge; and consequently a very fit person for shewing me how to correct my own work. He was the first who ever sat to me for a picture; and I found it was much easier to draw from the life than from any picture whatever, as nature was more striking than any imitation of it.

Lady Dipple came to his house in about half a year after I went thither. And as they thought I had a genius for painting, they consulted together about what might be the best way to put me forward. Mr. Baird thought it would be no difficult matter to make a collection for me among the neighbouring gentlemen, to put me to a painter at Edinburgh; but he found, upon trial, that nothing worth the while could be done among them. And as to himself, he could not do much that way, because he had but a small estate, and a very numerous family.

Lady Dipple then told me that she was to go to Edinburgh next spring, and that, if I would go thither she would give me a year's bed and board at her house *gratis*, and make all the interest she could for me among her acquaintance there. I thankfully accepted of her kind offer; and, instead of giving me one year, she gave me two. I carried with me a letter of recommendation from the Lord Pitsligo (a near neighbour of 'Squire Baird's) to Mr. John Alexander, a painter in Edinburgh; who allowed me to pass an hour every day at his house, for a month, to copy from his drawings; and said he would teach me to paint in oil colours, if I would serve him seven years, and my friends would maintain me all that time: but this was too much for me to desire them to do; nor did I choose to serve so long. I was then recommended to other painters, but they would do nothing without money. So I was quite at a loss what to do.

In a few days after this, I received a letter of recommendation from my good friend 'Squire Baird to the Reverend Dr. Robert Keith at Edinburgh, to whom I gave an account of my bad success among the painters there. He told me, that if I would copy from nature, I might do without their assistance, as all the rules for drawing signified but very little when one came to draw from the life; and, by what he had seen of my drawings brought from the north, he judged I might succeed very well in drawing pictures from the life, in Indian ink, on vellum. He then sat to me for his own picture, and sent me with it and a letter of recommendation to the Right Honourable the Lady Jane Douglas, who lived with her mother, the Marchioness of Douglas, at Merchiston-house, near Edinburgh. Both the Marchioness and Lady Jane behaved to me in the most friendly manner, on Dr. Keith's account, and sat for their pictures; telling me, at the same time, that I was in the very room in which Lord Napier invented and computed the logarithms; and that, if I thought it would inspire me, I should always have the same room whenever I came to Merchiston. I staid there several days, and drew several pictures of Lady Jane; of whom it was hard to say, whether the greatness of her beauty, or the goodness of her temper and dispositions, was the most predominant. She sent these pictures to ladies of her acquaintance, in order to recommend me to them; by which means I soon had as much business as I could possibly manage, so as not only to put a good deal of money in my own pocket, but also to spare what was sufficient to help to supply my father and mother in their old age. Thus a business was providentially put into my hands, which I followed for six-and-twenty years.

Lady Dipple, being a woman of the strictest piety, kept a watchful eye over me at first, and made me give her an exact account at night what families I had been in through the day, and of the money I had received. She took the money each night, desiring I would keep an account of what I had put into her hands; telling me that I should duly have, out of it, what I wanted for clothes, and to send to my father. But, in less than half a year, she told me that she would trust me with being my own banker; for she had made a good deal of private inquiry how I had behaved when I was out of her sight through the day; and was satisfied with my conduct.

During my two years' stay at Edinburgh, I somehow took a violent inclination to study anatomy, surgery, and physic, all from reading of books, and conversing with gentlemen on these subjects; which, for that time, put all thoughts of astronomy out of my mind, and I had no inclination to become acquainted with any one there who taught either mathematics or astronomy: for nothing would serve me but to be a doctor.

At the end of the second year I left Edinburgh, and went to see my father, thinking myself tolerably well qualified to be a physician in that part of the country; and I carried a good deal of medicines, plasters, &c. thither. But, to my mortification, I soon found that all my medical theories and study were of little use in practice. And then, finding that very few paid me for the medicines they had, and that I was far from being so successful as I could wish, I quite left off that business, and began to think of taking to the more sure one of drawing pictures again. For this purpose I went to Inverness, where I had eight months' business.

When I was there, I began to think of astronomy again; and was heartily sorry for having quite neglected it at Edinburgh, where I might have improved my knowledge, by conversing with those who were very able to assist me. I began to compare the ecliptic with its twelve signs (through which the sun goes in twelve months) to the circle of twelve hours on the dial-plate of the watch, the hour-hand to the sun, and the minute-hand to the moon, moving in the ecliptic; the one always overtaking the other, at a place forwarder than it did at their last conjunction before. On this, I contrived and finished a scheme on paper for shewing the motions and places of the sun and moon in the ecliptic on each day of the year, perpetually; and, consequently, the days of all the new and full moons.

To this I wanted to add a method for shewing the eclipses of the sun and moon; of which I knew the cause long before, by having observed that the moon was, for one half of her period, on the north side of the ecliptic, and for the other half on the south. But having not observed her course long enough among the stars by my above-mentioned thread, so as to delineate her path upon my celestial map, in order to find the two opposite points of the ecliptic in which her orbit crosses it, I was altogether at a loss how and where in the ecliptic (in my scheme) to place these intersecting points: this was in the year 1739.

At last, I recollected, that when I was with 'Squire Grant of Achoynaney in the year 1730, I had read, that on the 1st of January 1690, the moon's ascending node was in the 10th minute of the first degree of Aries; and that her nodes moved backward through the whole ecliptic in 18 years and 224 days, which was at the rate of 3 min. 11 sec. every 24 hours. But, as I scarce knew in the year 1730 what the moon's nodes meant, I took no farther notice of it at that time.

However, in the year 1739, I set to work at Inverness; and, after a tedious calculation of the slow motion of the nodes, from January 1690 to January 1740, it appeared to me, that (if I was sure I had remembered right) the moon's ascending node must be in 23 deg. 25 min. of Cancer at the beginning of the year 1740. And so I added the eclipse part to my scheme, and called it *The Astronomical Rotula.*

When I had finished it, I shewed it to the Reverend Mr. Alexander Macbean, one of the ministers at Inverness, who told me he had a set of almanacks by him for several years past, and would examine it by the eclipses mentioned in them. We examined it together, and found that it agreed throughout with the days of all the new and full moons and eclipses mentioned in these almanacks; which made me think I had constructed it upon true astronomical principles. On this, Mr. Macbean desired me to write to Mr. Maclaurin, professor of the mathematics at Edinburgh, and give him an account of the methods by which I had formed my plan, requesting him to correct it where it was wrong. He returned me a polite and friendly answer (although I had never seen him during my stay at Edinburgh), and informed me that I had only mistaken the mean place of the ascending node by a quarter of a degree; and that, if I would send the drawing of my rotula to him, he would examine it, and endeavour to procure me a subscription to defray the charges of engraving it on copperplates, if I chose to publish it. I then made a new and correct drawing of it, and sent it to him, who soon got me a very handsome subscription, by setting the example himself, and sending subscription-papers to others.

I then returned to Edinburgh, and had the rotula plates engraved there by Mr. Cooper.* It has gone through several impressions, and always sold very well till the year 1752, when the stile was changed, which rendered it quite useless. Mr. Maclaurin received me with the greatest civility when I first went to see him at Edinburgh. He then became an exceeding good friend to me, and continued so till his death.

One day I requested him to shew me his orrery, which he immediately did. I was greatly delighted with the motions of the earth and moon in it, and would gladly have seen the wheel-work, which was concealed in a brass box, and the box and planets above it were surrounded by an armillary sphere. But he told me, that he never had opened it; and I could easily perceive that it could not be opened but by the hand of some ingenious clockmaker, and not without a great deal of time and trouble.

After a great deal of thinking and calculation, I found that I could contrive the wheel-work for turning the planets in such a machine, and giving them their progressive motions; but should be very well satisfied if I could make an orrery to shew the motions of the earth and moon, and of the sun round its axis. I then employed a turner to make me a sufficient number of wheels and axles, according to patterns which I gave him in drawing: and, after having cut the teeth in the wheels by a knife, and put the whole together, I found that it answered all my expectations. It shewed the sun's motion round his axis, the diurnal and annual motions of the earth on its inclined axis, which kept its parallelism in its whole course round the sun; the motions and phases of the moon, with the retrograde motions of the nodes of her orbit; and consequently, all the variety of seasons, the different lengths of days and nights, the days of the new and full moons, and eclipses.

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* Cooper was master to the justly celebrated Mr. Robert Strange, who was at that time his apprentice.

When it was all completed, except the box that covers the wheels, I shewed it to Mr. Maclaurin, who commended it in presence of a great many young gentlemen who attended his lectures. He desired me to read them a lecture on it, which I did without hesitation, seeing I had no reason to be afraid of speaking before a great and good man who was my friend. Soon after that I sent it in a present to the reverend and ingenious Mr. Alexander Irvine, one of the ministers at Elgin in Scotland.

I then made a smaller and neater orrery, of which all the wheels were of ivory, and I cut the teeth in them with a file. This was done in the beginning of the year 1743; and, in May that year, I brought it with me to London, where it was soon after bought by Sir Dudley Rider. I have made six orreries since that time, and there are not any two of them in which the wheel-work is alike: for I could never bear to copy one thing of that kind from another, because I still saw there was great room for improvements.

I had a letter of recommendation from Mr. Baron Edlin, at Edinburgh, to the Right Honourable Stephen Poyntz, Esq. at St. James's, who had been preceptor to his Royal Highness the late Duke of Cumberland, and was well known to be possessed of all the good qualities that can adorn a human mind. To me his goodness was really beyond my power of expression; and I had not been a month in London till he informed me that he had wrote to an eminent professor of mathematics to take me into his house, and give me board and lodging, with all proper instructions to qualify me for teaching a mathematical school he (Mr. Poyntz) had in view for me, and would get me settled in it. This I should have liked very well, especially as I began to be tired of drawing pictures, in which, I confess, I never strove to excel, because my mind was still pursuing things more agreeable. He soon after told me he had just received an answer from the mathematical master, desiring I might be sent immediately to him. On hearing this, I told Mr. Poyntz, that I did not know how to maintain my wife during the time I must be under the master's tuition. What, says he, are you a married man? I told him I had been so, ever since May in the year 1739. He said he was sorry for it, because it quite defeated his scheme; as the master of the school he had in view for me must be a batchelor.

He then asked me, what business I intended to follow? I answered, that I knew of none besides that of drawing pictures. On this he desired me to draw the pictures of his lady and children, that he might shew them in order to recommend me to others; and told me, that, when I was out of business I should come to him, and he would find me as much as he could: and I soon found as much as I could execute: but he died in a few years after, to my inexpressible grief.

Soon afterwards, it appeared to me, that although the moon goes round the earth, and that the sun is far on the outside of the moon's orbit, yet the moon's motion must be in a line that is always concave toward the sun: and upon making a delineation, representing her absolute path in the heavens, I found it to be really so. I then made a simple machine for delineating both her path and the earth's, on a long paper laid on the floor I carried the machine and delineation to the late Martin Folkes, Esquire, President of the Royal Society, on a Thursday afternoon. He expressed great satisfaction at seeing it, as it was a new discovery; and took me that evening with him to the Royal Society, where I shewed the delineation, and the method of doing it.

When the business of the society was over, one of the members desired me to dine with him next Saturday at Hackney; telling me that his name was Ellicott, and that he was a watchmaker.

I accordingly went to Hackney, and was kindly received by Mr. John Ellicott, who then shewed me the very same kind of delineation, and part of the machine by which he had done it; telling me that he had thought of it twenty years before. I could easily see, by the colour of the paper, and of the ink lines upon it, that it must have been done many years before I saw it. He then told me what was very certain, that he had neither stolen the thought from me, nor had I from him. And from that time till his death, Mr. Ellicott was one of my best friends. The figure of this machine and delineation is in the seventh plate of my book of *Astronomy.*

Soon after the stile was changed, I had my rotula new engraved; but have neglected it too much by not fitting it up and advertising it. After this, I drew out a scheme, and had it engraved, for shewing all the problems of the rotula except the eclipses: and, in place of that, it shews the times of rising and setting of the sun, moon, and stars; and the positions of the stars for any time of the night.

In the year 1747, I published a Dissertation on the phenomena of the harvest moon, with the description of a new orrery, in which there are only four wheels. But having never had a grammatical education, nor time to study the rules of just composition, I acknowledge that I was afraid to put it to the press; and for the same cause, I ought to have the same fears still. But having the pleasure to find that this my first work was not ill received, I was emboldened to go on, in publishing my *Astronomy*, *Mechanical Lectures*, *Tables and Tracts relative to several Arts and Sciences*, *The Young Gentleman and Lady's Astronomy*, a small *Treatise on Electricity*, and my *Select Mechanical Exercises.*

In the year 1748, I ventured to read lectures on the eclipse of the sun that fell on the 14th of July in that year. Afterwards I began to read astronomical lectures on an orrery which I made, and of which the figures of all the wheel work are contained in the sixth and seventh plates of my *Mechanical Exercises.* I next began to make an apparatus for lectures on mechanics, and gradually increased the apparatus for other parts of experimental philosophy, buying from others what I could not make for myself, till I brought it to its present state. I then entirely left off drawing pictures, and employed myself in the much pleasanter business of reading lectures on mechanics, hydrostatics, hydraulics, pneumatics, electricity, and astronomy: in all which, my encouragement has been greater than I could have expected.

The best machine I ever contrived is the *Eclipsarcon*, of which there is a figure in the thirteenth plate of my *Astronomy.*

It shews the time, quantity, duration, and progress, of solar eclipses at all parts of the earth. My next best contrivance is the Universal dialling cylinder, of which there is a figure in the eighth plate of the *Supplement* to my *Mechanical Lectures.*

It is now thirty years since I came to London; and during all that time I have met with the highest instances of friendship from all ranks of people both in town and country, which I do here acknowledge with the utmost respect and gratitude; and particularly the goodness of our present gracious sovereign, who, out of his privy purse, allows me fifty pounds a-year, which is regularly paid, without any deduction.

To the preceding account of Mr. Ferguson's life, it may be proper to add that he was elected a member of the Royal Society of London, on the 24th November 1763, without paying the usual fees of admission. This honour, which had been conferred on the illustrious Newton, and on that ingenious and self-taught mathematician Mr. Thomas Simson of Woolwich, was generally reserved for such foreigners as were distinguished by their philosophical and literary attainments; and strongly marks the estimation in which Mr. Ferguson was held by that learned body. In 1770, he was chosen a member of the American Philosophical Society; and by the translation of his *Astronomy* into the German and Swedish languages, of his *Perspective* into French, and of his *Ladies' and Gentlemen's Astronomy* into German, his fame as a popular writer was extended beyond his native country. His lectures on experimental philosophy were frequently honoured with the presence of George III, who took great pleasure in conversing with Mr. Ferguson upon scientific subjects, and distinguished him by numerous marks of his royal favour. While he was thus enjoying, in the evening of his days, the respect of philosophers, and the general esteem of all who knew him, he was afflicted with a lingering illness, which closed his useful life on the 16th of November 1776, in the 66th year of his age.

Mr. Ferguson possessed a clear judgment, and was capable of thinking and writing on philosophical subjects with great accuracy and precision. He had a peculiar talent for simplifying what was complex, for rendering intelligible what was abstracted, and for bringing down to the lowest capacities what was naturally above them. His unwearied assiduity in the acquisition of knowledge, may be inferred from the great variety of his publications; and when we reflect upon the very unfavourable circumstances in which he was educated, and the little assistance which he received from others, we cannot fail to wonder at the style in which all his works are composed. On some occasions it is uncommonly correct and animated. When admiring the displays of wisdom and beneficence in the economy of nature, it often rises into a species of eloquence, characterised by the most artless simplicity, and infinitely more affecting than the laboured and polished periods of the professed orator. In his manners he was affable and mild; in his dispositions communicative and benevolent. He was distinguished by none of those peculiarities of temper, and eccentricities of conduct, which we generally observe in literary men. If Mr. Ferguson had any foibles, they 'leaned to virtue's side;' and even his wonderful simplicity of character, which, in a state of artificial manners, is too apt to be regarded as a failing, and exposed to ridicule and scorn, tended only to heighten the respect in which he was held. The religious character of our author corresponded with his general conduct. The anxieties and changes of his eventful life never effaced the religious impressions of his youth, but rather strengthened those principles of duty which the piety of his parents had early implanted; and confirmed him in the belief of those peculiar doctrines of our faith, which are the surest foundation of moral practice, and best fitted to inspire us with confidence, when the concerns of the present life must cease to interest us.—*Ed.*

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LECTURES
ON
SELECT SUBJECTS.

LECTURE I.

OF MATTER AND ITS PROPERTIES

As the design of the first part of this course is to explain and demonstrate those laws by which the material universe is governed, regulated, and continued; and by which the various appearances in nature are accounted for, it is requisite to begin with explaining the properties of matter.

Matter, what.

By the word *matter* is here meant every thing that has length, breadth, and thickness, and resists the touch.

Its properties.

The inherent properties of matter are solidity, inactivity, mobility, and divisibility.[1]

Solidity.

The *solidity* of matter arises from its having length, breadth, and thickness; and hence it is that all bodies are comprehended under some shape or other, and that each particular body hinders all others from occupying the same part of space which it possesses. Thus, if a piece of wood or metal be squeezed ever so hard between two plates, they cannot be brought into contact. And even water or air has this property; for if a small quantity of either be fixed between any other bodies, they cannot be brought to touch one another.

Inactivity.

A second property of matter is *inertia*, *inactivity*, or *passiveness;* by which it always endeavours to continue in the state that it is in, whether of rest or motion. And therefore, if one body contains twice or thrice as much matter as another body does, it will have twice or thrice as much inactivity; that is, it will require twice or thrice as much force to give it an equal degree of motion, or to stop it after it hath been put into such a motion.

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[1] Some late writers have increased the general properties of matter to twelve, namely, *extension*, *divisibility*, *figurability*, *impenetrability*, *porosity*, *rarefactibility*, *condensibility*, *compressibility*, *elasticity*, *dilatibility*, *mobility*, *inactivity*. It was formerly thought that fluids were incompressible, and that many bodies were not elastic. The compressibility of fluids, however, is now completely ascertained; and the softest bodies, even clay itself, have been found elastic.—*Ed.*

That matter can never put itself into motion is allowed by all men. For they see that a stone, lying on the plain surface of the earth, never removes itself from that place; nor does any one imagine it ever can. But most people are apt to believe that all matter has a propensity to fall from a state of motion into a state of rest; because they see that if a stone or a cannon-ball be put into ever so violent a motion, it soon stops; not considering that this stoppage is caused, 1. By the gravity or weight of the body, which sinks it to the ground in spite of the impulse; and 2. By the resistance of the air through which it moves, and by which its velocity is retarded every moment till it falls.

A bowl moves but a short way upon a bowling-green; because the roughness and unevenness of the grassy surface soon creates friction enough to stop it. But if the green were perfectly level, and covered with polished glass, and the bowl were perfectly hard, round, and smooth, it would go a great way farther, as it would have nothing but the air to resist it; if then the air were taken away, the bowl would go on without any friction, and consequently without any diminution of the velocity it had at setting out: and therefore, if the green were extended quite around the earth, the bowl would go on, round and round the earth, for ever.

If the bowl were carried several miles above the earth, and there projected in a horizontal direction, with such a velocity as would make it move more than a semidiameter of the earth in the time it would take to fall to the earth by gravity; in that case, and if there were no resisting medium in the way, the bowl would not fall to the earth at all, but would continue to circulate round it, keeping always in the same track, and returning to the same point from which it was projected, with the same velocity as at first. In this manner the moon goes round the earth, although she be as unactive and dead as any stone upon it.

Mobility. The third property of matter is *mobility*; for we find that all matter is capable of being moved, if a sufficient degree of force be applied to overcome its inactivity or resistance.

Divisibility.

The fourth property of matter is *divisibility*, of which there can be no end. For, since matter can never be annihilated by cutting or breaking, we can never imagine it to be cut into such small particles, but that if one of them be laid on a table, the uppermost side of it will be further from the table than the undermost side. Moreover, it would be absurd to say that the greatest mountain on earth has more halves, quarters, or tenth parts, than the smallest particle of matter has.

We have many surprising instances of the smallness to which matter can be divided by art, of which the two following are very remarkable.

1. If a pound of silver be melted with a single grain of gold, the gold will be equally diffused through the whole silver; so that taking one grain from any part of the mass (in which there can be no more than the 5760th part of a grain of gold), and dissolving it in *aqua fortis*, the gold will fall to the bottom.

2. The gold-beaters can extend a grain of gold into a leaf containing 50 square inches; and this leaf may be divided into 500,000 visible parts. For an inch in length can be divided into 100 parts, every one of which will be visible to the bare eye: consequently a square inch can be divided into 10,000 parts, and fifty square inches into 500,000. And if one of these parts be viewed with a microscope that magnifies the diameter of an object only ten times, it will magnify the area 100 times; and then the 100th part of a 500,000th part of a grain (that is, the fifty millionth part), will be visible. Such leaves are commonly used in gilding; and they are so very thin, that if 124,500 of them were laid upon one another, and pressed together, they would not exceed one inch in thickness.[2]

Yet all this is nothing in comparison of the lengths that nature goes in the division of matter; for Mr. Leewenhoek tells us, that there are more animals in the milt of a single cod-fish, than there are men upon the whole earth; and that, by comparing these animals in a microscope with grains of common sand, it appeared that one single grain is bigger than four millions of them.[3] Now each animal must have heart, arteries, veins, muscles, and nerves, otherwise they could neither live nor move. How inconceivably small then must the particles of their blood be, to circulate through the smallest ramifications and joinings of their arteries and veins! It has been found by calculation, that a particle of their blood must be as much smaller than a globe of the tenth part of an inch in diameter, as that globe is smaller than the whole earth; and yet, if these particles be compared with the particles of light, they will be found to exceed them as much in bulk as mountains do single grains of sand. For, the force of any body striking against an obstacle is directly in proportion to its quantity of matter multiplied into its velocity; and since the velocity of the particles of light is demonstrated to be at least a million times greater than the velocity of a cannon ball, it is plain, that if a million of these particles were as big as a single grain of sand, we durst no more open our eyes to the light, than we durst expose them to sand, shot point-blank from a cannon.

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[2] It would require 300,000 of these leaves to make an inch. The tenuity of gold leaf is so great, that it is decidedly transparent. When it is interposed between the eye and external objects, they are distinctly perceived of a greenish colour. When a small quantity of silver is combined with the gold, the colour of external objects becomes a fine blue, the deepness of the tint increasing with the quantity of silver. Dr. Wollaston has succeeded, by an ingenious process, in drawing platinum and gold wires so small as the *eighteen thousandth* part of an inch in diameter. See *Phil. Trans.* 1813, and the *Edinburgh Encyclopædia*, Art. *Micrometer*, vol. xiv, pp. 201 and 214.—*Ed.*

Plate I. Fig. 1.

The infinite divisibility of matter illustrated.

That matter is infinitely divisible in a mathematical sense, is easy to be demonstrated. For, let *A B* be the length of a particle to be divided; and let it be touched at opposite ends by the parallel lines *C D* and *E F*, which suppose to be infinitely extended beyond *D* and *F*. Set off the equal divisions *B G*, *G H*, *H I*, &c. on the line *E F*, toward the right hand from *B*; and take a point, as at *R*, anywhere toward the left hand from *A*, in the line *C D*; then, from this point, draw the right lines *R G*, *R H*, *R I*, &c. each of which will cut off a part from the particle *A B*. But after any finite number of such lines are drawn, there will still remain a part, as *A P*, at the top of the particle, which can never be cut off; because the lines *D R* and *E F* being parallel, no line can ever be drawn from the point *R* to any point of the line *E F* that will coincide with line *R D*. Therefore the particle *A B* contains more than any finite number of parts.

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[3] Captain Scoresby has shewn that a drop of green water from the Greenland

Attraction.

A fifth property of matter is *attraction*, which seems rather to be infused than inherent. Of this there are four kinds, viz. *cohesion*,[4] *gravitation*, *magnetism*, and *electricity*.

Cohesion.

The *attraction of cohesion* is that by which the small parts of matter are made to stick and cohere together. Of this we have several instances, some of which follow.

1. If a small glass tube, open at both ends, be dipt in water, the water will rise up in the tube to a considerable height above its level in the bason; which must be owing to the attraction of a ring of particles of the glass all round in the tube, immediately above those to which the water at any instant rises. And when it has risen so high, that the weight of the column balances the attraction of the tube, it rises no higher. This can be nowise owing to the pressure of the air upon the water in the basin; for, as the tube is open at top, it is full of air above the water, which will press as much upon the water in the tube as the neighbouring air does upon any column of an equal diameter in the basin.[5] Besides, if the same experiment be made in the exhausted receiver of an air-pump, there will be found no difference.

2. A piece of loaf-sugar will draw up a fluid, and a sponge will draw in water: and on the same principle sap ascends in trees.

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seas contains 26,450 animalcules; and he calculates that a gallon would contain a number of animalcules, exceeding by one-half the population of the whole globe. See *Edinburgh Philosophical Journal*, vol. iv, p. 113.—*Ed.*

[4] The attraction of cohesion is divided into two kinds, namely, the *attraction of aggregation*, and the *attraction of composition* or *affinity*. The tendency of the homogeneous parts of matter to each other, or the force by which they adhere, is called the *attraction of aggregation;* and the force which is exerted between the particles of heterogeneous bodies, and by which they unite and form a body differing more or less from its component parts, is called the *attraction of composition or affinity.*—*Ed.*

[5] An experiment more curious, and better fitted for illustrating this subject, may be made in the following manner.—Having procured two pieces of glass, about six inches square, join any two of their sides, and separate the opposite sides with a piece of wax, so that their surfaces may form an angle of about two or three degrees. Immerse this apparatus about an inch deep in a bason of water, and the water will rise between the plates of glass, and form a beautiful hyperbola, having for its asymptotes a line parallel to the surface of the water, and the common section of the two planes.—*Ed.*

3. If two drops of quicksilver be placed near each other, they will run together, and become one large drop.

4. If two pieces of lead be scraped clean, and pressed together with a twist, they will attract each other so strongly, as to require a force much greater than their own weight to separate them. And this cannot be owing to the pressure of the air, for the same thing will hold in an exhausted receiver.

5. If two polished plates of marble or brass be put together, with a little oil between them, to fill up the pores in their surfaces, and prevent the lodgment of any air, they will cohere so strongly, even if suspended in an exhausted receiver, that the weight of the lower plate will not be able to separate it from the upper one. In putting these plates together, the one should be rubbed upon the other, as a joiner does two pieces of wood when he glues them.[6]

6. If two pieces of cork, equal in weight, be put near each other in a bason of water, they will move equally fast toward each other with an accelerated motion, until they meet; and then, if either of them be moved, it will draw the other after it. If two corks of unequal weights be placed near each other, they will approach with accelerated velocities inversely proportional[7] to their weights; that is, the lighter cork will move as

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[6] The experiment thus stated does not properly exemplify the attraction of cohesion; for when oil, or any other fluid, is interposed between the plates of marble, it may reasonably be conjectured that the cohesion of the surfaces arises from the viscidity of the intermediate substance. Those who have been in the habit of constructing metals for reflecting telescopes, may probably have seen a striking exemplification of this species of attraction. We have repeatedly ground together a convex and a concave brass tool, with such accuracy, that the one would raise the other from the ground without the interposition of any liquid, though its weight was about two pounds, and the contiguous surfaces completely polished. When the tools of brass are small, the one will raise the other, even when a fibre of silk is placed between their surfaces.—*Ed.*

[7] When two variable or changeable quantities, A and B, are so related to each other, that when the one increases or diminishes, the other increases or diminishes in the same proportion, A is said to be *directly proportional to B.* Thus, if A represents the attraction of any planet, and B its quantity of matter, A is directly proportional to B; because if the quantity of matter is doubled, the force of attraction is also doubled, and *vice versa.* If when B increases, A diminishes, or when B diminishes A increases, A is then said to be *inversely proportional* to B. If A increases as the *square* of B increases, A is said to be *directly proportional to the square* of B. If A increases as the square of B diminishes, A is said to be *inversely proportional to the square* of B, or *inverse-*

much faster than the heavier, as the heavier exceeds the lighter in weight. This shews that the attraction of each cork is in direct proportion to its weight or quantity of matter.[8]

This kind of attraction reaches but to a very small distance; for, if two drops of quicksilver be rolled in dust, they will not run together, because the particles of dust keep them out of each other's attraction.

Repulsion.

Where the sphere of attraction ends, a *repulsive force* begins: thus, water repels most bodies till they are wet; and hence it is, that a small needle, if dry, swims upon water; and flies walk upon it without wetting their feet.[9]

The repelling force of the particles of a fluid is but small; and therefore, if a fluid be divided, it easily unites again. But if a glass, or any other hard substance, be broke into small parts, they cannot be made to stick together again without being first wetted; the repulsion being too great to admit of a reunion.

The repelling force between water and oil is so great, that we find it almost impossible to mix them so as not to separate again. If a ball of light wood be dipt in oil, and then put into water, the water will recede so as to form a channel of some depth all around the ball.

The repulsive force of the particles of air is so great, that they can never be brought so near together by condensation as to make them stick or cohere. Hence it is, that when the weight of the incumbent atmosphere is taken off from any small quan-

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ly as the square of *B*. In this case, if *A* be the force of gravity, and *B* the distance, then, because it is found by observation, that when the distance is doubled, gravity is four times weaker; and when tripled, nine times weaker, and so on, the force of gravity is said to be inversely as the square of the distance. It is well known, that the force of gravity depends upon two causes, upon the quantity of matter in the body which exerts the force, and on the distance of the body upon which that force is exerted. The force of gravity, therefore, is said to be *directly* as the quantity of matter, and *inversely as the square* of the distance. As these expressions occur frequently in the course of this work, it will be useful to the reader to make them familiar to his mind.—*Ed.*

[8] The approach of the corks, in this experiment, is not produced by their mutual attraction. Each piece of cork is encompassed with a ring of water, elevated above the level of the fluid in the bason by the attraction of cohesion; and it is in consequence of the mutual action of these fluid rings, that the floating corks rush into contact. See *Edinburgh Encyclopædia*, Art. *Hydrodynamics*, vol. xi, p. 478.—*Ed.*

[9] These phenomena do not arise from any repulsive force in the needle, or in the feet of flies, but from the cohesive force of the water.—*Ed.*

tity of air, that quantity will diffuse itself so as to occupy (in comparison) an infinitely greater portion of space than it did before.

Gravitation.

Attraction of gravitation is that power by which distant bodies tend toward one another. Of this we have daily instances in the falling of bodies to the earth. By this power in the earth it is, that bodies, on whatever side, fall in lines perpendicular to its surface; and consequently, on opposite sides, they fall in opposite directions; all toward the centre, where the force of gravity is as it were accumulated: and by this power it is, that bodies on the earth's surface are kept to it on all sides, so that they cannot fall from it. And as it acts upon all bodies in proportion to their respective quantities of matter, without any regard to their bulks or figures, it accordingly constitutes their weight. Hence,

If two bodies which contain equal quantities of matter, were placed at ever so great a distance from one another, and then left at liberty in free space; if there were no other bodies in the universe to affect them, they would fall equally swift toward one another by the power of gravity, with velocities accelerated as they approached each other; and would meet in a point which was half way between them at first. Or, if two bodies, containing unequal quantities of matter, were placed at any distance, and left in the same manner at liberty, they would fall toward one another with velocities which would be in an inverse proportion to their respective quantities of matter; and moving faster and faster in their mutual approach, would at last meet in a point as much nearer to the place from which the heavier body began to fall, than to the place from which the lighter body began to fall, as the quantity of matter in the former exceeded that in the latter.

All bodies that we know of have gravity or weight. For, that there is no such thing as positive levity, even in smoke, vapours, and fumes, is demonstrable by experiments on the air pump; which shews, that although the smoke of a candle ascends to the top of a tall receiver when full of air, yet, upon the air being exhausted out of the receiver, the smoke falls down to the bottom of it. So, if a piece of wood be immersed in a jar of water, the wood will rise to the top of the water, because it has a less degree of weight than its bulk of water has; but if the jar be emptied of water, the wood falls to the bottom.

Gravity demonstrated to be as the quantity of matter in bodies.

As every particle of matter has its proper gravity, the effect of the whole must be in proportion to the number of the attracting particles; that is, as the quantity of matter in the whole body. This is demonstrable by experiments on pendulums; for, if they are of equal lengths, whatever their weights be, they vibrate in equal times. Now it is plain, that if one be double or triple the weight of another, it must require a double or triple power of gravity to make it move with the same celerity; just as it would require a double or triple force to project a bullet of twenty or thirty pounds weight, with the same degree of swiftness that a bullet of ten pounds would require. Hence it is evident, that the power or force of gravity is always proportional to the quantity of matter in bodies, whatever their bulks or figures are.

It decreases as the square of the distance increases.

Gravity also, like all other virtues or emanations which proceed or issue from a centre, decreases as the distance multiplied by itself increases: that is, a body at twice the distance of another, attracts with only a fourth part of the force; at thrice the distance, with a ninth part; at four times the distance, with a sixteenth part; and so on. This, too, is confirmed by comparing the distance which the moon falls in a minute, from a right line touching her orbit, with the distance through which heavy bodies near the earth fall in that time; and also, by comparing the forces which retain Jupiter's moons in their orbits, with their repective distances from Jupiter. These forces will be explained in the next lecture.

The velocity which bodies near the earth acquire in descending freely by the force of gravity, is proportional to the times of their descent. For, as the power of gravity does not consist in a single impulse, but is always operating in a constant and uniform manner, it must produce equal effects in equal times; and consequently in a double or triple time, a double or triple effect: and so, by acting uniformly on the body, must accelerate its motion proportionably to the time of its descent.[1]

To be a little more particular on this subject, let us suppose that a body begins to move with a celerity constantly and gradually increasing, in such a manner as would carry it through a mile in a minute; at the end of this space it will have acquired such a degree of celerity, as is sufficient to carry it two miles the next minute, though it should then receive no new impulse from the cause by which its motion had been accelerated; but if the same accelerating cause continues, it will carry the body a mile farther; on which account it will have run through four miles at the end of two minutes; and then it will have acquired such a degree of celerity as is sufficient to carry it through a double space in as much more time, or eight miles in two minutes, even though the accelerating force should act upon it no more. But this force still continuing to operate in an uniform manner, will again, in an equal time, produce an equal effect; and so, by carrying it a mile farther, cause it to move through five miles the third minute; for the celerity already acquired, and the celerity still acquiring, will have each its complete effect. Hence we learn, that if the body should move one mile the first minute, it would move three miles the second, five the third, seven the fourth, nine the fifth, and so on in proportion.

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[1] It appears from the recent experiments of Captain Kater, on pendulums, that the space passed over by descending bodies, in the first second of time, in the latitude of London is 16.095 feet.—*Ed.*

And thus it appears, that the spaces described in successive equal parts of time, by an uniformly accelerated motion, are always as the odd numbers 1, 3, 5, 7, 9, &c. and consequently, the whole spaces are as the squares of the times, or of the last acquired velocities; for, the continued addition of the odd numbers yields the squares of all numbers from unity upward. Thus, 1 is the first odd number, and the square of 1 is 1; 3 is the second odd number, and this added to 1 makes 4, the square of 2; 5 is the third odd number, which added to 4 makes 9, the square of 3; and so on for ever. Since, therefore, the times and velocities proceed evenly and constantly, as 1, 2, 3, 4, &c. but the spaces described in equal times are as 1, 3, 5, 7, &c. it is evident that the space described

In 1 minute will be - - $1 =$ square of 1
In 2 minutes - - $1+3 = 4 =$ square of 2
In 3 minutes - $1+3+5 = 9 =$ square of 3
In 4 minutes - $1+3+5+7 = 16 =$ square of 4, &c.

N. B. The character + signifies *more*, and = *equal*.

As heavy bodies are uniformly accelerated by the power of gravity in their descent, it is plain that they must be uniformly retarded by the same power in their ascent. Therefore, the velocity which a body acquires by falling, is sufficient to carry it up again to the same height from whence it fell: allowance being made for the resistance of the air, or other medium in which the body is moved. Thus, the body *D* (Plate I, Fig. 2) in rolling down the inclined plane *A B*, will acquire such a velocity by the time it arrives at *B*, as will carry it up the inclined plane *B C*, almost to *C;* and would carry it quite up to *C*, if the body and plane were perfectly smooth, and the air gave no resistance. So, if a pendulum were put into motion, in a space quite free of air, and all other resistance, and had no friction on the point of suspension, it would move for ever: for the velocity it had acquired in falling through the descending part of the arc, would be still sufficient to carry it equally high in the ascending part thereof.

The descending velocity will give a power of equal ascent.

The *centre of gravity* is that point of a body in which the whole force of its gravity or weight is united. Therefore, whatever supports that point, bears the weight of the whole body; and while it is supported, the body cannot fall; because all its parts are in a perfect equilibrium about that point.[2]

The centre of gravity.

An imaginary line drawn from the centre of gravity of any and line of direction.

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[2] The following is a mechanical method of finding the centre of gravity:—1. If the body can be easily suspended by a thread or cord, then the centre of gravity will be situated in some point in the direction of the cord prolonged. Suspend the body at another part, so that the new direction of the cord may be nearly at right angles with its former direction; then, as the centre of gravity must lie somewhere in the new direction of the cord prolonged, the point where these two lines (formed by prolonging the cord) intersect each other, will determine the centre of gravity. 2. If the body is of such a size or quality that it cannot be conveniently suspended, place it upon an horizontal edge so that it may be in equilibrio; and the horizontal edge will make a line or mark on the body in the same direction with itself, and the centre of gravity will be in some point in this line. Balance the body a second time, so that the line upon the body may be nearly at right angles to the horizontal edge, which will make a new line or mark upon the body; the centre of gravity therefore will be somewhere in this new line, and consequently in the point where it intersects the former line. 3. If the body is so flexible that it can neither be suspended nor balanced, then let a board be balanced, as in case 2d, and upon it, when balanced, lay the body whose centre of gravity is to be found, in such a manner that the board may still be in equilibrio; then the centre of gravity will be in a line opposite to that which is made on the board by the horizontal edge; and by shifting the position of the board, and again balancing it, a new line will be found, the intersection of which, with the former line, will determine the centre of gravity.—*Ed.*

body toward the centre of the earth, is called the *line of direction.* In this line all heavy bodies descend, if not obstructed.

Since the whole weight of a body is united in its centre of gravity, as that centre ascends or descends, we must look upon the whole body to do so too. But as it is contrary to the nature of heavy bodies to ascend of their own accord, or not to descend when they are permitted, we may be sure, that unless the centre of gravity be supported, the whole body will tumble or fall. Hence it is, that bodies stand upon their bases when the line of direction falls within the base; for in this case the body cannot be made to fall, without first raising the centre of gravity higher than it was before. Thus, the inclining body *A B C D* (Fig. 3) whose centre of gravity is *E*, stands firmly on its base *C D I K*, because the line of direction *E F* falls within the base. But if a weight, as *A B G H*, be laid upon the top of the body, the centre of gravity of the whole body and weight together is raised up to *I*; and then, as the line of direction *I D* falls without the base at *D*, the centre of gravity *I* is not supported; and the whole body and weight tumble down together.

Hence appears the absurdity of people's rising hastily in a coach or boat when it is likely to overset; for, by that means they raise the centre of gravity so far as to endanger throwing it quite out of the base; and if they do, they overset the vehicle effectually. Whereas, had they clapt down to the bottom, they would have brought the line of direction, and consequently the centre of gravity, farther within the base, and by that means might have saved themselves.

The broader the base is, and the nearer the line of direction is to the middle or centre of it, the more firmly does the body stand. On the contrary, the narrower the base, and the nearer the line of direction is to the side of it, the more easily may the body be overthrown, a less change of position being sufficient to remove the line of direction out of the base in the latter case than in the former. And hence it is, that a sphere is so easily rolled upon a horizontal plane; and that it is so difficult, if not impossible, to make things which are sharp-pointed to stand upright on the point. From what hath been said, it plainly appears, that if the plane be inclined on which the heavy body is placed, the body will slide down upon the plane while the line of direction falls within the base; but it will tumble or roll down when that line falls without the base. Thus, the body *A* (Fig. 4) will only slide down the inclined plane *CD*, while the body *B* will roll down upon it.

When the line of direction falls within the base of our feet, we stand; and most firmly when it is in the middle: but when it is out of that base, we immediately fall. And it is not only pleasing, but even surprising, to reflect upon the various and unthought-of methods and postures which we use to retain this position, or to recover it when it is lost. For this purpose we bend our body forward when we rise from a chair, or when we go up stairs: and for this purpose a man leans forward when he carries a burthen on his back, and backward when he carries it on his breast; and to the right or left side as he carries it on the opposite side. A thousand more instances might be added.

The quantity of matter in all bodies is in exact proportion to their weights, bulk for bulk. Therefore, heavy bodies are as much more dense or compact than light bodies of the same bulk, as they exceed them in weight.

All bodies porous.

All bodies are full of pores, or spaces void of matter: and in gold, which is the heaviest of all known bodies, there is perhaps a greater quantity of space than of matter. For the particles of heat and magnetism find an easy passage through the pores of gold; and even water itself has been forced through them. Besides, if we consider how easily the rays of light pass through so solid a body as glass, in all manner of directions, we shall find reason to believe that bodies are much more porous than is generally imagined.[3]

The expansion of metals.

All bodies are some way or other affected by heat: and all metallic bodies are expanded in length, breadth, and thickness, thereby. The proportion of the expansion of several metals, according to the best experiments I have been able to make with my pyrometer, is nearly thus:—Iron and steel, as 3, copper 4½, brass 5, tin 6, lead 7. An iron rod 3 feet long is about a 70th part of an inch longer in summer than in winter.[4]

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[3] The word *porous* when employed to express a property of bodies, does not convey an accurate idea of their internal structure. According to Boscovich's beautiful theory of natural philosophy, which is now generally adopted, matter consists of physical points endued with certain powers of attraction and repulsion, varying both in kind and degree, with their respective distances. At a certain distance, one particle is attracted by another, at a different distance it is repelled, and there are certain positions, in which the particle is in a quiescent state, and affected neither by an attractive nor a repulsive force. By this means, a collection of these physical points may assume the various forms of solidity, fluidity, and elastic and non-elastic vapours, and may possess all those properties which regulate the phenomena of collision, and the diversified effects of chemical agency.—*Ed.*

The pyrometer.

The pyrometer here mentioned being (as far as I know) of a new construction, a description of it may perhaps be agreeable to the reader.

A A (Fig. 5) is a flat piece of mahogany, in which are fixed four brass studs, *B*, *C*, *D*, *L*, and two pins, one at *F* and the other at *H*. On the pin *F* turns the crooked index *E I*, and upon the pin *H* the straight index *G K*, against which a piece of watch-spring *R* bears gently, and so presses it toward the begining of the scale *M N*, over which the point of that index moves. This scale is divided into inches and tenth parts of an inch: the first inch is marked 1000, the second 2000, and so on. A bar of metal *O* is laid into notches in the top of the studs *C* and *D*; one end of the bar bearing against the adjusting screw *P*, and the other end against the crooked index *E I*, at a 20th part of its length from its centre of motion *F*. Now it is plain, that however much the bar *O* lengthens, it will move that part of the index *E I*, against which it bears, just as far: but the crooked end of the same index, near *H*, being 20 times as far from the centre of motion *F* as the point is against which the bar bears, it will move 20 times as far as the bar lengthens. And as this crooked end bears against the index *G K* at only a 20th part of the whole length *G S* from its centre of motion *H*, the point *S* will move through twenty times the space that the point of bearing near *H* does. Hence, as 20 multiplied by 20 produces 400, it is evident, that if the bar lengthens but a 400th part of an inch, the point *S* will move a whole inch on the scale;

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[4] The following table contains the principal results respecting the expansion of bodies in length, for a change of temperature of 180° of Fahrenheit's thermometer:—

Glass Rod,	.00080787	General Roy	Copperhammered,	.0017	Smeaton
Platina,	.0009918	Troughton			
Palladium,	.0010	Wollaston	Brass,	.001988	Troughton
Cast Iron,	.0011094	Roy	Silver,	.0020286	Troughton
Steel,	.0011898	Troughton	Speculum Metal,	.001933	Smeaton
Hard Steel,	.001225	Smeaton			
Iron Wire, hard drawn,	.00144	Troughton	Grain Tin, Lead,	.002483 .002867	Smeaton Smeaton
Bismuth,	.001392	Smeaton	Zinc,	.002942	Smeaton
Gold,	.0015	Ellicot			

A more copious table of expansions, with an account of Mr. Troughton's interesting experiments, will be found in the *Edinburgh Encyclopædia*, Art. *Expansion*, vol. ix, p. 251.—*Ed.*

and as every inch is divided into 10 equal parts, if the bar lengthens but the 10th part of the 400th part of an inch, which is only the 4000th part of an inch, the point *S* will move the 10th part of an inch, which is very perceptible.

To find how much a bar lengthens by heat, first lay it cold into the notches of the studs, and turn the adjusting screw *P* until the spring *R* brings the point *S* of the index *G K* to the beginning of the divisions of the scale at *M*: then, without altering the screw any farther, take off the bar, and rub it with a dry woollen cloth till it feels warm; and then, laying it on where it was, observe how far it pushes the point *S* upon the scale by means of the crooked index *E I*; and the point *S* will shew exactly how much the bar has lengthened by the heat of rubbing. As the bar cools, the spring *R* bearing against the index *K G*, will cause its point *S* to move gradually back toward *M* in the scale: and when the bar is quite cold the index will rest at *M*, where it was before the bar was made warm by rubbing. The indexes have small rollers under them at *I* and *K*, which, by turning round on the smooth wood as the indexes move, make their motions the easier, by taking off a great part of the friction, which would otherwise be on the pins *F* and *H*, and of the points of the indexes themselves on the wood.[5]

Magnetism.

Beside the universal properties above mentioned, there are bodies which have properties peculiar to themselves; such as the loadstone, in which the most remarkable are these:—1, It attracts iron and steel only. 2, It constantly turns one of its sides to the north and another to the south, when suspended by a thread that does not twist. 3, It communicates all its properties to a piece of steel when rubbed upon it, without losing any itself.

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[5] There is an obvious defect in the principle of this pyrometer. If we suppose, as in the text, that the distance of the point *n* (where the bar *O* bears against the crooked lever), from the centre of motion *F*, is a 20th part of the length of the lever, before the bar *O* begins to expand; it is evident that the quantity *n F* will increase as soon as the expansion begins, and that its length, when the expansion ends, will be in proportion to the increase of the bar by heat. But as *n F* is nearly a 20th part of the length of the crooked lever, the increase of *H m*, which is at first a 20th part of *H S*, will be 20 times greater than the increase of *F n*. As the arms of the two levers therefore are continually changing their proportion, every pyrometer, constructed upon the principle of the lever, must give a very inaccurate result, the error being always in excess. Brisson, in his *Traité Elementaire de Physique*, Paris, tom. ii, p. 213, has described a pyrometer upon the same principle; but as the number of levers is increased, the inaccuracy is much greater. For an account of Mushenbroeck's pyrometer, which seems to have been the earliest, see Desagulier's *Exper. Philos.* vol. i, p. 421. For an account of Ellicot's, see *Phil. Trans.* No. 443. An account of Smeaton's may be seen in the *Phil. Trans.* vol. xlviii; of De Luc's in vol. lxviii; and of Ramsden's in vol. lxxv. Of all these Mr. Ramsden's is by far the most accurate. The bar is placed in a trough of water, which is heated by twelve spirit lamps; and the expansion is measured by a microscope, furnished with a micrometer.—*Ed.*

According to Dr. Helsham's experiments, the attraction of the loadstone decreases as the square of the distance increases. Thus, if a loadstone be suspended at one end of a balance, and counterpoised by weights at the other end, and a flat piece of iron be placed beneath it, at the distance of four tenths of an inch, the stone will immediately descend and adhere to the iron. But if the stone be again removed to the same distance, and as many grains be put into the scale at the other end as will exactly counterbalance the attraction, then, if the iron be brought twice as near the stone as before, that is, only two tenth parts of an inch from it, there must be four times as many grains put into the scale as before, in order to be a just counterbalance to the attractive force, or to hinder the stone from descending and adhering to the iron. So, if four grains will do in the former case, there must be sixteen in the latter. But from some later experiments, made with the greatest accuracy, it is found that the force of magnetism decreases in a ratio between the reciprocal of the square, and the reciprocal of the cube of the distance; approaching to the one or the other, as the magnitudes of the attracting bodies are varied.*

Electricity.

Several bodies, particularly amber, glass, jet, sealing-wax, agate, topaz, and almost all precious stones, have a peculiar property of attracting and repelling light bodies when heated by rubbing. This is called *electrical attraction*, in which the chief things to be observed are—1, If a glass tube about an inch and a half diameter, and two or three feet long, be heated by rubbing, it will alternately attract and repel all light bodies when held near them; 2, It does not attract by being heated without rubbing; 3, Any light body, being once repelled by the tube, will never be attracted again till it has touched some other body; 4, If the tube be rubbed by a moist hand, or any thing that is wet, it totally destroys the electricity; 5, Any body, except air, being interposed, stops the electricity; 6, The tube attracts stronger when rubbed over with bee's-wax, and then with a dry woollen cloth; 7, When it is well rubbed, if a finger be brought near it, at about the distance of half an inch, the effluvia will snap against the finger, and make a little crackling noise; and if this be performed in a dark place, there will appear a little flash of light.[6]

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* See the Supplementary Chapter on MAGNETISM in vol. ii.

LECTURE II.

OF CENTRAL FORCES.

All bodies equally indifferent to motion or rest. Plate I. Fig. 6.

We have already mentioned it as a necessary consequence arising from the deadness or inactivity of matter, that all bodies endeavour to continue in the state they are in, whether of rest or motion. If the body *A* were placed in any part of free space, and nothing either drew or impelled it any way, it would for ever remain in that part of space, because it could have no tendency of itself to remove any way from thence. If it receives a single impulse any way, as suppose from *A* toward *B*, it will go on in that direction; for, of itself, it could never swerve from a right line, nor stop its course. When it has gone through the space *A B*, and met with no resistance, its velocity will be the same at *B* as it was at *A;* and this velocity, in as much more time, will carry it through as much more space, from *B* to *C;* and so on for ever. Therefore, when we see a body in motion, we conclude that some other substance must have given it that motion; and when we see a body fall from motion to rest, we conclude that some other body or cause must have stopt it.

All motion naturally rectilineal.

As all motion is naturally rectilineal, it appears, that a bullet projected by the hand, or shot from a cannon, would for ever continue to move in the same direction it received at first, if no other power diverted its course. Therefore, when we see a body move in a curve of any kind whatever, we conclude it must be acted upon by two powers at least; one putting it in motion, and another drawing it off from the rectilineal course it would otherwise have continued to move in: and whenever that power, which bent the motion of the body from a straight line into a curve, ceases to act, the body will again move on in a straight line touching that point of the curve in which it was when the action of that power ceased. For example, a pebble moved round in a sling ever so long a time, will fly off the moment it is set at liberty, by slipping one end of the sling cord, and will go on in a line touching the circle it described before; which line would actually be a straight one, if the earth's attraction did not affect the pebble, and bring it down to the ground. This shews, that the natural tendency of the pebble, when put into motion, is to continue moving in a straight line, although by the force that moves the sling it be made to revolve in a circle.

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[6] See Ferguson's *Introduction to Electricity* for a popular account of the principal experiments in that interesting branch of natural philosophy.—*Ed.*

The effects of combined forces.

The change of motion produced is in proportion to the force impressed; for the effects of natural causes are always proportionate to the force or power of those causes.

By these laws it is easy to prove, that a body will describe the diagonal of a square or parallelogram, by two forces conjoined, in the same time that it would describe either of the sides, by one force singly. Thus suppose the body *A* (Fig. 7) to represent a ship at sea, and that it is driven by the wind, in the right line *A B*, with such a force as would carry it uniformly from *A* to *B* in a minute: then suppose a stream or current of water running in the direction *A D*, with such a force as would carry the ship through an equal space from *A* to *D* in a minute. By these two forces, acting together at right angles to each other, the ship will describe the line *A C* in a minute: which line (because the forces are equal and perpendicular to each other) will be the diagonal of an exact square. To confirm this law by an experiment, let there be a wooden square *A B C D* (Fig. 8) so contrived, as to have the part *B E F C* made to draw out or push into the square at pleasure. To this part let the pulley *H* be joined, so as to turn freely on an axis, which will be at *H* when the piece is pushed in, and at *h* when it is drawn out. To this part let the ends of a straight wire *k* be fixed, so as to move along with it under the pulley; and let the ball *G* be made to slide easily on the wire. A thread *m* is fixed to this ball, and goes over the pulley to *I*; by this thread the ball may be drawn up on the wire, parallel to the side *A D*, when the part *B E F C* is pushed as far as it will go into the square. But, if this part be drawn out, it will carry the ball along with it, parallel to the bottom of the square *D C*. By this means, the ball *G* may either be drawn perpendicularly upward by pulling the thread *m*, or moved horizontally along by pulling out the part *B E F C* in equal times, and through equal spaces; each power acting equally and separately upon it. But if, when the ball is at *G*, the upper end of the thread be tied to the pin *I*, in the corner *A* of the fixed square, and the moveable part *B E F C* be drawn out, the ball will then be acted upon by both the powers together; for it will be drawn up by the thread toward the top of the square, and at the same time be carried with its wire *k* toward the right hand *B C*, moving all the while in the diagonal line *L*; and will be found at *g* when the sliding part is drawn out as far as it was before; which then will have caused the thread to draw up the ball to the top of the inside of the square, just as high as it was before, when drawn up singly by the thread without moving the sliding part.

If the acting forces are equal, but at oblique angles to each other, so will the sides of the parallelogram be; and the diagonal run through by the moving body will be longer or shorter, according as the obliquity is greater or smaller. Thus, if two equal forces act conjunctly upon the body *A* (Fig. 9), one having a tendency to move it through the space *A B* in the same time that the other has a tendency to move it through an equal space *A D*; it will describe the diagonal *A G C* in the same time that either of the single forces would have caused it to describe either of the sides. If one of the forces be greater than the other, then one side of the parallelogram will be so much longer than the other. For, if one force singly would carry the body through the space *A E*, in the same time that the other would have carried it through the space *A D*, the joint action of both will carry it in the same time through the space *A H F*, which is the diagonal of the oblique parallelogram *A D E F*.

If both forces act upon the body in such a manner as to move it uniformly, the diagonal described will be a straight line; but if one of the forces acts in such a manner as to make the body move faster and faster, then the line described will be a curve. And this is the case of all bodies which are projected in rectilineal directions, and at the same time acted upon by the power of gravity, which has a constant tendency to accelerate their motions in the direction wherein it acts.

The laws of the planetary motions.

From the uniform projectile motion of bodies in straight lines, and the universal power of gravity or attraction, arises the curvilineal motion of all the heavenly bodies. If the body *A* (Fig. 10) be projected along the straight line *A F H* in open space, where it meets with no resistance, and is not drawn aside by any power, it will go on for ever with the same velocity, and in the same direction. But if, at the same moment, when the projectile force is given it at *A*, the body *S* begins to attract it with a force duly adjusted,[1] and perpendicular to its motion at *A*, it will then be drawn from the straight line *A F H*, and forced to revolve about *S* in the circle *A T W;* in the same manner, and by the same law, that a pebble is moved round in a sling. And if, when the body is in any part of its orbit (as suppose at *K)* a smaller body, as *L*, within the sphere of attraction of the body *K*, be projected in the right line *L M*, with a force duly adjusted, and perpendicular to the line of attraction *L K;* then, the small body *L* will revolve about the large body *K* in the orbit *N O*, and accompany it in its whole course round the yet larger body *S*. But then, the body *K* will no longer move in the circle *A T W;* for that circle will now be described by the common centre of gravity between *K* and *L*. Nay, even the great body *S* will not keep in the centre; for it will be the common centre of gravity between all the three bodies *S*, *K*, and *L*, that will remain immoveable there. So, if we suppose *S* and *K* connected by a wire *P* that has no weight, and *K* and *L* connected by a wire *q* that has no weight, the common centre of gravity of all these three bodies will be a point in the wire *P* near *S;* which point being supported, the bodies will be all *in equilibrio* as they move round it. Though, indeed, strictly speaking, the common centre of gravity of all the three bodies will not be in the wire *P*, but when these bodies are all in a right line. Here *S* may represent the sun, *K* the earth, and *L* the moon.

In order to form an idea of the curves described by two bodies revolving about their common centre of gravity, while they themselves with a third body are in motion round the common centre of gravity of all the three; let us first suppose E (Fig. 11) to be the sun, and e the earth going round him without any moon; and their moving forces regulated as above. In this case, while the earth goes round the sun in the dotted circle $R\ T\ U\ V\ W\ X$, &c. the sun will go round the circle $A\ B\ D$, whose centre C is the common centre of gravity between the sun and earth: the right line $\beta\ \delta$ representing the mutual attraction between them, by which they are as firmly connected as if they were fixed at the two ends of an iron bar strong enough to hold them. So, when the earth is at e, the sun will be at E; when the earth is at T, the sun will be at F; and when the earth is at g, the sun will be at G, &c.

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[1] To make the projectile force a just balance to the gravitating power, so as to keep the planet moving in a circle, it must give such a velocity as the planet would acquire by gravity, when it had fallen through half the semi-diameter of that circle.

The curves described by bodies revolving about their common centre of gravity.
Plate I. Fig. 11.

Now, let us take in the moon q, (at the top of the figure), and suppose the earth to have no progressive motion about the sun; in which case, while the moon revolves about the earth in her orbit 𝔄 𝔅 ℭ 𝔇, the earth will revolve in the circle S 13, whose centre R is the common centre of gravity of the earth and moon; they being connected by the mutual attraction between them in the same manner as the earth and sun are.

But the truth is, that while the moon revolves about the earth, the earth is in motion about the sun; and now, the moon will cause the earth to describe an irregular curve, and not a true circle, round the sun; it being the common centre of gravity of the earth and moon that will then describe the same circle which the earth would have moved in, if it had not been attended by a moon. For, supposing the moon to describe a quarter of her progressive orbit about the earth in the time that the earth moves from e to f, it is plain, that when the earth comes to f, the moon will be found at r; in which time, their common centre of gravity will have described the arc R 1 T, the earth the curve R 5 f, and the moon the curve q 14 r, In the time that the moon describes another quarter of her orbit, the centre of gravity of the earth and moon will describe the arc T 2 U, the earth the curve f 6 g, the moon the curve r 15 s, and so on: and thus, while the moon goes once round the earth in her progressive orbit, their common centre of gravity describes the regular portion of a circle R 1 T 2 U 3 V 4 W, the earth the irregular curve R 5 f 6 g 7 h 8 i, and the moon the yet more irregular curve *q* 14 *r* 15 *s* 16 *t* 17 *u*; and then the same kind of tracks over again.

The centre of gravity of the earth and moon is 6000 miles from the earth's centre toward the moon: therefore the circle *S* 13 which the earth describes round that centre of gravity (in every course of the moon round her orbit) is 12,000 miles in diameter. Consequently the earth is 12,000 miles nearer the sun at the time of full moon, than at the time of new. [See the earth at *f* and at *h*.]

To avoid confusion in so small a figure, we have supposed the moon to go only twice and a half round the earth, in the time that the earth goes once round the sun; it being impossible to take in all the revolutions which she makes in a year, and to give a true figure of her path, unless we should make the semidiameter of the earth's orbit at least 95 inches; and then, the proportional semidiameter of the moon's orbit would be only a quarter of an inch. For a true figure of the moon's path, I refer the reader to my Treatise of Astronomy. (See Vol. I, p. 163, and Plate VII, Fig. 3.)

If the moon made any complete number of revolutions about the earth, in the time that the earth makes one revolution about the sun, the paths of the sun and moon would return into themselves at the end of every year; and so be the same over again; but they return not into themselves in less than 19 years nearly; in which time the earth makes nearly 19 revolutions about the sun, and the moon 235 about the earth.

If the planet *A* be attracted toward the sun, with such a force as would make it fall from *A* to *B* (Fig. 10) in the time that the projectile impulse would have carried it from *A* to *F*, it will describe the arc *A G* by the combined action of these forces, in the same time that the former would have caused it to fall from *A* to *B*, or the latter have carried it from *A* to *F*. But if the projectile force had been twice as great, that is, such as would have carried the planet from *A* to *H*, in the same time that now, by the supposition, it carries it only from *A* to *F*; the sun's attraction must then have been four times as strong as formerly, to have kept the planet in the circle *A T W*; that is, it must have been such as would have caused the planet to fall from *A* to *E*, which is four times the distance of *A* from *B*, in the time that the projectile force singly would have car-

A double projectile force balances a quadruple power of gravity.

ried it from A to H, which is only twice the distance of A from F.[2] Thus, a double projectile force will balance a quadruple power of gravity in the same circle, as appears plain by the figure, and shall soon be confirmed by an experiment.

Plate I. Fig. 12 The whirling-table described.

The whirling-table is a machine contrived for shewing experiments of this nature.[3] A A is a strong frame of wood, B a winch or handle fixed on the axis C of the wheel D, round which is the catgut string F, which also goes round the small wheels G and K, crossing between them and the great wheel D. On the upper end of the axis of the wheel G, above the frame, is fixed the round board d, to which the bearer M S X may be fastened occasionally, and removed when it is not wanted. On the axis of the wheel H is fixed the bearer N T Z; and it is easy to see that when the winch B is turned, the wheels and bearers are put into a whirling motion.

Each bearer has two wires, W, X, and Y, Z, fixed and screwed tight into them at the ends by nuts on the outside: and when these nuts are unscrewed, the wires may be drawn out in order to change the balls U and V, which slide upon the wires by means of brass loops fixed into the balls, which keep the balls from touching the wood below them. A strong silk line goes through each ball, and is fixed to it at any length from the centre of the bearer to its end, as occasion requires, by a nut screw at the top of the ball; the shank of the screw goes into the centre of the ball, and presses the line against the under side of the hole that it goes through. The line goes through the ball, and under a small pulley fixed in the middle of the bearer; then up through a socket in the round plate (see S and T) in the middle of each bearer; then through a slit in the middle of the square top (O and P) of each tower, and going over a small pulley on the top, comes down again the same way, and is at last fastened to the upper end of the socket fixed in the middle of the above-mentioned round plate. These plates S and T have each four round holes near their edges for letting them slide up and down upon the wires which make the corners of each tower. The balls and plates being thus connected, each by its particular line, it is plain, that if the balls be drawn outward, or toward the ends *M* and *N* of their respective bearers, the round plates *S* and *T* will be drawn up to the top of their respective towers *O* and *P*.

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[2] Here the arcs A G, A I, must be supposed to be very small; otherwise A E, which is equal to H I, will be more than quadruple of A B, which is equal to F G.

[3] This machine was invented and constructed by Mr. J. B. Haas, and has been varied both in shape and size, according to the peculiar taste of the instrument-maker.—*Ed.*

There are several brass weights, some of two ounces, some of three, and some of four, to be occasionally put within the towers *O* and *P*, upon the round plates *S* and *T*; each weight having a round hole in the middle of it, for going upon the sockets or axes of the plates, and is slit from the edge to the hole, for allowing it to be slipt over the aforesaid line which comes from each ball to its respective place.[4] (See Fig. 13.)

The experiments to be made by this machine are as follows.

The propensity of matter to keep the state it is in.

1. Take away the bearer *M X* (Fig. 12), and take the ivory ball *a*, to which the line or silk cord *b* is fastened at one end; and having made a loop on the other end of the cord, put the loop over a pin fixed in the centre of the board *d*. Then, turning the winch *B* to give the board a whirling motion, you will see that the ball does not immediately begin to move with the board, but, on account of its inactivity, it endeavours to continue in the state of rest which it was in before. Continue turning, until the board communicates an equal degree of motion with its own to the ball, and then turning on, you will perceive that the ball will remain upon one part of the board, keeping the same velocity with it, and having no relative motion upon it, as is the case with every thing that lies loose upon the plain surface of the earth, which, having the motion of the earth communicated to it, never endeavours to remove from that place. But stop the board suddenly by hand, and the ball will go on, and continue to revolve upon the board, until the friction thereof stops its motion; which shews, that matter being once put into motion would continue

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[4] A curious machine has been invented by Mr. Atwood, which exhibits the phenomena of accelerated and retarded motion, in a very simple and satisfactory manner. By means of it, the quantity of matter moved, the moving force, the space described, the time of description, and the velocity acquired, can be easily and accurately ascertained. It may be used also for estimating the velocities communicated by the percussion of elastic and non-elastic bodies, for determining the quantity of resistance occasioned by fluids, and for practically verifying the properties of rotatory motion. This useful machine is described in *Atwood's Treatise on Rectilineal and Rotatory Motion*, and with the recent improvements of Fortin in the *Edinburgh Encyclopædia*, Art. *Mechanics*, vol. xiii, p. 604.—*Ed.*

to move for ever, if it met with no resistance. In like manner, if a person stands upright in a boat before it begins to move, he can stand firm; but the moment the boat sets off, he is in danger of falling toward that place which the boat departs from; because, as matter, he has no natural propensity to move. But when he acquires the motion of the boat, let it be ever so swift, if it be smooth and uniform, he will stand as upright and firm as if he was on the plain shore; and if the boat strikes against any obstacle, he will fall toward that obstacle, on account of the propensity he has, as matter, to keep the motion which the boat has put him into.

2. Take away this ball, and put a longer cord to it, which may be put down through the hollow axis of the bearer *M X*, and wheel *G*, and fix a weight to the end of the cord below the machine; which weight, if left at liberty, will draw the ball from the edge of the whirling-board to its centre.

Bodies moving in orbits have a tendency to fly out of these orbits.

Draw off the ball a little from the centre, and turn the winch; then the ball will go round and round with the board, and will gradually fly off farther and farther from the centre, and raise up the weight below the machine; which shews, that all bodies revolving in circles have a tendency to fly off from these circles, and must have some power acting upon them from the centre of motion, to keep them from flying off. Stop the machine, and the ball will continue to revolve for some time upon the board; but as the friction gradually stops its motion, the weight acting upon it will bring it nearer and nearer to the centre in every revolution, until it brings it quite thither. This shews, that if the planets met with any resistance in going round the sun, its attractive power would bring them nearer and nearer to it in every revolution, until they fell upon it.

Bodies move faster in small orbits than in large ones.

3. Take hold of the cord below the machine with one hand, and with the other throw the ball upon the round board as it were at right angles to the cord, by which means it will go round and round upon the board. Then observing with what velocity it moves, pull the cord below the machine, which will bring the ball nearer to the centre of the board, and you will see that the nearer the ball is drawn to the centre, the faster it will revolve; as those planets which are nearest the sun revolve faster than those which are more remote; and not only go round sooner,

because they describe smaller circles, but even move faster in every part of their respective circles.

Their centrifugal forces shewn.

4. Take away this ball, and apply the bearer *M X*, whose centre of motion is in its middle at *w*, directly over the centre of the whirling-board *d*. Then put two balls (*V* and *U*) of equal weights upon their bearing wires, and having fixed them at equal distances from their respective centres of motion *w* and *x* upon their silk cords, by the screw nuts, put equal weights in the towers *O* and *P*. Lastly, put the catgut strings *E* and *F* upon the grooves *G* and *H* of the small wheels, which being of equal diameters, will give equal velocities to the bearers above, when the winch *B* is turned; and the balls *U* and *V* will fly off towards *M* and *N*, and will raise the weights in the towers at the same instant. This shews, that when bodies of equal quantities of matter revolve in equal circles with equal velocities, their centrifugal forces are equal.

5. Take away these equal balls, and instead of them put a ball of six ounces into the bearer *M X*, at a sixth part of the distance *w z* from the centre, and put a ball of one ounce into the opposite bearer, at the whole distance *x y*, which is equal to *w z* from the centre of the bearer; and fix the balls at these distances on their cords, by the screw nuts at top; and then the ball *U*, which is six times as heavy as the ball *V*, will be at only a sixth part of the distance from its centre of motion; and consequently will revolve in a circle of only a sixth part of the circumference of the circle in which *V* revolves. Now, let any equal weights be put into the towers, and the machine be turned by the winch; which (as the catgut string is on equal wheels below) will cause the balls to revolve in equal times; but *V* will move six times as fast as *U*, because it revolves in a circle of six times its radius; and both the weights in the towers will rise at once. This shews, that the centrifugal forces of revolving bodies (or their tendencies to fly off from the circles they describe) are in direct proportion to their quantities of matter multiplied into their respective velocities, or into their distances from the centres of their respective circles. For, supposing *U*, which weighs six ounces, to be two inches from its centre of motion *w*, the weight multiplied by the distance is 12; and supposing *V*, which weighs only one ounce, to be 12 inches distant from the centre of the motion *x*, the weight

one ounce, multiplied by the distance 12 inches, is 12. And as they revolve in equal times, their velocities are as their distances from the centre, namely, as 1 to 6.

If these two balls be fixed at equal distances from their respective centres of motion, they will move with equal velocities; and if the tower *O* has six times as much weight put into it as the tower *P* has, the balls will raise their weights exactly at the same moment. This shews, that the ball *U* being six times as heavy as the ball *V*, has six times as much centrifugal force, in describing an equal circle with an equal velocity.

A double velocity in the same circle, is a balance to a quadruple power of gravity.

6. If bodies of equal weights revolve in equal circles with unequal velocities, their centrifugal forces are as the squares of the velocities. To prove this law by an experiment, let two balls, *U* and *V*, of equal weights, be fixed on their cords at equal distances from their respective centres of motion *w* and *x*; and then let the catgut string *E* be put round the wheel *K* (whose circumference is only one half of the circumference of the wheel *H* or *G*), and over the pully *s* to keep it tight; and let four times as much weight be put into the tower *P*, as in the tower *O*. Then turn the winch *B*, and the ball *V* will revolve twice as fast as the ball *U* in a circle of the same diameter, because they are equidistant from the centres of the circles in which they revolve; and the weight in the towers will both rise at the same instant, which shews that a double velocity in the same circle will exactly balance a quadruple power of attraction in the centre of the circle. For the weights in the towers may be considered as the attractive forces in the centres, acting upon the revolving balls; which moving in equal circles, is the same thing as if they both moved in one and the same circle.

Kepler's problem.

7. If bodies of equal weights revolve in unequal circles, in such a manner that the squares of the times of their going round are as the cubes of their distances from the centres of the circles they describe, their centrifugal forces are inversely as the squares of their distances from those centres. For, the catgut string remaining as in the last experiment, let the distance of the ball *V* from the centre *x* be made equal to two of the cross divisions on its bearer; and the distance of the ball *U* from the centre *w* be three and a sixth part; the balls themselves being of equal weights, and *V* making two revolu-

tions by turning the winch, in the time that U makes one: so that if we suppose the ball V to revolve in one second, the ball U will revolve in two seconds, the squares of which are one and four; for the square of 1 is only 1, and the square of 2 is 4; therefore the square of the period or revolution of the ball V is contained four times in the square of the period of the ball U. But the distance of V is 2, the cube of which is 8, and the distance of U is $3\frac{1}{6}$, the cube of which is 32 very nearly, in which 8 is contained four times; and therefore, the squares of the periods of V and U are to one another as the cubes of their distances from x and w, which are the centres of their respective circles. And if the weight in the tower O be four ounces, equal to the square of 2, the distance of V from the centre x; and the weight in the tower P be ten ounces, nearly equal to the square of $3\frac{1}{6}$, the distance of U from w; it will be found upon turning the machine by the winch, that the balls U and V will raise their respective weights at the same instant of time; which confirms that famous observation of Kepler, viz. that the squares of the times in which the planets go round the sun are in the same proportion as the cubes of their distances from him; and that the sun's attraction is inversely as the square of the distance from his centre: that is, at twice the distance, his attraction is four times less; at thrice the distance, nine times less; at four times the distance, sixteen times less; and so on to the remotest part of the system.[5]

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[5] The whole doctrine of centrifugal forces may be summed up in the following eight propositions, some of which are mentioned in the text:—

1. The centrifugal forces of two unequal bodies, moving with the same velocity, and at the same distance from the central body, are to one another as the respective quantities of matter in the two bodies.

2. The centrifugal forces of two equal bodies which perform their revolution round the central body in the same time, but at different distances from it, are to one another as their respective distances from the central body.

3. The centrifugal forces of two bodies which perform their revolution in the same time, and whose quantities of matter are inversely as their distances from the centre, are equal to one another.

4. The centrifugal forces of two equal bodies, moving at equal distances from the central body, but with different velocities, are to one another as the squares of their velocities.

5. The centrifugal forces of two unequal bodies, moving at equal distances from the centre, with different velocities, are to one another in the compound ratio of their quantities of matter, and the squares of their velocities.

6. The centrifugal forces of two equal bodies, moving with equal velocities at different distances from the centre, are inversely as their distances from the centre.

8. Take off the catgut string *E* (Fig. 12) from the great wheel *D* and the small wheel *H*, and let the string *F* remain upon the wheels *D* and *G*. Take away also the bearer *M X* from the whirling-board *d*, and instead thereof put the machine *A B* (Fig. 14) upon it, fixing this machine to the centre of the board, by the pins *c* and *d*, in such a manner, that the end *e f* may rise above the board to an angle of 30 or 40 degrees. In the upper side of this machine are two glass tubes *a* and *b*, close stopt at both ends; and each tube is about three quarters full of water. In the tube *a* is a little quicksilver, which naturally falls down to the end *a* in the water, because it is heavier than its bulk of water; and in the tube *b* is a small cork which floats on the top of the water at *e*, because it is lighter; and it is small enough to have liberty to rise or fall in the tube. While the board *b* with this machine upon it continues at rest, the quicksilver lies at the bottom of the tube *a* and the cork floats on the water near the top of the tube *b*. But, upon turning the winch, and putting the machine in motion, the contents of each tube will fly off toward the uppermost ends, (which are farthest from the centre of motion), the heaviest with the greatest force. Therefore the quicksilver in the tube *a* will fly off quite to the end *f*, and occupy its bulk of space there, excluding the water from that place, because it is lighter than quicksilver; but the water in the tube *b* flying off to its higher end *e*, will exclude the cork from that place, and cause the cork to descend toward the lowermost end of the tube, where it will remain upon the lowest end of the water near *b*; for the heavier body having the greater centrifugal force, will therefore possess the uppermost part of the tube; and the lighter body will keep between the heavier and the lowermost part.

The absurdity of the Cartesian vortices.

This demonstrates the absurdity of the Cartesian doctrine of the planets moving round the sun in vortices; for, if the planet be more dense or heavy than its bulk of the vortex, it will fly off therein, farther and farther from the sun; if less dense, it will come down to the lowest part of the vortex at the sun; and

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7. The centrifugal forces of two unequal bodies, moving with equal velocities at different distances from the centre, are to one another as their quantities of matter multiplied by their respective distances from the centre.

8. The centrifugal forces of two unequal bodies, moving with unequal velocities at different distances from the central body, are in the compound ratio of their quantities of matter, the squares of their velocities, and their distances from the centre.—*Ed.*

the whole vortex itself must be surrounded with something like a great wall, otherwise it would fly quite off, planets and all together. But while gravity exists, there is no occasion for such vortices: and when it ceases to exist, a stone thrown upward will never return to the earth again.

If one body moves round another, both of them must move round their common centre of gravity.

9. If a body be so placed on the whirling-board of the machine (Fig. 12), that the centre of gravity of the body be directly over the centre of the board, and the board be put into ever so rapid a motion by the winch *B*, the body will turn round with the board, but will not move from the middle of it; for, as all parts of the body are *in equilibrio* round its centre of gravity, and the centre of gravity is at rest in the centre of motion, the centrifrugal force of all parts of the body will be equal at equal distances from its centre of motion, and therefore the body will remain in its place. But if the centre of gravity be placed ever so little out of the centre of motion, and the machine be turned swiftly round, the body will fly off toward that side of the board on which its centre of gravity lies. Thus, if the wire *C* (Fig. 15), with its little ball *B* be taken away from the demi-globe *A*, and the flat side *e f* of this demi-globe be laid upon the whirling-board of the machine, so that their centres may coincide; if then the board be turned ever so quick by the winch, the demi-globe will remain where it was placed. But if the wire *C* be screwed into the demi-globe at *d*, the whole becomes one body, whose centre of gravity is now at or near *d*. Let the pin *c* be fixed in the centre of the whirling-board, and the deep groove *b* cut in the flat side of the demi-globe be put upon the pin, so that the pin may be in the centre of *A* (See Fig. 16, where this groove is represented at *b)*, and let the whirling-board be turned by the winch, which will carry the little ball *B* (Fig 15), with its wire *C*, and the demi-globe *A*, all round the centre pin *c i;* and then the centrifugal force of the little ball *B*, which weighs only one ounce, will be so great as to draw off the demi-globe *A*, which weighs two pounds, until the end of the groove at *e* strikes against the pin *c*, and thus prevents the demi-globe *A* from going any farther; otherwise, the centrifugal force of *B* would have been great enough to have carried *A* quite off the whirling-board: which shews, that if the sun were placed in the very centre of the orbits of the planets, it could not possibly remain

there; for the centrifugal forces of the planets would carry them quite off, and the sun with them, especially when several of them happened to be in any one quarter of the heavens: for the sun and planets are as much connected by the mutual attraction that subsists between them, as the bodies *A* and *B* are by the wire *C* which is fixed into them both. And even if there were but one single planet in the whole heavens to go round ever so large a sun in the centre of its orbit, its centrifugal force would soon carry off both itself and the sun. For the greatest body placed in any part of free space might be easily moved; because if there were no other body to attract it, it could have no weight or gravity of itself; and consequently, though it could have no tendency of itself to remove from that part of space, yet it might be very easily moved by any other substance.

10. As the centrifugal force of the light body *B* will not allow the heavy body *A* to remain in the centre of motion, even though it be 24 times as heavy as *B;* let us now take the ball *A* (Fig. 17), which weighs six ounces, and connect it by the wire *C* with the ball *B*, which weighs only one ounce; and let the fork *E* be fixed into the centre of the whirling-board: then hang the balls upon the fork by the wire *C* in such a manner that they may exactly balance each other, which will be when the centre of gravity between them, in the wire at *d*, is supported by the fork. And this centre of gravity is as much nearer to the centre of the ball *A*, than to the centre of the ball *B*, as *A* is heavier than *B*, allowing for the weight of the wire on each side of the fork. This done, let the machine be put into motion by the winch; and the balls *A* and *B* will go round their common centre of gravity *d*, keeping their balance, because either will not allow the other to fly off with it. For, supposing the ball *B* to be only one ounce in weight, and the ball *A* to be six ounces, then, if the wire *C* were equally heavy on each side of the fork, the centre of gravity *d* would be six times as far from the centre of the ball *B* as from that of the ball *A*, and consequently *B* will revolve with a velocity six times as great as *A* does, which will give *B* six times as much centrifugal force as any single ounce of *A* has; but then, as *B* is only one ounce, and *A* six ounces, the whole centrifugal force of *A* will exactly balance the whole centrifugal force of *B:* and, therefore, each body will detain

the other so as to make it keep in its circle. This shews that the sun and planets must all move round the common centre of gravity of the whole system, in order to preserve that just balance which takes place among them. For the planets being as inactive and dead as the above balls, they could no more have put themselves into motion than these balls can; nor have kept their orbits without being balanced at first with the greatest degree of exactness upon their common centre of gravity, by the Almighty hand that made them, and put them in motion.

Perhaps it may be here asked, that since the centre of gravity between these balls must be supported by the fork *E* in this experiment, *what* prop it is that supports the centre of gravity of the solar system, and consequently bears the weight of all the bodies in it; and by what is the prop itself supported? The answer is easy and plain; for the centre of gravity of our balls must be supported, because they gravitate toward the earth, and would therefore fall to it: but as the sun and planets gravitate only toward one another, they have nothing else to fall to, and therefore have no occasion for any thing to support their common centre of gravity; and if they did not move round that centre, and consequently acquire a tendency to fly off from it by their motions, their mutual attractions would soon bring them together; and so the whole would become one mass in the sun: which would also be the case if their velocities round the sun were not quick enough to create a centrifugal force equal to the sun's attraction.

But after all this nice adjustment, it appears evident that the Deity cannot withdraw his regulating hand from his works, and leave them to be solely governed by the laws which he has impressed upon them at first. For if He should once leave them so, their order would in time come to an end; because the planets must necessarily disturb one another's motions by their mutual attractions, when several of them are in the same quarter of the heavens, as is often the case: and then, as they attract the sun more toward that quarter than when they are in a manner dispersed equably around him, if he was not at that time made to describe a portion of a larger circle round the common centre of gravity, the balance would then be immediately destroyed; and as it could never restore itself again, the whole system would begin to fall together, and would in time unite in a mass at the sun.[6] Of this disturbance we have a very remarkable instance in the comet which appeared lately, and which in going last up from the sun, went so near Jupiter, and was so affected by his attraction, as to have the figure of its orbit much changed; and not only so, but to have its period altered, and its course different in the heavens from what it was before.

11. Take away the fork and balls from the whirling-board, and place the trough *A B* (Fig. 18) thereon, fixing its centre to the centre of the whirling-board by the pin *H*. In this trough are two balls *D* and *E*, of unequal weights, connected by a wire *f*, and made to slide easily upon the wire *C* stretched from end to end of the trough, and made fast by nut screws on the outside of the ends. Let these balls be so placed upon the wire *C*, that their common centre of gravity *g* may be directly over the centre of the whirling-board. Then, turn the machine by the winch, ever so swiftly, and the trough and balls will go round their centre of gravity, so that neither of the balls will fly off; because, on account of the equilibrium, each ball detains the other with an equal force acting against it. But if the ball *E* be drawn a little more toward the end of the trough at *A*, it will remove the centre of gravity toward that end from the centre of motion; and then, upon turning the machine, the little ball *E* will fly off, and strike with a considerable force against the end *A*, and draw the great ball *B* into the middle of the trough. Or, if the great ball *D* be drawn toward the end *B* of the trough, so that the centre of gravity may be a little toward that end from the centre of motion, and the machine be turned by the winch, the great ball *D* will fly off, and strike violently against the end *B* of the trough, and will bring the little ball *E* into the middle of it. If the trough be not made very strong, the ball *D* will break through it.

Of the tides.

12. The reason why the tides rise at the same absolute time on opposite sides of the earth, and con-

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[6] The opinion here stated by our author is now completely exploded, by the recent discoveries in physical astronomy. All the inequalities in our system, arising from the mutual action of the planets, are periodical and compensatory. Every derangement rises to a maximum in the course of many centuries, and then diminishes; and this is balanced by derangements of an equal and opposite kind, so that the system resumes the same relative situation which it had in former ages.—*Ed.*

sequently in opposite directions, is made abundantly plain by a new experiment on the whirling-table. The cause of their rising on the side next the moon every one understands to be owing to the moon's attraction: but why they should rise on the opposite side at the same time, where there is no moon to attract them, is perhaps not so generally understood. For it would seem that the moon should rather draw the waters, as it were, closer to that side, than raise them upon it, directly contrary to her attractive force. Let the circle *a b c d* (Fig. 19) represent the earth, with its side *c* turned toward the moon, which will then attract the waters so as to raise them from *c* to *g*. But the question is, why should they rise as high at that very time on the opposite side, from *a* to *e*? In order to explain this, let there be a plate *A B* (Fig. 20), fixed upon one end of the flat bar *D C*, with such a circle drawn upon it as *a b c d* (in Fig. 19), to represent the round figure of the earth and sea; and such an ellipsis as *e f g h* to represent the swelling of the tide at *e* and *g*, occasioned by the influence of the moon. Over this plate *A B* let the three ivory balls *e*, *f*, *g*, be hung by the silk lines *h*, *i*, *k*, fastened to the tops of the crooked wires *H*, *I*, *K*, in such a manner, that the ball at *e* may hang freely over the side of the circle *e*, which is farthest from the moon *M* at the other end of the bar; the ball at *f* may hang freely over the centre, and the ball at *g* hang over the side of the circle *g* which is nearest the moon. The ball *f* may represent the centre of the earth, the ball *g* some water on the side next the moon, and the ball *e* some water on the opposite side. On the back of the moon *M* is fixed the short bar *N* parallel to the horizon, and there are three holes in it above the little weights *p*, *q*, *r*. A silk thread *o* is tied to the line *k* close above the ball *g*, and passing by one side of the moon *M*, goes through a hole in the bar *N*, and has the weight *p* hung to it. Such another thread *n* is tied to the line *i*, close above the ball *f*, and passing through the centre of the moon *M* and middle of the bar *N*, has the weight *q* hung to it, which is lighter than the weight *p*. A third thread *m* is tied to the line *h*, close above the ball *e*, and passing by the other side of the moon *M*, through the bar *N*, has the weight *r* hung to it, which is lighter than the weight *q*.

The use of these three unequal weights is to represent the moon's unequal attraction at different distances from her. With whatever force she attracts the centre of the earth, she attracts the side next her with a greater degree of force, and the side farthest from her with a less. So, if the weights are left at liberty, they will draw all the three balls toward the moon with different degrees of force, and cause them to make the appearance shewn in Fig. 21; by which means they are evidently farther from each other than they would be if they hung at liberty by the lines *h*, *i*, *k;* because the lines would then hang perpendicularly. This shews, that as the moon attracts the side of the earth which is nearest her with a greater degree of force than she does the centre of the earth, she will draw the water on that side more than she draws the centre, and so cause it to rise on that side; and as she draws the centre more than she draws the opposite side, the centre will recede farther from the surface of the water on that opposite side, and so leave it as high there as she raised it on the side next to her. For, as the centre will be in the middle between the tops of the opposite elevations, they must of course be equally high on both sides at the same time.

But upon this supposition the earth and moon would soon come together: and to be sure they would, if they had not a motion round their common centre of gravity, to create a degree of centrifugal force sufficient to balance their mutual attraction. This motion they have; for as the moon goes round her orbit every month, at the distance of 240,000 miles from the earth's centre, and of 234,000 miles from the centre of gravity of the earth and moon, so does the earth go round the same centre of gravity every month at the distance of 6,000 miles from it; that is, from it to the centre of the earth. Now as the earth is (in round numbers) 8,000 miles in diameter, it is plain that its side next the moon is only 2,000 miles from the common centre of gravity of the earth and moon; its centre 6,000 miles distant therefrom; and its farther side from the moon 10,000: therefore the centrifugal forces of these parts are as 2,000, 6,000, and 10,000; that is, the centrifugal force of any side of the earth, when it is turned from the moon, is five times as great as when it is turned toward the moon: and as the moon's attraction (expressed by the numbers 6,000), &c. at the earth's centre keeps the earth from flying out of this monthly circle, it must be greater than the centrifugal force of the waters on the side next her; and consequently, her greater degree of attraction on that side is sufficient to raise them; but as her attraction on the opposite side is less than the centrifugal force of the water there, the excess of this force is sufficient to raise the water just as high on the opposite side.[7] To prove this experimentally, let the bar *D C* (Fig. 20), with its furniture, be fixed upon the whirling-board of the machine (Fig. 12), by pushing the pin *P* into the centre of the board; which pin is in the centre of gravity of the whole bar with its three balls *e*, *f*, *g*, and moon *M*. Now if the whirling-board and bar be turned slowly round by the winch until the ball *f* hangs over the centre of the circle, as in Fig. 22, the ball *g* will be kept toward the moon by the heaviest weight *p* (Fig. 20), and the ball *e*, on account of its greater centrifugal force, and the lesser weight *r*, will fly off as far to the other side, as in Fig. 22. And thus, while the machine is kept turning, the balls *e* and *g* will hang over the end of the ellipsis *l f k*; so that the centrifugal force of the ball *e* will exceed the moon's attraction just as much as her attraction exceeds the centrifugal force of the ball *g*, while her attraction just balances the centrifugal force of the ball *f*, and makes it keep in its circle. And hence it is evident, that the tides must rise to equal heights at the same time on opposite sides of the earth. This experiment, to the best of my knowledge, is entirely new.

The earth's motion demonstrated.

From the principles thus established, it is evident that the earth moves round the sun, and not the sun round the earth; for the centrifugal law will never allow a great body to move round a small one in any orbit whatever; especially when we find that if a small body moves round a great one, the great one must also move round the common centre of gravity between them two; and it is well known that the quantity of matter in the sun is 227,000 times as great as the quantity of matter in the earth. Now, as the sun's distance from the earth is at least 81,000,000 of miles, if we divide that distance by 227,000, we shall have only 357 for the number of miles that the centre of gravity between the sun and earth is distant from the sun's centre. And as the sun's semidiameter is $\frac{1}{4}$ of a degree, which at so great a distance as that of the sun, must be no less than 381,500 miles; if this be divided by 357, the quotient will be nearly 1,069, which shews that the common centre of gravity between the sun and the earth is within the body of the sun; and is only the 1,069th part of his semi-diameter from the centre toward his surface.

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[7] Mr. Ferguson is here mistaken in referring the rise of the waters, on the side of the earth opposite the moon, to the excess of the centrifugal force above the earth's attraction. The reader will scarcely be surprised at this, when he is informed, that the celebrated mathematician Mr. Wallis committed the same oversight. The true cause of the rise of the sea is explained in the Appendix, vol. ii.—*Ed.*

All globular bodies, whose parts can yield, and which do not turn on their axes, must be perfect spheres, because all parts of their surfaces are equally attracted toward their centres. But all such globes which do turn on their axes will be oblate spheroids; that is, their surfaces will be higher, or farther from the centre, in the equatorial than in the polar regions. For, as the equatorial parts move quickest, they must have the greatest centrifugal force; and will therefore recede farthest from the axis of motion. Thus if two circular hoops *A B* and *C D* (Fig. 23), made thin and flexible, and crossing one another at right angles, be turned round their axis *E F* by means of the winch *m*, the wheel *n*, and pinion *o*, and the axis be loose in the pole or intersection *e*, the middle parts *A*, *B*, *C*, *D*, will swell out so as to strike against the sides of the frame at *F* and *G*, if the pole *e*, in sinking to the pin *E*, be not stopt by it from sinking farther: so that the whole will appear of an oval figure, the equatorial diameter being considerably longer than the polar. That our earth is of this figure, is demonstrable from the actual measurement of some degrees on its surface, which are found to be longer in the frigid than in the torrid zones;[8] and the difference is found to be such as proves the earth's equatorial diameter to be 36 miles longer than its axis.[9] Seeing, then, the earth is higher at the equator than at the poles, the sea, which like all other fluids naturally runs downwards (or toward the places which are nearest the earth's centre), would run toward the polar regions, and leave the equatorial parts dry, if the centrifugal force of the water, which carried it to those parts, and so raised them, did not detain and keep it from running back again toward the poles of the earth.

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[8] St. Pierre (Studies of Nature, vol. i.) has attempted to demonstrate, that if a degree of the meridian is longer at the pole than at the equator, the earth must be a prolate spheroid, *i. e.* lengthened out at the poles. Upon this principle he attempts to explain the tides and the currents in the ocean, which he supposes to be occasioned by the periodical melting of the polar glaciers. It is remarkable that this author, who certainly deserves praise as a naturalist, should have committed such a mistake. At the beginning of his demonstration, he supposes, that the figure of the earth is *circular*, or, in other words, he supposes that it is not an oblate spheroid, before he begins to demonstrate that it is a prolate one. His theory of the tides, therefore, must be false, as it is built upon a false principle.—*Ed.*

[9] From a comparison of the lengths of different degrees of the meridian, obtained by actual mensuration, it appears that the compression at the earth's poles is $\frac{1}{300}$; and since the diameter of the earth at the equator is 7934.9 English miles nearly, its diameter at the poles will be 7908.5 miles; less than the former by $\frac{1}{300}$ of the whole; or its equatorial axis will be longer than its polar axis by $26\frac{4}{10}$ miles.—*Ed.*

LECTURE III.

OF THE MECHANICAL POWERS.

The foundation of all mechanics.

If we consider bodies in motion, and compare them together, we may do this either with respect to the quantities of matter they contain, or the velocities with which they are moved. The heavier any body is, the greater is the power required either to move it or to stop its motion; and again, the swifter it moves, the greater is its force. So that the whole *momentum* or quantity of force of a moving body is the result of its quantity of matter multiplied by the velocity with which it is moved; and when the products arising from the multiplication of the particular quantities of matter in any two bodies by their respective velocities are equal, the *momenta* or entire forces are so too. Thus, suppose a body, which we shall call *A*, to weigh 40 pounds, and to move at the rate of two miles in a minute; and another body, which we shall call *B*, to weigh only four pounds, and to move 20 miles in a minute; the entire forces with which these two bodies would strike against any obstacle would be equal to each other, and therefore it would require equal powers to stop them: for 40 multiplied by 2 gives 80, the force of the body *A*; and 20 multiplied by 4 gives 80, the force of the body *B*.

Upon this easy principle depends the whole of mechanics: and it holds universally true, that when two bodies are suspended on any machine, so as to act contrary to each other, if the machine be put into motion, and the perpendicular ascent of one body multiplied into its weight, be equal to the perpendicular descent of the other body multiplied into its weight, those bodies, how unequal soever in their weights, will balance one another in all situations: for, as the whole ascent of one is performed in the same time with the whole descent of the other, their respective velocities must be directly as the spaces they move through;

and the excess of weight in one body is compensated by the excess of velocity in the other. Upon this principle, it is easy to compute the power of any mechanical engine, whether simple or compound; for it is but only finding how much swifter the power moves than the weight does (*i. e.* how much farther in the same time), and just so much is the power increased by the help of the engine.

How to compute the power of any mechanical engine.

In the theory of this science, we suppose all planes perfectly even, all bodies perfectly smooth, levers to have no weight, cords to be extremely pliable, machines to have no friction; and, in short, all imperfections must be set aside until the theory be established; and then proper allowances are to be made.

The mechanic powers, what.

The simple *machines*, usually called *mechanical powers*, are six in number,[1] viz. the *lever*, the *wheel and axle*, the *pulley*, the *inclined plane*, the *wedge*, and the *screw*. They are called mechanical powers, because they help us mechanically to raise weights, move heavy bodies, and overcome resistances, which we could not effect without them.

The *lever*.

1. A *lever* is a bar of iron or wood, one part of which being supported by a prop, all the other parts turn upon that prop as their centre of motion; and the velocity of every part or point is directly as its distance from the prop. Therefore, when the weight to be raised at one end is to the power applied to the other to raise it, as the distance of the power from the prop is to the distance of the weight from the prop, the power and weight will exactly balance or counterpoise each other; and as a common lever has next to no friction on its prop, a very little additional power will be sufficient to raise the weight.

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[1] Some writers on mechanics exclude the *inclined plane* from the number of the mechanical powers, while others add the *balance* to the number. All the mechanical powers may, with great propriety, be reduced to *two*, the LEVER and the INCLINED PLANE. The *pulley*, and the *wheel and axle*, are merely an assemblage of levers. The *wedge* is evidently composed of two inclined planes; and the *screw* is only a wedge wrapped round a cylinder. Some foreign mechanicians add the *rope* or *rope machine*, to the number of mechanical powers. The weight is suspended anywhere between the extremities of the rope, to each of which a power is applied after they have passed over a pulley. In this machine, the powers and weight will be *in equilibrio*, or will balance each other, when the powers are inversely as the sines of the angles which they make with the direction of the weight, or as the secants of the angles which the direction of the powers makes with the horizon.—*Ed.*

There are four kinds of levers. 1. The common sort, where the prop is placed between the weight and the power, but much nearer to the weight than to the power; 2. When the prop is at one end of the lever, the power at the other, and the weight between them; 3. When the prop is at one end, the weight at the other, and the power applied between them; 4. The bended lever, which differs only in form from the first sort, but not in property. Those of the first and second kind are often used in mechanical engines; but there are few instances in which the third sort is used.[2]

The *balance.*

A *common balance* is by some reckoned a lever of the first kind; but as both its ends are at equal distances from its centre of motion, they move with equal velocities; and therefore, as it gives no mechanical advantage, it cannot properly be reckoned among the mechanical powers.[3]

Plate II. Fig. 1. The first kind of lever.

A lever of the first kind is represented by the bar *A B C*, supported by the prop *D*. Its principal use is to loosen large stones in the ground, or raise great weights to small heights, in order to have ropes put under them for raising them higher by other machines. The parts *A B* and *B C*, on different sides of the prop *D*, are called the *arms* of the lever; the end *A* of the shorter arm *A B* being applied to the weight intended to be raised, or to the resistance to be overcome; and the power applied to the end *C* of the longer arm *B C*.

In making experiments with this machine, the shorter arm *A B* must be as much thicker than the longer arm *B C*, as will be sufficient to balance it on the prop. This supposed, let *P* represent a power, whose gravity is equal to 1 ounce; and *W* a weight, whose gravity is equal to 12 ounces. Then, if the power be 12 times as far from the prop as the weight is, they will exactly counterpoise; and a small addition to the power *P* will cause it to descend, and raise the weight *W*; and the velocity with which the power descends will be to the velocity with which the weight rises, as 12 to 1; that is, directly as their distances from the prop, and consequently as the spaces through which they move. Hence it is plain, that a man, who by his natural strength, without the help of any machine, could support a hundred-weight, will by the help of this lever be enabled to support twelve hundred. If the weight be less, or the power greater, the prop may be placed so much farther from the weight, and then it can be raised to a proportionably greater height. For, universally, if the intensity of the weight multiplied into its distance from the prop, be equal to the intensity of the power multiplied into its distance from the prop, the power and weight will exactly balance each other; and a little addition to the power will raise the weight. Thus, in the present instance, the weight W is 12 ounces, and its distance from the prop is 1 inch; and 12 multiplied by 1 is 12; the power P is equal to 1 ounce, and its distance from the prop is 12 inches, which multiplied by 1 is 12 again; and therefore there is an equilibrium between them. So, if a power equal to 2 ounces be applied at the distance of six inches from the prop, it will just balance the weight W; for 6 multiplied by 2 is 12, as before. And a power equal to 3 ounces, placed at 4 inches distant from the prop would be the same; for 3 times 4 is 12; and so on in proportion.

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[2] For an account of what is called *Mechanical Arithmetic*, which is performed by moving weights on the arm of a lever, see Adam's Natural Philosophy, vol. iii, p. 276. The *arithmetical balance* was invented by Cassini, and is described in the *Journal des Sçavans*, 1676, p. 303, 12mo. By means of it, all the principal rules of arithmetic may be easily solved.—*Ed.*

[3] The following curious property of the balance is mentioned by Helsham. If a man placed in one scale, and counterpoised by a weight in the other, press the beam upwards, he will thus cause the scale in which he stands to preponderate.—*Ed.*

The *steelyard.*

The *statera*, or Roman *steelyard*, is a lever of this kind, and is used for finding the weights of different bodies by one single weight placed at different distances from the prop or centre of motion D. For, if a scale hangs at A, the extremity of the shorter arm $A\,B$ is of such a weight as will exactly counterpoise the longer arm $B\,C$; if this arm be divided into as many equal parts as it will contain, each equal to $A\,B$, the single weight P (which we may suppose to be 1 pound) will serve for weighing any thing as heavy as itself, or as many times heavier as there are divisions in the arm $B\,C$, or any quantity between its own weight and that quantity. As, for example, if P be 1 pound, and placed at the first division 1 in the arm $B\,C$, it will balance 1 pound in the scale at A; if it be removed to the second division at 2, it will balance 2 pounds in the scale; if to the third, 3 pounds; and so on to the end of the arm $B\,C$. If each of these integral divisions be subdivided into as many equal parts as a pound contains ounces, and the weight P be placed at any of these subdivisions, so as to counterpoise what is in the scale, the pounds and odd ounces therein will by that means be ascertained.

To this kind of lever may be reduced several sorts of instruments, such as scissars, pincers, snuffers, which are made of two levers, acting contrary to one another; their prop or centre of motion being the pin which keeps them together.

In common practice, the longer arm of this lever greatly exceeds the weight of the shorter, which gains great advantage, because it adds so much to the power.

The second kind of lever.

A lever of the second kind has the weight between the prop and the power. In this, as well as the former, the advantage gained is as the distance of the power from the prop to the distance of the weight from the prop: for the respective velocities of the power and weight are in that proportion; and they will balance each other when the intensity of the power multiplied by its distance from the prop is equal to the intensity of the weight multiplied by its distance from the prop. Thus, if *A B* (Fig. 2) be a lever on which the weight *W* of 6 ounces hangs at the distance of 1 inch from the prop *G*, and a power *P* equal to the weight of 1 ounce hangs at the end *B*, 6 inches from the prop, by the cord *C D* going over the fixed pulley *E*, the power will just support the weight: and a small addition to the power will raise the weight 1 inch for every 6 inches that the power descends.

This lever shews the reason why two men carrying a burthen upon a stick between them bear unequal shares of the burthen in the inverse proportion of their distances from it. For it is well known, that the nearer any of them is to the burthen, the greater share he bears of it; and if he goes directly under it he bears the whole. So, if one man be at *G*, and the other at *P*, having the pole or stick *A B* resting on their shoulders; if the burthen or weight *W* be placed five times as near the man at *G*, as it is to the man at *P*, the former will bear five times as much weight as the latter. This is likewise applicable to the case of two horses of unequal strength to be so yoked as that each horse may draw a part proportional to his strength; which is done by so dividing the beam they pull, that the point of traction may be as much nearer to the stronger horse than to the weaker, as the strength of the former exceeds that of the latter.

To this kind of lever may be reduced oars, rudders of ships, doors turning upon hinges, cutting-knives which are fixed at the point of the blade, and the like.

The third kind of lever.

If in this lever we suppose the power and weight to change places, so that the power may be between the weight and the prop, it will become a lever of the third kind; in which, that there may be a balance between the power and the weight, the intensity of the power must exceed the intensity of the weight, just as much as the distance of the weight from the prop exceeds the distances of the power from it. Thus, let *E* (Fig. 3) be the prop of the lever *A B*, and *W* a weight of 1 pound, placed 3 times as far from the prop as the power *P* acts at *F*, by the cord *C* going over the fixed pulley *D*; in this case, the power must be equal to 3 pounds, in order to support the weight.

To this sort of lever are generally referred the bones of a man's arm, and almost all the moveable bones of animals; for when we lift a weight by the hand, the muscle that exerts its force to raise that weight, is fixed to the bone about one tenth part as far below the elbow as the hand is: and the elbow being the centre round which the lower part of the arm turns, the muscle must therefore exert a force ten times as great as the weight that is raised.

As this kind of lever is a disadvantage to the moving power, it is never used but in cases of necessity, such as that of a ladder, which being fixed at one end, is by the strength of a man's arms reared against a wall; and in clock-work, where all the wheels may be reckoned levers of this kind, because the power that moves every wheel, except the first, acts upon it near the centre of motion, by means of a small pinion, and the resistance it has to overcome, acts against the teeth round its circumference.

The fourth kind of lever.

The fourth kind of lever differs nothing from the first, but in being bent, for the sake of convenience. *A C B* (Fig. 4) is a lever of this sort, bent at *C*, which is its prop, or centre of motion. *P* is a power acting upon the longer arm *A C* at *F*, by means of the cord *D E* going over the pulley *G*; and *W* is a weight or resistance acting upon the end *B* of the shorter arm *B C*. If the power is to the weight, as *C B* is to *C F*, they are *in equilibrio*. Thus, suppose *W* to be 5 pounds acting at the distance of one foot from the centre of motion *C*, and *P* to be 1 pound acting at *F*, five feet from the centre *C*, the power and weight will just balance each other. A hammer when used in drawing a nail is a lever of this sort.[4]

The *wheel and axle.*

2. The second mechanical power is the *wheel and axle*,[5] in which the power is applied to the circumference of the wheel, and the weight is raised by a rope, which coils about the axle as the wheel is turned round. Here it is plain that the velocity of the power must be to the velocity of the weight, as the circumference of the wheel is to the circumference of the axle: and, consequently, the power and weight will balance each other, when the intensity of the power is to the intensity of the weight, as the circumference of the axle is to the circumference of the wheel. Let *A B* (Fig. 5) be a wheel, *C D* its axle, and suppose the circumference of the wheel to be eight times as great as the circumference of the axle; then a power *P* equal to one pound hanging by the cord *I*, which goes round the wheel, will balance a weight *W* of eight pounds hanging by the rope *K*, which goes round the axle. And as the friction on the pivots or gudgeons of the axle is but small, a small addition to the power will cause it to descend, and raise the weight; but the weight will rise with only an eighth part of the velocity wherewith the power descends, and, consequently, through no more than an eighth part of an equal space, in the same time. If the wheel be pulled round by the handles *S*, *S*, the power will be increased in proportion to their length; and, by this means, any weight may be raised as high as the operator pleases.

To this sort of engine belong all cranes for raising great weights;[6] and in this case, the wheel may have cogs all round it instead of handles, and a small lantern or trundle may be made to work in the cogs, and be turned by a winch; which will make the power of the engine to exceed the power of the man who works it, as much as the number of revolutions of the winch exceed those of the axle *D*, when multiplied by the excess of the length of the winch above the length of the semi-diameter of the axle, added to the semi-diameter or half thickness of the rope *K*, by which the weight is drawn up. Thus, suppose the diameter of the rope and axle taken together, to be 13 inches, and, consequently, half their diameters to be 6½ inches; so that the weight *W* will hang at 6½ inches perpendicular distance from below the centre of the axle. Now, let us suppose the wheel *A B*, which is fixed on the axle, to have 80 cogs, and to be turned by means of a winch 6½ inches long, fixed on the axis of a trundle of 8 staves or rounds, working in the cogs of the wheel: here it is plain, that the winch and trundle would make 10 revolutions for one of the wheel *A B*, and its axis *D*, on which the rope *K* winds in raising the weight *W;* and the winch being no longer than the sum of the semi-diameters of the great axle and rope, the trundle could have no more power on the wheel than a man could have by pulling it round by the edge, because the winch would have no greater velocity than the edge of the wheel has, which we here suppose to be ten times as great as the velocity of the rising weight; so that, in this case, the power gained would be as 10 to 1. But if the length of the winch be 13 inches, the power gained will be as 20 to 1; if 19½ inches (which is long enough for any man to work by), the power gained would be as 30 to 1: that is, a man could raise 30 times as much by such an engine, as he could do by his natural strength without it; because the velocity of the handle of the winch would be 30 times as great as the velocity of the rising weight; the absolute force of any engine being in proportion of the velocity of the power to the velocity of the weight raised by it. But then, just as much power or advantage as is gained by the engine, so much time is lost in working it.[7] In this sort of machines it is requisite to have a ratchet-wheel *G* on one end of the axle, with a catch *H* to fall into its teeth, which will at any time support the weight, and keep it from descending, if the person who turns the handle should, through inadvertency or carelessness, quit his hold while the weight is raising; and, by this means, the danger is prevented which might otherwise happen by the running down of the weight when left at liberty.

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[4] Whatever be the form of the lever, or the direction of the power and the weight, the mechanical energy of the power or the weight is always represented by a line drawn from the *fulcrum*, at right angles, to the direction in which the forces are exerted. A new and beautiful property of the rectilineal lever is mentioned by Æpinus, in the *Nov. Comment. Petropol.* tom. viii, p. 271; but, as it cannot be illustrated without the aid of geometry, we must rest satisfied with referring the reader to the place where it is to be found. Van Swinden, *Positiones Physicæ*, p. 187, has extended this property to levers of all kinds.—*Ed.*

[5] Sometimes called the *axis in Peritrochio.* To this may be reduced all cranes for raising weights, the capstane, the windlass, and the crank.—*Ed.*

[6] The description of a simple and powerful capstane, which can be converted into a crane, will be found in the Appendix, vol. ii.—*Ed.*

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[7] This maxim, which has found its way into many treatises on mechanics, is certainly erroneous. If the impelling force of any machine is properly proportioned to the resistance to be overcome, the *work* performed increases nearly in the proportion of the power employed, when the resistance arises merely from the inertia of the work; but when the resistance is chiefly occasioned by other causes, the work performed increases nearly in the duplicate ratio (or as the square) of the power employed.—*Ed.*

The *pulley*.

3. The third mechanical power or engine consists either of one *moveable pulley*, or a *system of pulleys*, some in a block or case which is fixed, and others in a block which is moveable, and rises with the weight. For though a single pulley that only turns on its axis, and moves not out of its place, may serve to change the direction of the power, yet it can give no mechanical advantage thereto; but is only as the beam of a balance, whose arms are of equal length and weight. Thus, if the equal weights *W* and *P* (Fig. 6) hang by the cord *B B* upon the pulley *A*, whose frame *b* is fixed to the beam *H I*, they will counterpoise each other, just in the same manner as if the cord were cut in the middle, and its two ends hung upon the hooks fixed in the pulley at *A* and *A*, equally distant from its centre.

But if a weight *W* hangs at the lower end of the moveable block *p* of the pulley *D*, and the cord *G F* goes under that pulley, it is plain that the half *G* of the cord bears one half of the weight *W*, and the half *F* the other; for they bear the whole between them. Therefore, whatever holds the upper end of either rope, sustains one half of the weight; and if the cord at *F* be drawn up so as to raise the pulley *D* to *C*, the cord will then be extended to its whole length, all but that part which goes under the pulley: and consequently the power that draws the cord will have moved twice as far as the pulley *D* with its weight *W* rises; on which account, a power whose intensity is equal to one half of the weight will be able to support it, because if the power moves (by means of a small addition), its velocity will be double the velocity of the weight; as may be seen by putting the cord over the fixed pulley *C* (which only changes the direction of the power, without giving any advantage to it), and hanging on the weight *P*, which is equal only to one half the weight *W*; in which case there will be an equilibrium, and a little addition to *P* will cause it to descend, and raise *W* through a space equal to one half of that through which *P* descends. Hence, the advantage gained will be always equal to twice the number of pulleys in the moveable or undermost block. So that when the upper or fixed block *u* contains two pulleys, which only turn on their axes, and the lower or moveable block *U* contains two pulleys, which not only turn upon their axes, but also rise with the block and weight; the advantage gained by this is as 4 to the working power. Thus, if one end of the rope *K M O Q* be fixed to a hook at *I*, and the rope passes over the pulleys *N* and *R*, and under the pulleys *L* and *P*, and has a weight *T*, of one pound, hung to its other end at *T*, this weight will balance and support a weight *W* of four pounds, hanging by a hook at the moveable block *U*, allowing the said block as a part of the weight: and if as much more power be added, as is sufficient to overcome the friction of the pulleys, the power will descend with four times as much velocity as the weight rises, and consequently through four times as much space.

The two pulleys in the fixed block *X*, and the two in the moveable block *Y*, are in the same case with those last mentioned; and those in the lower block give the same advantage to the power.[8]

As a system of pulleys has no great weight, and lies in a small compass, it is easily carried about, and can be applied in a great many cases, for raising weights, where other engines cannot. But they have a great deal of friction on three accounts: 1. Because the diameters of their axes bear a very considerable proportion to their own diameters;[9] 2. Because in working they are apt to rub against one another,[1] or against the sides of the block; 3. Because of the stiffness of the rope that goes over and under them.

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[8] A very curious and useful combination of pulleys is described by Mr. Smeaton, in the xlviith volume of the *Philosophical Transactions*. The model which he shewed to the society consisted of twenty shieves, five on each pin. With this model, which could be carried in the pocket, he raised six hundred weight; and if a larger tackle of the same kind was properly executed, a ton might easily be raised by one man.—*Ed.*

[9] An ingenious method of removing the friction of the pulley upon its axis, was lately invented by Mr. John Gamett. This method consists in interposing six or more cylindrical rollers between the axis and the internal cavity of the pulley. The extremities of the pulley's axis are fixed as usual in a block, and the ends of the axes of the rollers turn in large holes, made in flat rings of brass or iron. This method has been adopted on board of ships; and it has been found that *three* men are able to draw as much weight by a set of pulleys of that construction as *five* men are able to raise by a similar set of common pulleys.—*Ed.*

[1] A considerable improvement in the construction of the pulley has been made by

The *inclined plane.*

Plate II. Fig. 7.

4. The fourth mechanical power is the *inclined plane,* and the advantage gained by it is as great as its length exceeds its perpendicular height. Let *A B* be a plane parallel to the horizon, and *C D* a plane inclined to it; and suppose the whole length *C D* to be three times as great as the perpendicular height *G f F*: in this case, the cylinder *E* will be supported upon the plane *C D*, and kept from rolling down upon it, by a power equal to a third part of the weight of the cylinder. Therefore, a weight may be rolled up this inclined plane with a third part of the power which would be sufficient to draw it up by the side of an upright wall. If the plane was four times as long as high, a fourth part of the power would be sufficient; and so on, in proportion. Or, if a weight was to be raised from a floor to the height *G F*, by means of the machine *A B C D* (which would then act as a half wedge, where the resistance gives way only on one side), the machine and weight would be *in equilibrio* when the power applied at *G F* was to the weight to be raised, as *G F* to *G B*; and if the power be increased, so as to overcome the friction of the machine against the floor and weight, the machine will be driven, and the weight raised; and when the machine has moved its whole length upon the floor, the weight will be raised to the whole height from *G* to *F*.

The force wherewith a rolling body descends upon an inclined plane, is to the force of its absolute gravity, by which it would descend perpendicularly in a free space, as the height of the plane is to its length: for, suppose the plane *A B* (Fig. 8) to be parallel to the horizon, the cylinder *C* will keep at rest upon any part of the plane where it is laid. If the plane be so elevated, that its perpendicular height *D* (Fig. 9) is equal to half its length *A B*, the cylinder will roll down upon the plane with a force equal to half its weight; for it would require a power (acting in the direction of *A B*), equal to half its weight, to keep it from rolling. If the plane *A B* (Fig. 10)

Mr. J. White, for which he has received a patent. The apparatus consists of two pulleys, one fixed, and the other moveable, each of which has six concentric grooves, of different diameters, capable of receiving a rope round them, that they may act in the same manner as separate pulleys, whose diameters are equal to those of the grooves. By this construction, lateral friction is destroyed, and that species of shaking motion removed with which the common pulley is attended.—*Ed.*

be elevated, so as to be perpendicular to the horizon, the cylinder *C* will descend with its whole force of gravity, because the plane contributes nothing to its support or hindrance; and therefore, it would require a power equal to its whole weight to keep it from descending.

Let the cylinder *C* (Fig. 11) be made to turn upon slender pivots in the frame *D*, in which there is a hook *e*, with a line *G* tied to it: let this line go over the fixed pulley *H*, and have its other end tied to the hook in the weight *I*. If the weight of the body *I*, be to the weight of the cylinder *C*, added to that of its frame *D*, as the perpendicular height of the plane *L M* is to its length *A B*, the weight will just support the cylinder upon the plane, and a small touch of a finger will either cause it to ascend or descend with equal ease: then, if a little addition be made to the weight *I*, it will descend, and draw the cylinder up the plane. In the time that the cylinder moves from *A* to *B*, it will rise through the whole height of the plane *M L;* and the weight will descend from *H* to *K*, through a space equal to the whole length of the plane *A B*.

If the machine be made to move upon rollers or friction-wheels, and the cylinder be supported upon the plane *C B* by a line *G* parallel to the plane, a power somewhat less than that which drew the cylinder up the plane will draw the plane under the cylinder, provided the pivots of the axes of the friction-wheels be small, and the wheels themselves be pretty large. For, let the machine *A B C*, Fig. 12 (equal in length and height to *A B M*, Fig. 11), move upon four wheels, two whereof appear at *D* and *E ;* and the third under *C*, while the fourth is hid from sight by the horizontal board *a*. Let the cylinder *F* be laid upon the lower end of the inclined plane *C B*, and the line *G* be extended from the frame of the cylinder about six feet, parallel to the plane *C B ;* and, in that direction, fixed to a hook in the wall; which will support the cylinder, and keep it from rolling off the plane. Let one end of the line *H* be tied to a hook at *C* in the machine, and the other end to a weight *K*, somewhat less than that which drew the cylinder up the plane before. If this line be put over the fixed pulley *I*, the weight *K* will draw the machine along the horizontal plane *L*, and under the cylinder *F ;* and when the machine has been drawn a little more than the whole length *C A*, the cylinder will be raised to *d*, equal to the perpendicular height *A B* above the horizontal part at *A*. The reason why the machine must be drawn further than the whole length *C A* is, because the weight *F* rises perpendicular to *C B*.

To the inclined plane may be reduced all hatchets, chisels, and other edge-tools which are chamfered only on one side.

The *wedge*.

5. The fifth mechanical power or machine is the *wedge*, which may be considered as two equally inclined planes *D E F* and *C E F* (Fig. 14), joined together at their bases *e E F O*: then *D C* is the whole thickness of the wedge at its back *A B C D*, where the power is applied; *E F* is the depth or height of the wedge; *D F* the length of one of its sides, equal to *C F* the length of the other side; and *O F* is its sharp edge, which is entered into the wood intended to be split by the force of a hammer or mallet striking perpendicularly on its back. Thus *A B b* (Fig. 15) is a wedge driven into the cleft *C D E* of the wood *F G*.

When the wood does not cleave at any distance before the wedge, there will be an equilibrium between the power impelling the wedge downward, and the resistance of the wood acting against the two sides of the wedge when the power is to the resistance as half the thickness of the wedge at its back is to the length of either of its sides; because the resistance then acts perpendicular to the sides of the wedge. But, when the resistance on each side acts parallel to the back, the power that balances the resistances on both sides will be as the length of the whole back of the wedge is to double its perpendicular height.

When the wood cleaves at any distance before the wedge (as it generally does), the power impelling the wedge will not be to the resistance of the wood, as the length of the back of the wedge is to the length of both its sides; but as half the length of the back is to the length of either side of the cleft, estimated from the top or acting part of the wedge. For, if we suppose the wedge to be lengthened down from *b* to the bottom of the cleft at *E*, the same proportion will hold, namely, that the power will be to the resistance as half the length of the back of the wedge is to the length of either of its sides; or, which amounts to the same thing, as the whole length of the back is to the length of both the sides.

In order to prove what is here advanced concerning the wedge, let us suppose the wedge to be divided lengthwise into two equal parts; and then it will become two equal inclined planes; one of which, as *a b c* (Fig. 13), may be made use of as a half wedge for separating the moulding *c d* from the wainscot *A B*. It is evident, that when this half wedge has been driven its whole length *a c* between the wainscot and moulding, its side *a c* will be at *e d*, and the moulding will be separated to *f g* from the wainscot. Now, from what has been already proved of the inclined plane, it appears, that to have an equilibrium between the power impelling the half wedge, and the resistance of the moulding, the former must be to the latter, as *a b* to *a c*; that is, as the thickness of the back which receives the stroke is to the length of the side against which the moulding acts. Therefore, since the power upon the half wedge is to the resistance against its side, as the half back *a b* is to the whole side *a c*, it is plain, that the power upon which the whole wedge (where the whole back is double the half back), must be to the resistance against both its sides, as the thickness of the whole back is to the length of both the sides, supposing the wedge at the bottom of the cleft; or as the thickness of the whole back to the length of both sides of the cleft, when the wood splits at any distance before the wedge. For, when the wedge is driven quite into the wood, and the wood splits at ever so small a distance before its edge, the top of the wedge then becomes the acting part, because the wood does not touch it anywhere else: and since the bottom of the cleft must be considered as that part where the whole stickage or resistance is accumulated, it is plain, from the nature of the lever, that the farther the power acts from the resistance, the greater is the advantage.

Some writers have advanced, that the power of the wedge is to the resistance to be overcome, as the thickness of the back of the wedge is to the length only of one of its sides, which seems very strange;[2] for, if we suppose *A B* (Fig. 16) to be a strong inflexible bar of wood or iron fixed into the ground at *C B*, and *D* and *E* to be two blocks of marble lying on the ground on opposite sides of the bar; it is evident that the block *D* may be separated from the bar to the distance *d*, equal to *a b*, by driving the inclined plane or half wedge *a b o* down between them; and the block *E* may be separated to an equal distance on the other side, in like manner, by the half wedge *c d o*. But the power impelling each half wedge will be to the resistance of the block against its side, as the thickness of that half wedge is to its perpendicular height, because the block will be driven off perpendicularly to the side of the bar *A B*. Therefore the power to drive both the half wedges is to both the resistances, as both the half backs is to the perpendicular height of each half wedge. And if the bar be taken away, the blocks put close together, and the two half wedges joined to make one; it will require as much force to drive it down between the blocks, as is equal to the sum of the separate powers acting upon the half wedges when the bar was between them.

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[2] If two bodies are forced from one another by means of a wedge, and in a direction parallel to its back, the power will be to the resistance as half the back or head of the wedge is to its length. But if one of the bodies only is moveable, while the other from which it is separated retains its place, the power will be to the resistance as the back is to the length of the wedge. The controversies about the power of the wedge would never have been agitated, had this simple distinction been attended to.—*Ed.*

To confirm this by an experiment, let two cylinders, as *A B* and *C D* (Fig. 17), be drawn towards one another by lines running over fixed pulleys, and a weight of 40 ounces hanging at the lines belonging to each cylinder; and let a wedge of 40 ounces weight, having its back just as thick as either of its sides is long, be put between the cylinders, which will then act against each side with a resistance equal to 40 ounces, while its own weight endeavours to bring it down and separate them. And here, the power of the wedge's gravity impelling it downward, will be to the resistance of both the cylinders against the wedge, as the thickness of the wedge is to double its perpendicular height; for there will then be an equilibrium between the weight of the wedge and the resistance of the cylinders against it, and it will remain at any height between them; requiring just as much power to push it upward as to pull it downward. If another wedge of equal weight and depth with this, and only half as thick, be put between the cylinders, it will require twice as much weight to be hung at the ends of the lines which draw them together, to keep the wedge from going down between them: that is, a wedge of 40 ounces, whose back is only equal to half its perpendicular height, will require 80 ounces to each cylinder, to keep it in an equilibrium between them; and twice 80 is 160, equal to four times 40.

So that the power will be always to the resistance, as the thickness of the back of the wedge is to twice its perpendicular height, when the cylinders move off in a line at right angles to that perpendicular.

The best way, though perhaps not the neatest, that I know of, for making a wedge with its appurtenances for such experiments, is as follows: Let *K I L M* and *L M N O* (Fig. 17), be two flat pieces of wood, each about fifteen inches long, and three or four in breadth, joined together by a hinge at *L M*; and let *P* be a graduated arch of brass, on which the said pieces of wood may be opened to any angle, not more than 60 degrees, and then fixed at the given angle by means of the two screws *a* and *b*. Then, *I K N O* will represent the back of the wedge, *L M* its sharp edge which enters the wood, and the outsides of the pieces *K I L M* and *L M N O* the two sides of the wedge against which the wood acts in cleaving. By means of the said arch, the wedge may be opened so as to adjust the thickness of its back in any proportion to the length of either of its sides, but not to exceed that length; and any weight as *p* may be hung to the wedge upon the hook *M*, which weight, together with the weight of the wedge itself, may be considered as the impelling power; which is all the same in the experiment, whether it be laid upon the back of the wedge to push it down, or hung to its edge to pull it down. Let *A B* and *C D* be two wooden cylinders, each about two inches thick, where they touch the outsides of the wedge; and let their ends be made like two round flat plates, to keep the wedge from slipping off edgewise from between them. Let a small cord, with a loop on one end of it, go over a pivot in the end of each cylinder, and the cords *S* and *T* belonging to the cylinder *A B* go over the fixed pulleys *W* and *X*, and be fastened at their other ends to the bar *w x*, on which any weight as *Z* may be hung at pleasure. In like manner, let the cords *Q* and *R* belonging to the cylinder *C D* go over the fixed pulleys *V* and *U* to the bar *v u*, on which a weight *Y* equal to *Z* may be hung. These weights, by drawing the cylinders toward one another, may be considered as the resistance of the wood acting equally against opposite sides of the wedge; the cylinders themselves being suspended near, and parallel to each other, by their pivots and loops on the lines *E*, *F*, *G*, *H*; which lines may be fixed to hooks in the ceiling of the room. The longer these lines are, the better; and they should never be less than four feet each. The farther also the pulleys *V*, *U*, and *X*, *W*, are from the cylinders, the truer will the experiments be; and they may turn upon pins fixed into the wall.

In this machine, the weights *Y* and *Z*, and the weight *p*, may be varied at pleasure, so as to be adjusted in proportion of double the wedge's perpendicular height to the thickness of its back; and when they are so adjusted, the wedge will be *in equilibrio* with the resistance of the cylinders.

The wedge is a very great mechanical power, since not only wood but even rocks can be split by it; which would be impossible to effect by the lever, wheel and axle, or pulley; for the force of the blow, or stroke, shakes the cohering parts, and thereby makes them separate more easily.

The *screw*.

6. The sixth and last mechanical power is the *screw*, which cannot properly be called a simple machine, because it is never used without the application of a lever or winch to assist in turning it; and then it becomes a compound engine of a very great force either in pressing the parts of bodies closer together, or in raising great weights. It may be conceived to be made by cutting a piece of paper *A B C* (Fig. 18) into the form of an inclined plane or half wedge, and then wrapping it round a cylinder *A B* (Fig. 19). And here it is evident, that the winch *E* must turn the cylinder once round before the weight of resistance *D* can be moved from one spiral winding to another, as from *d* to *c*; therefore, as much as the circumference of a circle described by the handle of the winch, is greater than the interval or distance between the spirals, so much is the force of the screw. Thus, supposing the distance between the spirals to be half an inch, and the length of the winch to be twelve inches, the circle described by the handle of the winch where the power acts will be 76 inches nearly, or about 152 half inches, and consequently 152 times as great as the distance between the spirals; and therefore a power at the handle, whose intensity is equal to no more than a single pound, will balance 152 pounds acting against the screw; and as much additional force, as is sufficient to overcome the friction, will raise the 152 pounds; and the velocity of the power will be to the velocity of the weight, as 152 to 1. Hence it appears, that

the longer the winch is, and the nearer the spirals are to one another, so much the greater is the force of the screw.[3]

A machine for shewing the force or power of the screw may be contrived in the following manner:—Let the wheel *C* (Fig. 20) have a screw *a b* on its axle, working in the teeth of the wheel *D*, which suppose to be 48 in number. It is plain, that for every time the wheel *C* and screw *a b* are turned round by the winch *A*, the wheel *D* will be moved one tooth by the screw; and therefore, in 48 revolutions of the winch, the wheel *D* will be turned once round. Then, if the circumference of a circle described by the handle of the winch *A* be equal to the circumference of a groove *e* round the wheel *D*, the velocity of the handle will be 48 times as great as the velocity of any given point in the groove. Consequently if a line *G* goes round the groove *e*, and has a weight of 48 pounds hung to it below the pedestal *E F*, a power equal to one pound at the handle will balance and support the weight. To prove this by experiment, let the circumferences of the grooves of the wheels *C* and *D* be equal to one another; and then if a weight *H* of one pound be suspended by a line going round the groove of the wheel *C*, it will balance a weight of 48 pounds hanging by the line *G*; and a small addition to the weight *H* will cause it to descend, and so raise up the other weight.

If the line *G*, instead of going round the groove *e* of the wheel *D*, goes round its axle *I*, the power of the machine will be as much increased as the circumference of the groove *e* exceeds the circumference of the axle; which, supposing it to be six times, then one pound at *H* will balance 6 times 48, or 288 pounds hung to the line on the axle: and hence the power or advantage of this machine will be as 288 to 1; that is to say, a man, who by his natural strength could lift a hundred weight, will be able to raise 288 hundred, or $14\frac{8}{20}$ ton weight by this engine.

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[3] A new and very ingenious method of applying the screw, so as to make it act with the greatest accuracy, has been invented by Mr. Hunter, surgeon. The apparatus consists of two screws, one of which moves in the other, and has a thread fewer in an inch than the male screw of the one in which it moves. If we suppose the one screw *A* to have 20 threads in an inch, and the other *B* to have 21; and if the screw *B* is so fixed that it can move forwards without moving round; then one turn of *A* will make the screw *B* advance $\frac{1}{21} \times \frac{1}{20}$ or $\frac{1}{420}$ of an inch. Thus the compound screw produces an effect much superior to the common one; for the latter must have 420 threads in an inch before it could produce an effect equal to the former, which would weaken it to such a degree, that it would be unable to resist any considerable force. As it is impossible in the short compass of a note to convey an accurate account of this excellent improvement, we must refer the reader to the 71st vol. of the Philosophical Transactions, or the *Edinburgh Encyclopædia*, Art. *Mechanics*, vol. xiii, p. 531.—*Ed.*

Plate III. Fig. 1.

A combination of all the mechanical powers.

But the following engine is still more powerful, on account of its having the addition of four pulleys: and in it we may look upon all the mechanical powers as combined together, even if we take in the balance. For, as the axle *D* of the bar *A B* enters its middle at *C*, it is plain that if equal weights are suspended upon any two pins equidistant from the axis *C*, they will counterpoise each other. It becomes a lever by hanging a small weight *P* upon the pin *n*, and a weight as much heavier upon either of the pins *b*, *c*, *d*, *e*, or *f*, as is in proportion to the pins being so much nearer the axis. The wheel and axle *F G* is evident; so is the screw *E*, which takes in the inclined plane, and with it the half wedge. Part of a cord goes round the axle, the rest under the lower pulleys *K*, *m*, over the upper pulleys *L*, *n*, and then it is tied to a hook at *m* in the lower or moveable block on which the weight *W* hangs.

In this machine, if the wheel *F* has 30 teeth, it will be turned once round in thirty revolutions of the bar *A B*, which is fixed on the axis *D* of the screw *E*: if the length of the bar is equal to twice the diameter of the wheel, the pins *a* and *n* at the ends of the bar will move 60 times as fast as the teeth of the wheel do; and, consequently, one ounce at *P* will balance 60 ounces hung upon a tooth at *q* in the horizontal diameter of the wheel. Then, if the diameter of the wheel *F* is ten times as great as the diameter of the axle *G*, the wheel will have 10 times the velocity of the axle; and therefore one ounce *P* at the end of the lever *A C* will balance 10 times 60 or 600 ounces hung to the rope *H* which goes round the axle. Lastly, if four pulleys be added, they will make the velocity of the lower block *K*, and weight *W*, four times less than the velocity of the axle: and this being the last power in the machine, which is four times as great as that gained by the axle, it makes the whole power of the machine 4 times 600, or 2,400. So that a man who could lift one hundred weight in his arms by his natural strength, would be able to raise 2,400 times as much by this engine. But it is here as in all other mechanical cases; for the time lost is always as much as the power gained, because the velocity with which the power moves will ever exceed the velocity with which the weight rises, as much as the intensity of the weight exceeds the intensity of the power.

The friction of the screw itself is very considerable; and there are few compound engines but what, upon account of the friction of the parts against one another, will require a third part more of power to work them when loaded, than what is sufficient to constitute a balance between the weight and the power.[4]

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[4] The following valuable Tables on the strength of different materials are taken from Mr. Thomas Tredgold's excellent work on Carpentry, recently published.

Cohesive Force of a Square Inch of different Woods, pulled in the Direction of their Lengths.

	Cohesion of a Square In. in Pounds.	Experimentalists.		Cohesion of a Square In. in Pounds.	Experimentalists.
Oak,	17.300	Muschenbroek.	Ash,	6.070	Emerson.
Do.	13.950	Rondelet.	Elm,	13.489	Muschenbroek.
Do. dry English, from	12.000	Barlow.	Do.	6.070	Emerson.
to	8.889		Do.	4.455	Anderson.
Do.	7.850	Anderson.	Acacia,	20.582	Muschenbroek.
Do.	6.114	Emerson.	Mahogany,	8.000	Barlow.
Beech,	17.709	Muschenbroek.	Walnut,	8.130	Muschenbroek.
Do.	11.500	Barlow.	Do.	5.360	Emerson.
Do.	6.300	Anderson.	Teak,	15.000	Barlow.
Do.	6.070	Emerson.	Poplar, from	6.641	Muschenbr.
Alder,	14.186	Muschenbroek.	to	4.596	
Do.	5.094	Anderson.	Fir, from	13.448	Barlow.
Do.	4.290	Emerson.	to	11.000	
Sycamore,	5.000	Emerson.	Do.	8.506	Muschenbroek.
Chesnut, Spanish,	13.300	Rondelet.	Do.	5.000	Emerson.
Ash, from	17.850	Barlow.	Pitch pine,	7.818	Muschenbroek.
to	15.784		Norway pine,	7.287	Rondelet.
Do.	12.000	Muschenbroek.	Larch,	10.220	Rondelet.
Do.	6.300	Anderson.	Cedar,	4.973	Muschenbroek.

Cohesive Force of a Square Inch, pulled asunder in a Direction perpendicular to the Length of the Fibres.

	Pounds.	
Oak,	2.316	Mr. Tredgold.
Poplar,	1.782	
Larch, from	970	
to	1.700	

Cohesive Force of a Square Inch of Iron.

	Pounds.	Experimentalists.		Pounds.	Experimentalists.
Iron wire,	113.077	Sickingen.	English iron,	61.600	Telford.
Do.	93.964	Telford.	Do.	55.772	Rennie.
Swedish iron,	78.850	Muschenbrock.	Welsh iron,	64.960	Telford.
Do.	72.064	Rennie.	Do.	55.776	Brown.
Do.	64.960	Telford.	French iron,	61.041	Perronet.
Do.	53.244	Brown.	Russian iron,	59.472	Brown.
German iron,	69.133	Muschenbrock.	Cast iron,	68.295	Muschenbrock.
English iron, from	66.000	Rumford.	Do.	19.488	Rennie.
to	55.000		Do. Welsh,	16.255	Brown.

LECTURE IV.

OF MILLS, CRANES, WHEEL-CARRIAGES, AND THE ENGINE FOR DRIVING PILES.

As these engines are so universally useful, it would be needless to make any apology for describing them.

Plate III. Fig. 2. *A common mill.*

In a common *breast mill,*[1] where the fall of water may be about ten feet, *A A* is the great wheel, which is generally about 17 or 18 feet in diameter, reckoned from the outermost edge of any float-board at *a* to that of its opposite float at *b*. To this wheel the water is conveyed through a channel, and, by falling upon the float-boards, turns it round.

On the axis *B B* of this wheel, and within the mill-house, is a wheel *D*, about 8 or 9 feet diameter, having 61 cogs,[2] which turn a trundle *E*, containing ten upright staves or rounds;[3] and when these are the number of cogs and rounds, the trundle will make $6\frac{1}{10}$ revolutions for one revolution of the wheel.

The trundle is fixed upon a strong iron axis called the

Cohesive Force of different Metals compared with Iron, according to Muschenbroek.

	Pounds.		Pounds.
Iron bar, Swedish, -	97.7	Tin, grain, - - -	7.5
Copper, Swedish, - -	43.0	—— block, - - -	4.4
——— Anglesea, - -	39.5	—— English block, -	6.0
——— Japan, - -	22	Zinc, - - - - -	3
Silver, cast, - - -	48.3	Lead, - - -	1
Gold, cast, - -	25.6		

Emerson makes the relative strengths of iron, brass, and lead, as 107, 50, and $6\frac{1}{2}$.—*Ed.*

[1] Water mills are divided into *breast mills*, *undershot mills*, and *overshot mills*, a particular account of which will be found in the Appendix, Vol. II.

[2] The wooden teeth of a large wheel are denominated *cogs.*—*Ed.*

[3] When a large wheel gives motion to a smaller one, the name of the small wheel varies with its shape. When the small wheel is solid and oblong, and its teeth longer than their distance from the axis, it is called a *pinion*, and its teeth are named *leaves*. When the small wheel is shaped like *E* (Fig. 2, Plate III), it is called a *trundle*, sometimes a *lantern*, and sometimes a *drum;* and the cylindrical bars of trundles similar to those at *E* are called *staves* or *rounds*. In a combination of wheels, that which is acted upon by the power, or by some other wheel, is called a *leader;* and the other wheel on the same axis is called a *follower*. When there is only one wheel on the axis, and when this wheel drives another, it acts both as a *leader* and a *follower.*—*Ed.*

spindle, the lower end of which turns in a brass foot, fixed at *F*,[4] in the horizontal beam *S T* called the bridge-tree; and the upper part of the spindle turns in a wooden bush fixed into the nether millstone, which lies upon beams in the floor *Y Y*. The top part of the spindle above the bush is square, and goes into a square hole in a strong iron cross *a b c d* (see Fig. 3), called the rynd; under which, and close to the bush, is a round piece of thick leather upon the spindle, which it turns round at the same time as it does the rynd.

The rynd is let into grooves in the under surface of the running millstone *G* (Fig. 2), and so turns it round in the same time that the trundle *E* is turned round by the cog-wheel *D*. This millstone has a large hole quite through its middle, called the eye of the stone, through which the middle part of the rynd and upper end of the spindle may be seen, while the four ends of the rynd lie hid below the stone in their grooves.

The end *T* of the bridge-tree *T S* (which supports the upper millstone *G* upon the spindle) is fixed into a hole in the wall; and the end *S* is let into a beam *Q R* called the brayer, whose end *R* remains fixed in a mortise; and its other end *Q* hangs by a strong iron rod *P* which goes through the floor *Y Y*, and has a screw-nut on its top at *O*; by the turning of which nut, the end *Q* of the brayer is raised or depressed at pleasure; and, consequently, the bridge-tree *T S* and upper millstone. By this means, the upper millstone may be set as close to the under one, or raised as high from it, as the miller pleases. The nearer the millstones are to one another, the finer they grind the corn, and the more remote from one another, the coarser.[5]

The upper millstone *G* is inclosed in a round box *H*, which does not touch it anywhere; and is distant about an inch from its edge all around. On the top of this box stands a frame for holding the hopper *k k*, to which is hung the shoe *I* by two lines fastened to the hind part of it, fixed upon hooks in the hopper, and by one end of the crook-string *K* fastened to the fore part of it at *i*; the other end being twisted round the pin *L*. As the pin is turned one way, the string draws up the shoe closer to the hopper, and so lessens the aperture between them; and, as the pin is turned the other way, it lets down the shoe, and enlarges the aperture.

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[4] The extremities of an axle or spindle, such as *F* and *C* (Fig. 2, Plate III), are called *gudgeons* when the wheels are large, and *pivots* in small pieces of machinery.—*Ed.*

[5] As a good quality of flour cannot be obtained unless the under millstone is kept perfectly level and firm on its bearings, Mr. T. Austin has contrived the following method of effecting this. The millstone is fixed in a strong cast-iron frame, bedded in plaster of Paris, and this frame is adjusted by three vertical screws, on the points of which it rests, and by four horizontal screws. See the *Transactions of the Society of Arts*, vol. xxxviii, p. 68.

If the shoe be drawn up quite to the hopper, no corn can fall from the hopper into the mill; if it be let a little down, some will fall; and the quantity will be more or less, according as the shoe is more or less let down. For the hopper is open at bottom, and there is a hole in the bottom of the shoe, not directly under the bottom of the hopper, but forwarder toward the end *i*, over the middle of the eye of the millstone.

There is a square hole in the top of the spindle, in which is put the feeder *e* (Fig 3): this feeder (as the spindle turns round) jogs the shoe three times in each revolution, and so causes the corn to run constantly down from the hopper through the shoe, into the eye of the millstone, where it falls upon the top of the rynd, and is, by the motion of the rynd, and the leather under it, thrown below the upper stone, and ground between it and the lower one. The violent motion of the stone creates a centrifugal force in the corn going round with it, by which means it gets farther and farther from the centre, as in a spiral, in every revolution, until it be thrown quite out; and, being then ground, it falls through a spout *M*, called the mill-eye, into the trough *N*.

When the mill is fed too fast, the corn bears up the stone, and is ground too coarse; and, besides, it clogs the mill, so as to make it go too slow. When the mill is too slowly fed, it goes too fast, and the stones, by their attrition, are apt to strike fire against one another. Both which inconveniencies are avoided by turning the pin *L* backward or forward, which draws up or lets down the shoe; and so regulates the feeding as the miller sees convenient.

The heavier the running millstone is, and the greater the quantity of water that falls upon the wheel, so much the faster will the mill bear to be fed; and, consequently, so much the more it will grind: and, on the contrary, the lighter the stone, and the less the quantity of water, so much slower must the feeding be. But when the stone is considerably worn, and become light, the mill must be fed slowly at any rate; otherwise the stone will be too much borne up by the corn under it, which will make the meal coarse.

The quantity of power required to turn a heavy millstone is but very little more than what is sufficient to turn a light one: for as it is supported upon the spindle by the bridge-tree *S T*, and the end of the spindle that turns in the brass foot therein being but small, the odds arising from the weight is but very inconsiderable in its action against the power or force of the water: and, besides, a heavy stone has the same advantage as a heavy fly; namely, that it regulates the motion much better than a light one.

In order to cut and grind the corn, both the upper and under millstones have channels or furrows cut into them, proceeding obliquely from the centre towards the circumference: and these furrows are cut perpendicularly on one side, and obliquely on the other, into the stone, which gives each furrow a sharp edge, and in the two stones they come, as it were, against one another like the edges of a pair of scissars; and so cut the corn, to make it grind the easier when it falls upon the places between the furrows. These are cut the same way in both stones when they lie upon their backs, which makes them run cross ways to each other when the upper stone is inverted by turning its furrowed surface toward that of the lower. For, if the furrows of both stones lay the same way, a great deal of the corn would be driven onward in the lower furrows, and so come out from between the stones without being either cut or bruised.*

When the furrows become blunt and shallow by wearing, the running stone must be taken up, and both stones new dressed with a chisel and hammer; and every time the stone is taken up, there must be some tallow put round the spindle upon the bush, which will soon be melted by the heat that the spindle acquires from its turning and rubbing against the bush, and so will get in between them, otherwise the bush would take fire in a very little time.

The bush must embrace the spindle quite close, to prevent any shake in the motion, which would make some parts of the stones grate and fire against each other, while other parts of them would be too far asunder, and by that means spoil the meal in grinding.

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* See the *Appendix*, Vol. II.

Whenever the spindle wears the bush so as to begin to shake in it, the stone must be taken up, and a chisel driven into several parts of the bush; and when it is taken out, wooden wedges must be driven into the holes; by which means the bush will be made to embrace the spindle close all around it again. In doing this, great care must be taken to drive equal wedges into the bush on opposite sides of the spindle, otherwise it will be thrown out of the perpendicular, and so hinder the upper stone from being set parallel to the under one, which is absolutely necessary for making good work. When any accident of this kind happens, the perpendicular position of the spindle must be restored by adjusting the bridge-tree *S T* by proper wedges put between it and the brayer *Q R*.

It often happens that the rynd is a little wrenched in laying down the upper stone upon it, or is made to sink a little lower upon one side of the spindle than on the other; and this will cause one edge of the upper stone to drag all around upon the other, while the opposite edge will not touch. But this is easily set to rights, by raising the stone a little with a lever, and putting bits of paper, cards, or thin chips, between the rynd and the stone.

The diameter of the upper stone is generally about six feet, the lower stone about an inch more; and the upper stone, when new, contains about 22½ cubic feet, which weighs somewhat more than 19,000 pounds. A stone of this diameter ought never to go more than 60 times round in a minute; for, if it turns faster, it will heat the meal.

The grinding surface of the under stone is a little convex from the edge to the centre, and that of the upper stone a little more concave: so that they are farthest from one another in the middle, and come gradually nearer toward the edges. By this means, the corn at its first entrance between the stones is only bruised; but as it goes farther on towards the circumference or edge, it is cut smaller and smaller; but at last finely ground just before it comes out from between them.

The water wheel must not be too large, for, if it be, its motion will be too slow; nor too little, for then it will want power. And for a mill to be in perfection, the floats of the wheel ought to move with a third part of the velocity of the water, and the stone to turn round once in a second of time.

In order to construct a mill in this perfect manner, observe the following rules :—

1. Measure the perpendicular height of the fall of water, in feet, above that part of the wheel on which the water begins to act; and call that the height of the fall.

2. Multiply this constant number 64,2882 by the height of the fall in feet, and the square root of the product shall be the velocity of the water at the bottom of the fall, or the number of feet that the water there moves per second.

3. Divide the velocity of the water by 3, and the quotient shall be the velocity of the float-boards of the wheel, or the number of feet they must each go through in a second, when the water acts upon them, so as to have the greatest power to turn the mill.

4. Divide the circumference of the wheel in feet, by the velocity of its floats in feet per second, and the quotient shall be the number of seconds in which the wheel turns round.

5. By this last number of seconds divide 60; and the quotient shall be the number of turns of the wheel in a minute.

6. Divide 60 (the number of revolutions the millstone ought to have in a minute) by the number of turns of the wheel in a minute, and the quotient shall be the number of turns the millstone ought to have for one turn of the wheel.

7. Then, as the number of turns of the wheel in a minute is to the number of turns of the millstone in a minute, so must the number of staves in the trundle be to the number of cogs in the wheel, in the nearest whole numbers that can be found.

By these rules I have calculated the following Table to a water-wheel, 18 feet diameter, which I apprehend may be a good size in general.

To construct a mill by this table, find the height of the fall of water in the first column, and against that height, in the sixth column, you have the number of cogs in the wheel, and staves in the trundle, for causing the millstone to make about 60 revolutions in a minute, as near as possible, when the wheel goes with a third part of the velocity of the water; and it appears by the seventh column, that the number of cogs in the wheel, and staves in the trundle, are so near the truth for the required purpose, that the least number of revolutions of the millstone in a minute is between 59 and 60, and the greatest number never amounts to 61.

The Mill-Wright's Table.

Height of the fall of water.	Velocity of the water *per* second.	Velocity of the wheel *per* second.	Revolutions of the wheel *per* minute.	Revolutions of the mill-stone for one of the wheel.	Cogs in the wheel and staves in the trundle.		Revolutions of the mill-stone *per* minute by these staves & cogs.
Feet.	Feet. 100 parts of a foot.	Feet. 100 parts of a foot.	Rev. 100 parts of a rev.	Rev. 100 parts of a rev.	Cogs.	Staves.	Rev. 100 parts of a rev.
1	8.02	2.67	2.83	21.20	127	6	59.92
2	11.34	3.78	4.00	15.00	105	7	60.00
3	13.89	4.63	4.91	12.22	98	8	60.14
4	16.04	5.35	5.67	10.58	95	9	59.87
5	17.93	5.98	6.34	9.46	85	9	59.84
6	19.64	6.55	6.94	8.64	78	9	60.10
7	21.21	7.07	7.50	8.00	72	9	60.00
8	22.68	7.56	8.02	7.48	67	9	59.67
9	24.05	8.02	8.51	7.05	70	10	59.57
10	25.35	8.45	8.97	6.69	67	10	60.09
11	26.59	8.86	9.40	6.38	64	10	60.16
12	27.77	9.26	9.82	6.11	61	10	59.90
13	28.91	9.64	10.22	5.87	59	10	60.18
14	30.00	10.00	10.60	5.66	56	10	59.36
15	31.05	10.35	10.99	5.46	55	10	60.48
16	32.07	10.69	11.34	5.29	53	10	60.10
17	33.06	11.02	11.70	5.13	51	10	59.67
18	34.02	11.34	12.02	4.99	50	10	60.10
19	34.95	11.65	12.37	4.85	49	10	60.61
20	35.86	11.95	12.68	4.73	47	10	59.59
1	2	3	4	5	6		7

Such a mill as this, with a fall of water about 7½ feet, will require about 32 hogsheads every minute to turn the wheel with a third part of the velocity with which the water falls; and to overcome the resistance arising from the friction of the geers, and attrition of the stones in grinding the corn.

The greater fall the water has, the less quantity of it will serve to turn the mill. The water is kept up in the mill-dam, and let out by a sluice called the penstock, when the mill is to go. When the penstock is drawn up by means of a lever, it opens a passage through which the water flows to the wheel; and when the mill is to be stopt, the penstock is let down, which stops the water from falling upon the wheel.

A less quantity of water will turn an over-shot mill (where the wheel has buckets instead of float-boards) than a breast-mill, where the fall of the water seldom exceeds half the height *A b* of the wheel: so that, where there is but a small quantity of water, and a fall great enough for the wheel to lie under it, the bucket or overshot wheel is always used. But where there is a large body of water, with a little fall, the breast or float-board wheel must take place. Where the water runs only upon a little declivity, it can act but slowly upon the under part of the wheel at *b*; in which case, the motion of the wheel will be very slow: and therefore, the floats ought to be very long, though not high, that a large body of water may act upon them, so that what is wanting in velocity may be made up in power; and then the cog-wheel may have a greater number of cogs in proportion to the rounds in the trundle, in order to give the mill-stone a sufficient degree of velocity.

They who have read what is said in the first lecture, concerning the acceleration of bodies falling freely by the power of gravity acting constantly and uniformly upon them, may perhaps ask, why should the motion of the wheel be equable, and not accelerated, seeing the water acts constantly and uniformly upon it? The plain answer is, that the velocity of the wheel can never be so great as the velocity of the water that turns it; for if it should become so great, the power of the water would be quite lost upon the wheel, and then there would be no proper force to overcome the friction of the geers and attrition of the stones. Therefore, the velocity with which the wheel begins to move, will increase no longer than till its *momentum* or force is balanced by the resistance of the working parts of the mill: and then the wheel will go on with an equable motion.[6]

A *hand-mill.*

If the cog-wheel *D* be made about 18 inches diameter, with 30 cogs, the trundle as small in proportion, with 10 staves, and the millstones be each about two feet in diameter, and the whole work be put into a strong frame of wood, as represented in the figure, the engine will be a hand-mill for grinding corn or malt in private families: and then, it may be turned by a winch instead of the wheel *A A*; the millstone making three revolutions for every one of the winch. If a heavy fly be put upon the axle *B*, near the winch, it will help to regulate the motion.[7]

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[6] Our author's explanation of this remarkable fact, viz. that the best constructed machines acquire in a short time an uniform motion, is far from being satisfactory. The question, indeed, is extremely difficult; and from our imperfect knowledge of the nature of friction, it does not admit of a scientific explanation. When a pendulum clock is stripped of its pallets, and allowed to run down, it acquires an uniform motion in a very short time, though there is little friction, and though the moving power acts with the greatest uniformity. Dr. Robison observes, that the "uniform "motion of machines arises from a diminution of the impelling power by an in- "crease of velocity; but that there is something yet unexplained in the nature of "friction, which takes away some of the acceleration."—*Ed.*

If the cogs of the wheel and rounds of the trundle could be put in as exactly as the teeth are cut in the wheels and pinions of a clock, then the trundle might divide the wheel exactly; that is to say, the trundle might make a given number of revolutions for one of the wheel, without a friction. But as any exact number is not necessary in mill-work, and the cogs and rounds cannot be set in so truly as to make all the intervals between them equal, a skilful mill-wright will always give the wheel what he calls a *hunting cog*; that is, one more than what will answer to an exact division of the wheel by the trundle. And then, as every cog comes to the trundle, it will take the next staff or round, behind the one which it took in the former revolution; and by that means will wear all the parts of the cogs and rounds which work upon one another equally, and to equal distances from one another in a little time; and so make a true uniform motion throughout the whole work. Thus, in the above water-mill (Fig. 4), the trundle has 10 staves, and the wheel 61 cogs.

Plate III. Fig. 4.

Sometimes, where there is a sufficient quantity of water, the cog-wheel *A A* turns a large trundle *B B*, on whose axis *C* is fixed the horizontal wheel *D*, with cogs all around its edge, turning two trundles *E* and *F* at the same time; whose axes or spindles *G* and *H* turn two millstones *I* and *K*, upon the fixed stones *L* and *M*. And when there is not work for them both, either may be made to lie quiet, by taking out one of the staves of its trundle, and turning the vacant place toward the cog-wheel *D*: and there may be a wheel fixed on the upper end of the great upright axle *C* for turning a couple of boultingmills, and other work for drawing up the sacks, fanning and cleaning the corn, sharpening of tools, &c.

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[7] If the fly be put upon the axle *B*, its velocity will not be sufficient to make it of any use as a regulator. In an ingenious hand-mill, invented by Mr. Lloyd, this defect is remedied by fixing a hollow circular fly upon the upper millstone, with iron straps. Its diameter is two feet, the breadth of its rim three inches and a half, and its depth five inches. It is divided into six cavities, into which a quantity of lead shot is put to bring it to a proper weight. See Bailey's *Designs of Machines, approved by the Society of Arts*, vol. i, p. 176; and vol. ii, p. 44. In some hand-mills, the millstone moves in a vertical direction, like a common grindstone, without either a cog-wheel or trundle.—*Ed.*

A *horse-mill.*

If, instead of the cog-wheel *A A* and trundle *B B*, horizontal levers be fixed into the axle *C*, below the wheel *D*, then horses may be put to these levers for turning the mill; which is often done where water cannot be had for that purpose.

A *wind-mill.*

The working parts of a wind-mill differ very little from those of a water-mill; only the former is turned by the action of the wind upon four sails, every one of which ought (as is generally believed) to make an angle of $54\frac{4}{5}$ degrees, with a plane perpendicular to the axis on which the arms are fixed for carrying of them. It being demonstrable, that when the sails are set to such an angle, and the axis turned endwise toward the wind, the wind has the greatest power upon the sails. But this angle answers only to the case of a vane or sail just beginning to move:[8] for, when the vane has a certain degree of motion, it yields to the wind; and then that angle must be increased to give the wind its full effect.

Again, the increase of this angle should be different, according to the different velocities from the axis to the extremity of the vane. At the axis it should be $54\frac{4}{5}$ degrees, and thence continually decreasing, giving the vane a twist, and so causing all the ribs of the vane to lie in different planes.

Lastly, these ribs ought to decrease in length from the axis to the extremity, giving the vane a curvilineal form; so that no part of the force of any one rib be spent upon the rest, but all move on independent of each other. All this is required to give the sails of a wind-mill their true form; and we see both the twist and the diminution of the ribs exemplified in the wings of birds.

It is almost incredible to think with what velocity the tips of the sails move when acted upon by a moderate gale of wind. I have several times counted the number of revolutions made by the sails in ten or fifteen minutes; and from the length of the arms from tip to tip, have computed that if a hoop of that diameter was to run upon the ground with the same velocity that it would move with if put upon the sail-arms, it would go upwards of 30 miles in an hour.

As the ends of the sails nearest the axis cannot move with the same velocity that the tips or farthest ends do, although the wind acts equally strong upon them, perhaps a better position than that of stretching them along the arms directly from the centre of motion, might be to have them set perpendicularly across the farther ends of the arms, and there adjusted lengthwise to the proper angle. For, in that case, both ends of the sails would move with the same velocity; and being farther from the centre of motion, they would have so much the more power: and then there would be no occasion for having them so large as they are generally made; which would render them lighter, and consequently there would be so much the less friction on the thick neck of the axle where it turns in the wall.*

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[8] See Maclaurin's Fluxions, near the end.

A crane.

A *crane* is an engine by which great weights are raised to certain heights, or let down to certain depths. It consists of wheels, axles, pulleys, ropes, and a gib or gibbet.

Plate III. Fig. 5.

When the rope *H* is hooked to the weight *K*, a man turns the winch *A*, on the axis whereof is the trundle *B*, which turns the wheel *C*, on whose axis *D* is the trundle *E*, which turns the wheel *F* with its upright axis *G*, on which the great rope *H H* winds as the wheel turns; and going over a pulley *I* at the end of the arm *d* of the gib *c c d e*, it draws up the heavy weight *K*; which, being raised to a proper height, as from a ship to the quay, is then brought over the quay by pulling the wheel *Z* round by the handles *z*, *z*, which turns the gib by means of the half wheel *b* fixed to the gib-post *c c*, and the strong pinion *a* fixed on the axis of the wheel *Z*. This wheel gives the man that turns it an absolute command over the gib, so as to prevent it from taking any unlucky swing, such as often happens when it is only guided by a rope tied to its arm *d*; and people are frequently hurt, and sometimes killed by such accidents.

The great rope goes between two upright rollers *i* and *k*; which turn upon gudgeons in the fixed beams *f* and *g*; and as the gib is turned toward either side, the rope bends upon the roller next that side. Were it not for these rollers, the gib would be quite unmanageable; for the moment it were turned ever so little toward any side, the weight *K* would begin to descend, because the rope would be shortened between the pulley *I* and axis *G*; and so the gib would be pulled violently to that side, and either be broke to pieces, or break every thing that came in its way. These rollers must be placed so that the sides of them, round which the rope bends, may keep the middle of the bended part directly even with the centre of the hole in which the upper gudgeon of the gib turns in the beam *f*. The truer these rollers are placed, the easier the gib is managed, and the less apt to swing either way by the force of the weight *K*.

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* See the *Appendix*, Vol. II.

A ratchet-wheel *Q* is fixed upon the axis *D*, near the trundle *E;* and into this wheel the catch or click *R* falls. This hinders the machinery from running back by the weight of the burthen *K*, if the man who raises it should happen to be careless, and so leave off working at the winch *A* sooner than he ought to do.

When the weight *K* is raised to its proper height from the ship, and brought over the quay by turning the gib about, it is let down gently upon the quay, or into a cart standing thereon, in the following manner :—A man takes hold of the rope *t t* (which goes over the pulley *v*, and is tied to a hook at *S* in the catch *R*), and so disengages the catch from the ratchet-wheel *Q;* and then the man at the winch *A* turns it backward, and lets down the weight *K*. But if the weight pulls too hard against this man, another lays hold of the stick *V*, and by pulling it downward, draws the gripe *U* close to the wheel *Y*, which, by rubbing hard against the gripe, hinders the too quick descent of the weight; and not only so, but even stops it at any time, if required. By this means, heavy goods may be either raised or let down at pleasure, without any danger of hurting the men who work the engine.

When part of the goods are craned up, and the rope is to be let down for more, the catch *R* is first disengaged from the ratchet-wheel *Q*, by pulling the cord *t;* then the handle *q* is turned half round backward, which by the crank *n n* in the piece *o* pulls down the frame *h* between the guides *m* and *m* (in which it slides in a groove), and so disengages the trundle *B* from the wheel *C:* and then, the heavy hook *β* at the end of the rope *H* descends by its own weight, and turns back the great wheel *F* with its trundle *E*, and the wheel *C;* and this last wheel acts like a fly against the wheel *F* and hook *β*, and so hinders it from going down too quick; while the weight *X* keeps up the gripe *U* from rubbing against the wheel *Y*, by means of a cord going from the weight, over the pulley *w* to the hook *W* in the gripe; so that the gripe never touches the wheel, unless it be pulled down by the handle *V*.

When the crane is to be set at work again, for drawing up another burthen, the handle *q* is turned half round forward;

which by the crank *n n*, raises up the frame *h*, and causes the trundle *B* to lay hold of the wheel *C;* and then, by turning the winch *A*, the burthen of goods *K* is drawn up as before.

The crank *n n* turns pretty stiff in the mortise near *o*, and stops against the farther end of it when it has got just a little beyond the perpendicular; so that it can never come back of itself: and therefore the trundle *B* can never come away from the wheel *C*, until the handle *q* be turned half round backward.

The great rope runs upon rollers in the lever *L M*, which keeps it from bending between the axle at *G* and the pulley *I*. This lever turns upon the axis *N*, by means of the weight *O*, which is just sufficient to keep its end *L* up to the rope; so that, as the great axle turns, and the rope coils round it, the lever rises with the rope, and prevents the coilings from going over one another.

The power of this crane may be estimated thus:—Suppose the trundle *B* to have 13 staves or rounds, and the wheel *C* to have 78 spur cogs; the trundle *E* to have 14 staves, and the wheel *F* 56 cogs. Then, by multiplying the staves of the trundles, 13 and 14, into one another, their product will be 182; and by multiplying the cogs of the wheels, 78 and 56, into one another, their product will be 4,368; and dividing 4,368 by 182, the quotient will be 24; which shews that the winch *A* makes 24 turns for one turn of the wheel *F* and its axle *G*, on which the great rope or chain *H I H* winds. So that, if the length or radius of the winch *A* were only equal to half the diameter of the great axle *G*, added to half the thickness of the rope *H*, the power of the crane would be as 24 to 1; but the radius of the winch being double the above length, it doubles the said power, and so makes it as 48 to 1; in which case, a man may raise 48 times as much weight by this engine as he could do by his natural strength without it, making proper allowance for the friction of the working parts. Two men may work at once, by having another winch on the opposite end of the axis of the trundle under *B;* and this will make the power double.

If this power be thought greater than what may be generally wanted, the wheels may be made with fewer cogs in proportion to the staves in the trundles; and so the power may be of whatever degree is judged to be requisite. But if the weight be so great as will require yet more power to raise it, suppose a double quantity, then the rope *H* may be put under a moveable pulley, as δ, and the end of it tied to a hook in the gib at ε; which will give a double power to the machine, and so raise a double weight hooked to the block of the moveable pulley.

When only small burthens are to be raised, this may be quickly done by men pushing the axle *G* round by the long spokes *y, y, y, y;* having first disengaged the trundle *B* from the wheel *C;* and then, this wheel will only act as a fly upon the wheel *F;* and the catch *R* will prevent its running back, if the men should inadvertently leave off pushing, before the burthen be unhooked from β.

Lastly, when very heavy burthens are to be raised, which might endanger the breaking of the cogs in the wheel *F*, their force against these cogs may be much abated by men pushing round the long spokes *y, y, y, y,* while the man at *A* turns the winch.

I have only shewn the working parts of this crane, without the whole of the beams which support them; knowing that these are easily supposed, and that if they had been drawn, they would have hid a great deal of the working parts from sight, and also confused the figure.

Another crane.

Another very good *crane* is made in the following manner.—*A A* (Fig. 6) is a great wheel, turned by men walking within it at *H*. On the part *C*, of its axle *B C*, the great rope *D* is wound as the wheel turns; and this rope draws up goods in the same way as the rope *H H* does in the above-mentioned crane, the gib-work here being supposed to be of the same sort. But these cranes are very dangerous to the men in the wheel; for, if any of the men should chance to fall, the burthen will make the wheel run back, and throw them all about within it; which often breaks their limbs, and sometimes kills them. The late ingenious Mr. Padmore of Bristol (whose contrivance the fore-mentioned crane is, so far as I can remember its construction, after seeing it once about twelve years ago),[9] observing this dangerous construction, contrived a method of remedying it, by putting cogs all around the outside of the wheel, and applying a trundle *E* to turn it; which increases the power as much as the number of cogs in the wheel is greater than the number of staves in the trundle; and by putting a ratchet-wheel *F* on the axis of the trundle (as in the above-mentioned crane), with a catch to fall into it, the great wheel is stopt from running back by the force of the weight, even if all the men in it should leave off working: and by one man working at the winch *I*, or two men at the opposite winches when needful, the men in the wheel are much assisted, and much greater weights are raised, than could be by men only within the wheel. Mr. Padmore put also a gripe-wheel *G* upon the axis of the trundle, which being pinched in the same manner as described in the former crane, heavy burthens may be let down without the least danger. And before this contrivance, the lowering of goods was always attended with the utmost danger to the men in the wheel; as every one must be sensible of, who has seen such engines at work.

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[9] Since the first edition of this book was printed, I have seen the same crane again; and find, that though the working parts are much the same as above described, yet the method of raising or lowering the trundle *B*, and the catch *R*, are better contrived than I have described them.

And it is surprising that the masters of wharfs and cranes should be so regardless of the limbs, or even lives, of their workmen, that, excepting the late Sir James Creed of Greenwich, and some gentlemen at Bristol, there is scarce an instance of any who has used this safe contrivance.

Wheel-carriages.

The structure of *wheel-carriages* is generally so well known, that it would be needless to describe them; and, therefore, we shall only point out some inconveniences attending the common method of placing the wheels, and loading the waggons.

In coaches, and all other four-wheeled carriages, the fore-wheels are made of a less size than the hind ones, both on account of turning short, and to avoid cutting the braces; otherwise, the carriage would go much easier, if the fore-wheels were as high as the hind ones, and the higher the better, because they would sink to less depths in little hollowings in the roads, and be the more easily drawn out of them. But carriers and coachmen give another reason for making the fore-wheels much lower than the hind-wheels; merely, that when they are so, the hind-wheels help to push on the fore ones, which is too unphilosophical and absurd to deserve to deserve a refutation; and yet, for their satisfaction, we shall shew, by experiment, that it has no existence but in their own imaginations.

It is plain that the small wheels must turn as much oftener round than the great ones, as their circumferences are less: and, therefore, when the carriage is loaded equally heavy on both axles, the fore-axle must sustain as much more friction, and consequently wear out as much sooner, than the hind-axle, as the fore-wheels are less than the hind ones. But the great misfortune is, that all the carriers, to a man, do obstinately persist, against the clearest reason and demonstration, in putting the heavier part of the load upon the fore-axle of the waggon; which not only makes the friction greatest where it ought to be least, but also presses the fore-wheels deeper into the ground than the hind-wheels, notwithstanding the fore-wheels, being less than the hind ones, are with so much the greater difficulty drawn out of a hole or over an obstacle, even supposing the weights on their axles were equal. For the difficulty, with equal weights, will be as the depth of the hole or height of the obstacle is to the semidiameter of the wheel. Thus, if we suppose the small wheel *D* (Fig. 7) of the waggon *A B* to fall into a hole of the depth *E F*, whch is equal to the semidiameter of the wheel, and the waggon to be drawn horizontally along, it is evident, that the point *E* of the small wheel will be drawn directly against the top of the hole; and, therefore, all the power of horses and men will not be able to draw it out, unless the ground gives way before it. Whereas, if the hind-wheel *G* falls into such a hole, it sinks not near so deep in proportion to its semidiameter; and, therefore, the point *G* of the large wheel will not be drawn directly, but obliquely, against the top of the hole; and so will be easily got out of it. Add to this, that as a small wheel will often sink to the bottom of a hole, in which a great wheel will go but a very little way, the small wheels ought in all reason to be loaded with less weight than the great ones; and then the heavier part of the load would be less jolted upward and downward, and the horses tired so much the less, as their draught raised the load to less heights.

It is true, that when the waggon-road is much up hill, there may be danger in loading the hind-part much heavier than the fore-part; for then the fore-part would overhang the hind-axle, especially if the load be high, and endanger tilting up the fore-wheels from the ground. In this case, the safest way would be to load it equally heavy on both axles; and then, as much more of the weight would be thrown upon the hind-axle than upon the fore one, as the ground rises from a level below the carriage. But as this seldom happens, and, when it does, a small temporary weight laid upon the pole between the horses would overbalance the danger; and this weight might be thrown into the waggon when it comes to level ground; it is strange, that an advantage so plain and obvious as would arise from loading the hind-wheels heaviest, should not be laid hold of, by complying with this method.

To confirm these reasonings by experiment, let a small model of a waggon be made, with its fore-wheels $2\frac{1}{2}$ inches in diameter, and its hind-wheels $4\frac{1}{2}$; the whole model weighing about 20 ounces. Let this little carriage be loaded anyhow with weights, and have a small cord tied to each of its ends, equally high from the ground it rests upon; and let it be drawn along a horizontal board, first by a weight in a scale hung to the cord at the fore-part; the cord going over a pulley at the end of the board, to facilitate the draught, and the weight just sufficient to draw it along. Then, turn the carriage, and hang the scale and weight to the hind cord, and it will be found to move along with the same velocity as at first; which shews, that the power required to draw the carriage is the same, whether the great or small wheels are foremost; and therefore the great wheels do not help in the least to push on the small wheels in the road.

Hang the scale to the fore-cord, and place the fore-wheels (which are the small ones) in two holes, cut three-eight parts of an inch deep into the board; then put a weight of 32 ounces into the carriage, over the fore-axle, and an equal weight over the hind one: this done, put 44 ounces into the scale, which will be just sufficient to draw out the fore-wheels: but if this weight be taken out of the scale, and one of 16 ounces put into its place, if the hind-wheels are placed in the holes, the 16 ounce weight will draw them out; which is little more than a third part of what was necessary to draw out the fore-wheels. This shews, that the larger the wheels are, the less power will draw the carriage, especially on rough ground.

Put 64 ounces over the axle of the hind-wheels, and 32 over the axle of the fore ones, in the carriage, and place the fore-wheels in the holes; then put 38 ounces into the scale, which will just draw out the fore-wheels; and when the hind ones come to the hole, they will find but very little resistance, because they sink but a little way into it.

But shift the weights in the carriage, by putting the 32 ounces upon the hind-axle, and the 64 ounces upon the fore one, and place the fore-wheels in the holes: then, if 76 ounces be put into the scale, it will be found no more than sufficient to draw out these wheels; which is double the power required to draw them out, when the lighter part of the load was put upon them; which is a plain demonstration of the absurdity of putting the heaviest part of the load in the fore-part of the waggon.

Every one knows what an outcry was made by the generality, if not the whole body of the carriers, against the broad-wheel act; and how hard it was to persuade them to comply with it, even though the government allowed them to draw with more horses, and carry greater loads, than usual. Their principal objection was, that as a broad wheel must touch the ground in a great many more points than a narrow wheel, the friction must of course be just so much the greater; and, consequently, there must be so many more horses than usual to draw the waggon. I believe that the majority of people were of the same opinion, not considering, that if the whole weight of the waggon and load in it bears upon a great many points, each sustains a proportionably less degree of weight and friction, than when it bears only upon a few points; so that what is wanting in one is made up in the other; and therefore will be just equal under equal degrees of weight, as may be shewn by the following plain and easy experiment.

Let one end of a piece of packthread be fastened to a brick, and the other end to a common scale for holding weights; then, having laid the brick edgewise on a table, and let the scale hang under the edge of the table, put as much weight into the scale as will just draw the brick along the table. Then taking back the brick to its former place, lay it flat on the table, and leave it to be acted upon by the same weight in the scale as before, which will draw it along with the same ease as when it lay upon its edge. In the former case, the brick may be considered as a narrow wheel on the ground; and in the latter as a broad wheel. And since the brick is drawn along with equal ease whether its broad side or narrow edge touches the table, it shews that a broad wheel might be drawn along the ground with the same ease as a narrow one (supposing them equally heavy), even though they should drag, and not roll, as they go along.

As narrow wheels are always sinking into the ground, especially when the heaviest part of the load lies upon them, they must be considered as going constantly up hill, even on level ground; and their sides must sustain a great deal of friction, by rubbing against the ruts made by them. But both these inconveniences are avoided by broad wheels; which instead of cutting and ploughing up the roads, roll them smooth, and harden them, as experience testifies in places where they have been used, especially either on wettish or sandy ground; though, after all, it must be confessed, that they will not do in stiff clayey cross roads; because they would soon gather up as much clay as would be almost equal to the weight of an ordinary load.

If the wheels were always to go upon smooth and level ground, the best way would be to make the spokes perpendicular to the naves, that is, to stand at right angles to the axles; because they would then bear the weight of the load perpendicularly, which is the strongest way for wood. But because the ground is generally uneven, one wheel often falls into a cavity or rut when the other does not; and then it bears much more of the weight than the other does: in which case, concave or dishing wheels are best, because, when one falls into a rut, and the other keeps upon high ground, the spokes become perpendicular in the rut, and therefore have the greatest strength when the obliquity of the load throws most of its weight upon them; while those on the high ground have less weight to bear, and therefore need not be at their full strength: so that the usual way of making the wheels concave is by much the best.

The axles of the wheels ought to be perfectly straight, that the rim of the wheels may be parallel to each other; for then they will move easiest, because they will be at liberty to go on straight forward. But in the usual way of practice, the axles are bent downward at their ends, which brings the sides of the wheels next the ground nearer to one another than their opposite or higher sides are: and this not only makes the wheels to drag sideways as they go along, and gives the load as much greater power of crushing them than when they are parallel to each other; but also endangers the overturning of the carriage when any wheel falls into a hole or rut; or when the carriage goes in a road which has one side lower than the other, as along the side of a hill. Thus (in the hind view of a waggon or cart) let $A E$ and $B F$ be the great wheels parallel to each other, on their straight axle K, and $H C I$ (Fig. 8) the carriage loaded with heavy goods from C to G. Then, as the carriage goes on in the oblique road $A a B$, the centre of gravity of the whole machine and load will be at C;

and the line of direction *C d D* falling within the wheel *B F*, the carriage will not overset. But if the wheels (Fig. 9) be inclined to each other on the ground, as *A E* and *B F* are, and the machine be loaded as before, from *C* to *G*, the line of direction *C d D* falls without the wheel *B F*, and the whole machine tumbles over. When it is loaded with heavy goods (such as lead or iron) which lie low (Fig. 8), it may travel safely upon an oblique road so long as the centre of gravity is at *C*, and the line of direction *C d* falls within the wheels; but if it be loaded high with lighter goods (such as wool-packs) from *C* to *L* (Fig. 10), the centre of gravity is raised from *C* to *K*, which throws the line of direction *K k* without the lowest edge of the wheel *B F*, and then the load oversets the waggon.

If there be some advantage from small fore-wheels, on account of the carriage turning more easily and short than it can be made to do when they are large, there is at least as great a disadvantage attending them, which is, that as their axle is below the level of the horse's breast, the horses not only have the loaded carriage to draw along, but also part of its weight to bear, which tires them sooner, and makes them grow much stiffer in their hams, than they would do if they drew on a level with the fore-axle: and, for this reason, we find coach-horses soon become unfit for riding. So that, on all accounts, it is plain, that the fore-wheels of all carriages ought to be so high as to have their axles even with the breast of the horses; which would not only give the horses a fair draught, but likewise keep them longer fit for drawing the carriage.[10]

Plate IV. Fig. 1, 2.

We shall conclude this lecture with a description of Mr. Vauloue's curious engine, which was made use of for driving the piles of Westminster bridge; and the reader may cast his eyes upon the first and second figures of the plate, in which the same letters of reference are annexed to the same parts, in order to explain those in the second, which are either partly or wholly hid in the first.

The *pile-engine.*

A is the great upright shaft or axle, on which are the great wheel *B* and drum *C*, turned by horses joined to the bars *S*, *S*. The wheel *B* turns the trundle *X*, on the top of whose axis is the fly *O*, which serves to regulate the motion, and also to act against the horses, and keep them from falling when the heavy ram *Q* is discharged to drive the pile *P* down into the mud in the bottom of the river. The drum *C* is loose upon the shaft *A*, but is locked to the wheel *B* by the bolt *Y*. On this drum the great rope *H H* is wound; one end of the rope being fixed to the drum, and the other to the follower *G*, to which it is conveyed over the pulleys *I* and *K*. In the follower *G* is contained the tongs *F*, (Fig. 3) that take hold of the ram *Q* by the staple *R* for drawing it up. *D* is a spiral or fusee fixed to the drum, on which is wound the small rope *T* that goes over the pulley *U*, under the pulley *V*, and is fastened to the top of the frame at 7. To the pulley block *V* is hung the counterpoise *W*, which hinders the follower from accelerating as it goes down to take hold of the ram; for, as the follower tends to acquire velocity in its descent, the line *T* winds downward upon the fusee, on a larger and larger radius, by which means the counterpoise *W* acts stronger and stronger against it; and so allows it to come down with only a moderate and uniform velocity. The bolt *Y* (Fig. 1 and 2) locks the drum to the great wheel, being pushed upward by the small lever 2, which goes through a mortise in the shaft *A*, turns upon a pin in the bar 3 fixed to the great wheel *B*, and has a weight 4, which always tends to push up the bolt *Y* through the wheel into the drum. *L* is the great lever turning on the axis *m*, and resting upon the forcing bar 5, 5, which goes down through a hollow in the shaft *A*, and bears up the little lever 2.

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[10] Mr. Edgeworth recommends the application of springs to heavy waggons, &c. because they facilitate the draught, by permitting the load to rise gradually over an obstacle, without obstructing the velocity of the carriage.—*Ed.*

By the horses going round, the great rope *H* is wound about the drum *C*, and the ram *Q* is drawn up by the tongs *F* in the follower *G*, until the tongs come between the inclined planes *E*; which, by shutting the tongs at the top, opens it at the foot, and discharges the ram, which falls down between the guides *b b* upon the pile *P*, and drives it by a few strokes as far into the mud as it can go; after which, the top part is sawed off close to the mud, by an engine for that purpose. Immediately after the ram is discharged, the piece 6 upon the follower G takes hold of the ropes *a a*, which raise the end of the lever *L*, and cause its end *N* to descend and press down the forcing bar 5 upon the little lever 2, which by pulling down the bolt *Y*, unlocks the drum *C* from the great wheel *B*; and then the follower, being at liberty, comes down by its own weight to the ram;

and the lower ends of the tongs slip over the staple *R*, and the weight of their heads causes them to fall outward, and shuts upon it. Then the weight 4 pushes up the bolt *Y* into the drum, which locks it to the great wheel, and so the ram is drawn up as before.

As the follower comes down, it causes the drum to turn backward, and unwinds the rope from it, while the horses, great wheel, trundle, and fly, go on with an uninterrupted motion; and as the drum is turning backward, the counterpoise *W* is drawn up, and its rope *T* wound upon the spiral fusee *D*.

There are several holes in the under side of the drum, and the bolt *Y* always takes the first one that it finds when the drum stops by the falling of the follower upon the ram; until which stoppage, the bolt has not time to slip into any of the holes.

This engine was placed upon a barge on the water, and so was easily conveyed to any place desired. I never had the good fortune to see it, but drew this figure from a model which I made from a print of it, being not quite satisfied with the view which the print gives. I have been told that the ram was a ton weight, and that the guides *b b*, between which it was drawn up and let fall down, were 30 feet high. I suppose the great wheel may have had 100 cogs, and the trundle 10 staves or rounds; so that the fly would make 10 revolutions for one of the great wheel.[11]

LECTURE V.

OF HYDROSTATICS, AND HYDRAULIC MACHINES.

THE science of *hydrostatics* treats of the nature, gravity, pressure, and motion of fluids in general, and of weighing solids in them.[1]

Definition of a fluid.

A fluid is a body that yields to the least pressure or difference of pressures. Its particles are so small, that they cannot be discerned by the best microscopes; they are hard, since no fluid, except air or steam, can be pressed into a less space than it naturally possesses;[2] and they must be round and smooth, seeing they are so easily moved among one another.

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[11] An account of Camus's pile-engine, Bunce's and others, will be found in the *Edinburgh Encyclopædia*, Art. *Mechanics*, vol. xiii, p. 615.—*Ed.*

[1] *Hydrodynamics* is the science which treats of the pressure, equilibrium, and motion of fluids. It is divided into two branches, Hydrostatics and Hydraulics. *Hydrostatics* treats of the pressure and equilibrium of fluids; and *Hydraulics* treats of the motion of fluids, and the machines in which they are chiefly concerned.—*Ed.*

All bodies, both fluid and solid, press downward by the force of gravity: but fluids have this wonderful property, that their pressure upward and sidewise is equal to their pressure downward; and this is always in proportion to their perpendicular height, without any regard to their quantity; for as each particle is quite free to move, it will move toward that part or side on which the pressure is least: and hence, no particle or quantity of a fluid can be at rest, till it is every way equally pressed.

Plate IV. Fig. 4. Fluids press as much upward as downward.

To shew by experiment that fluids press upward as well as downward, let *A B* be a long upright tube filled with water near to its top; and *C D* a small tube open at both ends, and immersed into the water in the large one: if the immersion be quick, you will see the water rise in the small tube to the same height that it stands in the great one, or until the surfaces of the water in both are on the same level; which shews that the water is pressed upward into the small tube by the weight of what is in the great one; otherwise it could never rise therein, contrary to its natural gravity, unless the diameter of the bore were so small, that the attraction of the tube would raise the water; which will never happen if the tube be as wide as that in a common barometer. And, as the water rises no higher in the small tube than till its surface be on a level with the surface of the water in the great one, this shews that the pressure is not in proportion to the quantity of water in the great tube, but in proportion to its perpendicular height therein; for there is much more water in the great tube all around the small one, than what is raised to the same height in the small one, as it stands within the great.

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[2] It is now found from many unquestionable experiments, made both in this country and on the continent, that most fluids are to a certain degree compressible. Mr. Canton having placed a glass tube filled with water, beneath the receiver of an air-pump, and removed the pressure of the atmosphere, found that the water expanded itself, and rose in the tube. When the same glass tube was placed under the receiver of a condensing engine, and the air in the receiver greatly condensed, the water was compressed, and sunk in the tube. When other fluids were subjected to similar experiments, he found them compressible in the following proportions:—Spirit of wine 66 millionth parts of the whole; oil of olives 48 millionth parts; rain water 46; sea water 40; and mercury 3.—For a farther account of the experiments of this ingenious philosopher, see *Phil. Trans.* vols. lii & liv. Mr. Zimmerman also found, that sea water was compressed $\frac{1}{340}$th part of its bulk, when inclosed in the cavity of a strong iron cylinder, and under the influence of a force equal to a column of sea water a thousand feet high. More recently Mr. Perkins has found that water may be compressed $\frac{1}{212}$th part of its bulk. See the *Phil. Trans.* 1820, p. 324, and the *Edinburgh Philosophical Journal*, vol. iv, p. 175 and 438. The elasticity, and consequently the compressibility, of fluids is also evident from the reflection of stones, and even cannon balls, which strike the surface of stagnant water in an oblique direction.—*Ed.*

Take out the small tube, and let the water run out of it; then it will be filled with air. Stop its upper end with the cork *C*, and it will be full of air all below the cork: this done, plunge it again to the bottom of the water in the great tube, and you will see the water rise up in it only to the height *E;* which shews that the air is a body, otherwise it could not hinder the water from rising up to the same height as it did before, namely, to *A;* and in so doing, it drove the air out at the top; but now the air is confined by the cork *C:* and it also shews that the air is a compressible body, for if it were not so, a drop of water could not enter into the tube.

The pressure of fluids being equal in all directions, it follows that the sides of a vessel are as much pressed by a fluid in it, all around in any given ring of points, as the fluid below that ring is pressed by the weight of all that stands above it. Hence the pressure upon every point in the sides, immediately above the bottom, is equal to the pressure upon every point of the bottom. To shew this by experiment, let a hole be made at *e* (Fig. 5) in the side of the tube *A B*, close by the bottom, and another hole of the same size in the bottom at *C;* then pour water into the tube, keeping it full as long as you choose the holes should run, and have two basons ready to receive the water that runs through the two holes, until you think there is enough in each bason; and you will find by measuring the quantities, that they are equal, which shews that the water runs with equal speed through both holes; which it could not have done, if it had not been equally pressed through them both. For, if a hole of the same size be made in the side of the tube, as about *f*, and if all three are permitted to run together, you will find that the quantity run through the hole at *f* is much less than what was run in the same time through either of the holes *C* or *c*.

In the same figure, let a tube be turned up from the bottom at *e* into the shape *D E*, and the hole at *C* be stopt with a cork: then pour water into the tube to any height, as *A g*, and it will spout up in a jet *E F G*, nearly as high as it is kept in the tube *A B*, by continuing to pour in as much there as runs through the hole *E*; which will be the case while the surface *A g* keeps at the same height: and if a little ball of cork *G* be laid upon the top of the jet, it will be supported thereby, and dance upon it. The reason why the jet rises not quite so high as the surface of the water *A g*, is owing to the resistance it meets with in the open air; for, if a tube, either great or small, was screwed upon the pipe at *E*, the water would rise in it until the surface of the water in both tubes were on the same level; as will be shewn by the next experiment.

The *hydrostatic paradox.*

Any quantity of a fluid, however small, may be made to balance and support any quantity, however great. This is deservedly termed the hydrostatical paradox, which we shall first shew by an experiment, and then account for it upon the principle above mentioned; namely, that *the pressure of fluids is directly as their perpendicular height, without any regard to their quantity.*

Let a small glass tube *D C G* (Fig. 6) open throughout, and bent at *B*, be joined to the end of a great one *A I* at *c d*, where the great one is also open; so that these tubes in their openings may freely communicate with each other. Then pour water through a small-necked funnel into the small tube at *H*; this water will run through the joining of the tubes at *c d*, and rise up into the great tube; and if you continue pouring until the surface of the water comes to any part, as *A*, in the great tube, and then leave off, you will see that the surface of the water in the small tube will be just as high at *D*; so that the perpendicular height of the water will be the same in both tubes, however small the one be in proportion to the other.[3] This shews, that the small column *D C G* balances and supports the great column *A c d*, which it could not do if their pressures were not equal against one another in the recurved bottom at *B*. If the small tube be made longer, and inclined in the situation *G E F*, the surface of the water in it will stand at *F*, on the same level with the surface *A* in the great tube; that is, the water will have the same perpendicular height in both tubes, although the column in the small tube is longer than that in the great one; the former being oblique, and the latter perpendicular.

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[3] This is true only when the bore of the smaller tube *D C G* is sufficiently large. For if this tube be of the capillary kind, the attraction of cohesion will raise the water in it to a much greater height than the column of water in the large tube *A I*.—*Ed.*

Since, then, the pressure of fluids is directly as their perpendicular heights, without any regard to their quantities, it appears that whatever the figure or size of vessels be, if they are of equal heights, and if the areas of their bottoms are equal, the pressures of equal heights of water are equal upon the bottoms of these vessels, even though the one should hold a thousand, or ten thousand, times as much water as would fill the other. To confirm this part of the hydrostatical paradox by an experiment, let two vessels be prepared of equal heights, but very unequal contents, such as *A B* in Fig. 7, and *A B* in Fig. 8. Let each vessel be open at both ends, and their bottoms *D d*, *D d*, be of equal widths. Let a brass bottom *C C* be exactly fitted to each vessel, not to go into it, but for it to stand upon; and let a piece of wet leather be put between each vessel and its brass bottom, for the sake of closeness. Join each bottom to its vessel by a hinge *D*, so that it may open like the lid of a box; and let each bottom be kept up to its vessel by equal weights *E* and *E*, hung to lines which go over the pulleys *F* and *F* (whose blocks are fixed to the sides of the vessels at *f*), and the lines tied to hooks at *d* and *d*, fixed in the brass bottoms opposite to the hinges *D* and *D*. Things being thus prepared and fitted, hold the vessel *A B* (Fig. 8) upright in your hands over a bason on a table, and cause water to be poured into the vessel slowly, till the pressure of the water bears down its bottom at the side *d*, and raises the weight *E;* and then part of the water will run out at *d*. Mark the height at which the surface *H* of the water stood in the vessel, when the bottom began to give way at *d;* and then, holding up the other vessel *A B* (Fig. 7) in the same manner, cause water to be poured into it at *H;* and you will see that when the water rises to *A* in this vessel, just as high as it did in the former, its bottom will also give way at *d*, and it will lose part of the water.

The natural reason of this surprising phenomenon is, that since all parts of a fluid at equal depths below the surface are equally pressed in all manner of directions, the water immediately below the fixed part *B f* (Fig. 7) will be pressed as much upward against its lower surface within the vessel, by the action of the column *A g*, as it would be by a column of the same height, and of any diameter whatever (as was evident by the experiment of the tube, Fig. 3); and therefore, since action and re-action are equal and contrary to each other, the water immediately below the surface *B f* will be pressed as much downward by it, as if it was immediately touched and pressed by a column of the height *g A*, and of the diameter *B f*: and therefore, the water in the cavity *B D d f* will be pressed as much downward upon its bottom *C C* as the bottom of the other vessel (Fig. 8) is pressed by all the water above it.

To illustrate this a little farther, let a hole be made at *f*, (Fig. 7) in the fixed top *B f*, and let a tube *G* be put into it; then, if water be poured into the tube *A*, it will (after filling the cavity *B d*) rise up into the tube *G*, until it comes to a level with that in the tube *A*, which is manifestly owing to the pressure of the water in the tube *A*, upon that in the cavity of the vessel below it. Consequently, that part of the top *B f*, in which the hole is now made, would, if corked up, be pressed upward with a force equal to the weight of all the water which is supported in the tube *G*; and the same thing would hold at *g*, if a hole were made there. And so if the whole cover or top *B f* were full of holes, and had tubes as high as the middle one *A g* put into them, the water in each tube would rise to the same height as it is kept into the tube *A*, by pouring more into it, to make up the deficiency that it sustains by supplying the others, until they are all full; and then the water in the tube *A* would support equal heights of water in all the rest of the tubes. Or, if all the tubes except *A*, or any other one, were taken away, and a large tube equal in diameter to the whole top *B f* were placed upon it, and cemented to it, and then if water were poured into the tube that was left in either of the holes, it would ascend through all the rest of the holes, until it filled the large tube, to the same height that it stands in the small one, after a sufficient quantity had been poured into it; which shews, that the top *B f* was pressed upward by the water under it, and before any hole was made in it, with a force equal to that wherewith it is now pressed downward by the weight of all the water above it in the great tube. And therefore, the re-action of the fixed top *B f* must be as great, in pressing the water downward upon the bottom

C C, as the whole pressure of the water in the great tube would have been, if the top had been taken away, and the water in that tube left to press directly upon the water in the cavity *B D d f.*

The hydrostatic bellows.

Perhaps the best machine in the world for demonstrating the upward pressure of fluids, is the hydrostatic bellows *A* (Fig. 9); which consists of two thick oval boards, each about 16 inches broad, and 18 inches long, covered with leather, to open and shut like a common bellows, but without valves; only a pipe *B*, about three feet high, is fixed into the bellows at *e*. Let some water be poured into the pipe at *C*, which will run into the bellows, and separate the boards a little. Then lay three weights, *b*, *c*, *d*, each weighing 100 pounds, upon the upper board, and pour more water into the pipe *B*, which will run into the bellows, and raise up the board with all the weights upon it; and if the pipe be kept full, until the weights are raised as high as the leather which covers the bellows will allow them, the water will remain in the pipe, and support all the weights, even though it should weigh no more than a quarter of a pound, and they 300 pounds; nor will all their force be able to cause them to descend and force the water out at the top of the pipe.

The reason of this will be made evident, by considering what has been already said of the result of the pressure of fluids of equal heights without any regard to the quantities. For, if a hole be made in the upper board, and a tube be put into it, the water will rise in the tube to the same height that it does in the pipe; and would rise as high (by supplying the pipe) in as many tubes as the board would contain holes. Now, suppose only one hole to be made in any part of the board, of an equal diameter with the bore of the pipe *B*, and that the pipe holds just a quarter of a pound of water, if a person claps his finger upon the hole, and the pipe be filled with water, he will find his finger to be pressed upward with a force equal to a quarter of a pound. And as the same pressure is equal upon all equal parts of the board, each part whose area is equal to the area of the hole, will be pressed upward with a force equal to that of a quarter of a pound: the sum of all which pressures against the under side of an oval board 16 inches broad, and 18 inches long, will amount to 300 pounds; and therefore so much weight will be raised up and supported by a quarter of a pound of water in the pipe.

How a man may raise himself upward by his breath.

Hence, if a man stands upon the upper board, and blows into the bellows through the pipe *B*, he will raise himself upward upon the board; and the smaller the bore of the pipe is, the easier he will be able to raise himself. And then, by clapping his finger upon the top of the pipe, he can support himself as long as he pleases, provided the bellows be air-tight, so as not to lose what is blown into it.

How solid lead may be made to swim in water.

Upon this principle of the upward pressure of fluids, a piece of lead may be made to swim in water, by immersing it to a proper depth, and keeping the water from getting above it. Let *C D* (Fig. 10) be a glass tube, open throughout, and *E F G* a flat piece of lead, exactly fitted to the lower end of the tube, not to go within it, but for it to stand upon, with a wet leather between the lead and the tube, to make close work. Let this leaden bottom be half an inch thick, and held close to the tube by pulling the packthread *I H L* upward at *L* with one hand, while the tube is held in the other by the upper end *C*. In this situation, let the tube be immersed in water in the glass vessel *A B*, to the depth of six inches below the surface of the water at *K*, and then, the leaden bottom *E F G* will be plunged to the depth of somewhat more than eleven times its own thickness: holding the tube at that depth, you may let go the thread at *L*; and the lead will not fall from the tube, but will be kept to it by the upward pressure of the water below it, occasioned by the height of the water at *K* above the level of the lead. For as lead is 11.33 times as heavy as its bulk of water, and is in this experiment immersed to a depth somewhat more than 11.33 times its thickness, and no water getting into the tube between it and the lead, the column of water *E a b c G* below the lead is pressed upward against it by the water *K D E G L* all around the tube; which water being a little more than 11.33 times as high as the lead is thick, is sufficient to balance and support the lead at the depth *K E*. If a little water be poured into the tube upon the lead, it will increase the weight upon the column of water under the lead, and cause the lead to fall from the tube to the bottom of the glass vessel, where it will lie in the situation *b d*. Or, if the tube be raised a little in the water, the lead will fall by its own weight, which will then be too great for the pressure of the water around the tube upon the column of water below it.

How light wood may be made to lie at the bottom of water.

Let two pieces of wood be planed quite flat, so that no water may get in between them when they are put together; let one of the pieces, as *b d*, be cemented to the bottom of the vessel *A B* (Fig. 7), and the other piece be laid flat and close upon it, and held down to it by a stick, while water is poured into the vessel; then remove the stick, and the upper piece of wood will not rise from the lower one; for, as the upper one is pressed down both by its own weight and the weight of all the water over it, while the contrary pressure of the water is kept off by the wood under it, it will lie as still as a stone would do in its place. But if it be raised ever so little at any edge, some water will then get under it, which being acted upon by the water above, will immediately press it upward; and as it is lighter than its bulk of water, it will rise and float upon the surface of the water.

All fluids weigh just as much in their own elements as they do in open air. To prove this by experiment, let as much shot be put into a phial, as, when corked, will make it sink in water; and, being thus charged, let it be weighed, first in air, and then in water, and the weights in both cases written down. Then, as the phial hangs suspended in water, and counterpoised, pull out the cork, that water may run into it, and it will descend, and pull down that end of the beam. This done, put as much weight into the opposite scale as will restore the equipoise; which weight will be be found to answer exactly to the additional weight of the phial when it is again weighed in air, with the water in it.

The velocity of spouting water.

The velocity with which water spouts out at a hole in the side or bottom of a vessel, is as the square root[4] of the depth or distance of the hole below the surface of the water. For, in order to make double the quantity of a fluid run through one hole as through another of the same size, it will require four times the pressure of the other, and therefore must be four times the depth of the other below the surface of the water; and for the same reason, three times the quantity running in an equal time through the same sort of hole, must run with three times the velocity, which will require nine times the pressure; and consequently must be nine times as deep below the surface of the fluid; and so on. To prove this by an experiment, let two pipes, as *C* and *g* (Fig. 11), of equal-sized bores, be fixed into the side of the vessel *A B*; the pipe *g* being four times as deep below the surface of the water at *b* in the vessel as the pipe *C* is: and while these pipes run, let water be constantly poured into the vessel, to keep the surface still at the same height. Then, if a cup that holds a pint be so placed as to receive the water that spouts from the pipe *C*, and at the same moment a cup that holds a quart be so placed as to receive the water that spouts from the pipe *g*, both cups will be filled at the same time by their respective pipes.[5]

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[4] The square root of any number is that which being multiplied by itself produces the said number. Thus, 2 is the square root of 4, and 3 is the square root of 9; for 2 multiplied by 2 produces 4, and 3 multiplied by 3 produces 9, &c.

The horizontal distance to which water will spout from pipes.

The horizontal distance to which a fluid will spout from a horizontal pipe, in any part of the side of an upright vessel below the surface of the fluid, is equal to twice the length of a perpendicular to the side of the vessel, drawn from the mouth of the pipe to a semicircle described upon the altitude of the fluid: and therefore, the fluid will spout to the greatest distance possible from a pipe, whose mouth is at the centre of the semicircle; because a perpendicular to its diameter (supposed parallel to the side of the vessel) drawn from that point, is the longest that can possibly be drawn from any part of the diameter to the circumference of the semicircle. Thus, if the vessel *A B* (Fig. 11) be full of water, the horizontal pipe *D* be in the middle of its side, and the semicircle *N d c b* be described upon *D* as a centre, with the radius or semidiameter *D g N* or *D f b*, the perpendicular *D d* to the diameter *N D b* is the longest that can be drawn from any part of the diameter to the circumference *N d c b*. And if the vessel be kept full, the jet *G* will spout from the pipe *D*, to the horizontal distance *N M*, which is double the length of the perpendicular *D d*. If two other pipes, as *C* and *E*, be fixed into the side of the vessel at equal distances above and below the pipe *D*, the perpendiculars Cc and Ee, from these pipes to the semicircle, will be equal; and the jets *F* and *H* spouting from them will each go to the horizontal distance NK; which is double the length of either of the equal perpendiculars Cc or Ee.

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[5] From a variety of accurate experiments on the motion of fluids, the following results have been deduced.

1. That the quantities of water expended, in equal times, by different orifices, at the same height in the reservoir, are to one another, very nearly, as the areas of the orifices, or the squares of their diameters.

2. That the quantities of water expended in equal times by the same orifice at different heights in the reservoir are very nearly as the square roots of the corresponding altitudes of the water in the reservoir, above the centre of the similar orifices.

3. That, in general, the quantities of water expended in equal times, by different orifices, and at different heights in the reservoir, are to one another in the compound ratio of the areas of the orifices, and the square roots of the heights in the reservoir.—*Ed.*

How water may be conveyed over hills and valleys.

Fluids by their pressure may be conveyed over hills and valleys in bended pipes, to any height not greater than the level of the spring from whence they flow.[6] But when they are designed to be raised higher than the springs, forcing engines must be used, which shall be described when we come to treat of pumps.[7]

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[6] It frequently happens, that in the winter season a supply of water is cut off by the congelation of the water in the pipes; and the tubes themselves are often burst by the expansion that takes place during the freezing of the included water.—For remedying these inconveniences, Mr. Wright of Kennington recommends the application of an air-valve, by means of which the conduit pipes may be kept empty, when there is no occasion for a supply of water.—For a description of this valve, and for farther information upon this useful subject, we must refer the reader to the *Philosophical Magazine* for July 1804, No. 74, p. 147.—*Ed.*

[7] In pipes employed for conveying water, their thickness should be in the compound ratio of their diameters, and the perpendicular height of the water directly, and as the strength of the materials inversely; that is, if a be the perpendicular altitude of the water in the pipe, d its diameter, and s the relative strength of the materials, $\frac{ad}{s}$ will represent the thickness of the pipe. Now Roemer (*Mem. Adopt. par l'Acad.* t. v, p. 116) found that a leaden pipe, 16 inches in diameter, and filled with water to the height of 50 feet, should be $6\frac{1}{3}$ lines thick. Let it be required, therefore, to find the thickness of an iron pipe for holding a column of water, 20 feet high and 10 inches in diameter. Then since in Roemer's standard experiment, $a = 600$ inches, $d = 16$ inches, and $s = 1$, the relative strength of lead, $\frac{ad}{s} = \frac{600 \times 16}{1}$; and since, in the present example, $a = 240$ inches, $d = 10$ inches, and $s = 79$, the relative strength of iron, $\frac{ad}{s} = \frac{240 \times 10}{79}$. But the pipe employed by Roemer required a thickness of $6\frac{1}{3}$ lines. Therefore, as $\frac{600 \times 16}{1} : 6\frac{1}{3} = \frac{240 \times 10}{79} : \frac{1}{50}$ of a line, the thickness of an iron pipe required. When the conduit pipes are horizontal, and made of lead, their thickness,
according to Bossuet, should be........................... $2\frac{1}{2}$, 3, 4, 5, 6, 7, 8, lines,
when their diameters are...... .... 1, $1\frac{1}{2}$, 2, 3, $4\frac{1}{2}$, 6, 7, inches;
and when these pipes are made of iron, their thickness
should be... 1, 3, 4, 5, 6, 7, 8, lines,
when their diameters are,.................................. 1, 2, 4, 6, 8, 10, 12, inches.
It is curious, and well deserving the attention of plumbers, that the tenacity of lead is increased five times by adding 1 part of zinc to 8 of lead.—*Ed.*

The *syphon.*

A *syphon*, generally used for decanting liquors, is a bended pipe, whose legs are of unequal lengths; and the shortest leg must always be put into the liquor intended to be decanted, that the perpendicular altitude of the column of liquor in the other leg may be longer than the column in the immersed leg, especially above the surface of the water; for, if both columns were equally high in that respect, the atmosphere, which presses as much upward as downward, and therefore acts as much upward against the column in the leg that hangs without the vessel, as it acts downward upon the surface of the liquor in the vessel, would hinder the running of the liquor through the syphon, even though it were brought over the bended part by suction: so that there is nothing left to cause the motion of the liquor, but the superior weight of the column in the longer leg, on account of its having the greater perpendicular height.

Let *D* (Fig. 12) be a cup filled with water to *C*, and *A B C* a syphon, whose shorter leg *B C F* is immersed in the water from *C* to *F*. If the end of the other leg were no lower than the line *A C*, which is level with the surface of the water, the syphon would not run, even though the air should be drawn out of it at the mouth *A*. For although the suction would draw some water at first, yet the water would stop at the moment the suction ceased; because the air would act as much upward against the water at *A*, as it acted downward for it by pressing on the surface at *C*. But if the leg *A B* comes down to *G*, and the air be drawn out at *G* by suction, the water will immediately follow, and continue to run, until the surface of the water in the cup comes down to *F;* because, till then, the perpendicular height of the column *B A G* will be greater than that of the column *C B;* and, consequently, its weight will be greater, until the surface comes down to *F;* and then the syphon will stop, though the leg *C F* should reach to the bottom of the cup. For which reason, the leg that hangs without the cup is always made long enough to reach below the level of its bottom, as from *d* to *E;* and then, when the syphon is emptied of air by suction at *E*, the water immediately follows, and by its continuity brings away the whole from the cup; just as pulling one end of a thread will make the whole clue follow.

If the perpendicular height of a syphon, from the surface of the water to its bended top at *B*, be more than 33 feet, it will draw no water, even though the other leg were much longer, and the syphon quite emptied of air; because the weight of a column of water 33 feet high, is equal to the weight of as thick a column of air, reaching from the surface of the earth to the top of the atmosphere; so that there will then be an equilibrium, and, consequently, though there would be weight enough of air upon the surface *C* to make the water ascend in the leg *C B* almost to the height *B*, if the syphon were emptied of air, yet that weight would not be sufficient to force the water over the bend; and, therefore, it could never be brought over into the leg *B A G*.[8]

Tantalus's cup.

Let a hole be made quite through the bottom of the cup *A* (Fig. 13), and the longer leg of the bended syphon *D E B G* be cemented into the hole, so that the end *D* of the shorter leg *D E* may almost touch the bottom of the cup within.

Then, if water be poured into this cup, it will rise in the shorter leg by its upward pressure, driving out the air all the way before it through the longer leg; and when the cup is filled above the bend of the syphon at *F*, the pressure of the water in the cup will force it over the bend of the syphon; and it will descend in the longer leg *C B G*, and run through the bottom, until the cup be emptied.

This is generally called *Tantalus's cup*, and the legs of the syphon in it are almost close together; and a little hollow statue, or figure of a man, is sometimes put over the syphon to conceal it; the bend *E* being within the neck of the figure as high as the chin. So that poor thirsty *Tantalus* stands up to the chin in water, imagining it will rise a little higher, and he may drink; but instead of that, when the water comes up to his chin, it immediately begins to descend, and so, as he cannot stoop to follow it, he is left as much pained with thirst as ever.

The *fountain at command*. Plate V. Fig. 1.

The device called *the fountain at command*, acts upon the same principle with the syphon in the cup. Let two vessels *A* and *B* be joined together by the pipe *C* which opens into them both. Let *A* be open at top, *B* close both at top and bottom, save only a small hole at *b* to let the air get out of the vessel *B*, and *A* be of such a size as to hold about six times as much water as *B*. Let a syphon *D E F* be soldered to the vessel *B* at *e*, so that the part *D E e* may be within the vessel, and *F* without it; the end *D* almost touching the bottom of the vessel, and the end *F* below the level of *D;* the vessel *B* hanging to *A* by the pipe *C* (soldered into both), and the whole supported by the pillars *G* and *H* upon the stand *I*. The bore of the pipe must be considerably less than the bore of the syphon.

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[8] An improvement on the syphon, by James Hunter, Esq. of Thurston, will be found described in the *Edinburgh Philosophical Journal*, vol. i, p. 204.

The whole being thus constructed, let the vessel *A* be filled with water, which will run through the pipe *C*, and fill the vessel *B*. When *B* is filled above the top of the sypon at *E*, the water will run through the syphon, and be discharged at *F*. But as the bore of the syphon is larger than the bore of the pipe, the syphon will run faster than the pipe, and will soon empty the vessel *B;* upon which the water will cease from running through the syphon at *F*, until the pipe *C* refills the vessel *B*, and then it will begin to run as before. And thus the syphon will continue to run and stop alternately until all the water in the vessel *A* has run through the pipe *C*. So that after a few trials, one may easily guess about what time the syphon will stop, and when it will begin to run; and then, to amuse others, he may call out *stop*, or *run*, accordingly.

Intermitting springs. Fig. 2.

Upon this principle, we may easily account for *intermitting*, or *reciprocating springs*. Let *A A* be part of a hill, within which there is a cavity *B B;* and from this cavity a vein or channel running in the direction *B C D E*. The rain that falls upon the side of the hill will sink and strain through the small pores and crannies *G*, *G*, *G*, *G*, and fill the cavity with water *K*. When the water rises to the level *H H C*, the vein *B C D E* will be filled to *C*, and the water will run through *C D F* as through a syphon; which running will continue until the cavity be emptied, and then it will stop until the cavity be filled again.

The *common pump.*

The *common pump*[9] (improperly called the *sucking pump)* with which we draw water out of wells, is an engine both pneumatic and hydraulic. It consists of a pipe open at both ends, in which is a moveable piston or bucket, as big as the bore of the pipe in that part wherein it works;

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[9] The pump was invented about 120 years before Christ, by *Ctesibius*, a mathematician in Alexandria.—*Ed.*

and is leathered round, so as to fit the bore exactly; and may be moved up and down, without suffering any air to come between it and the pipe or pump barrel.

We shall explain the construction both of this and the forcing pump by pictures of glass models, in which both the action of the pistons and motion of the valves are seen.[1]

Hold the model *D C B L* (Fig. 3) upright in the vessel of water *K*, the water being deep enough to rise at least as high as from *A* to *L*. The valve *a* on the moveable bucket *G* and the valve *b* on the fixed box *H* (which box quite fills the bore of the pipe or barrel at *H)* will each lie close by its own weight, upon the hole in the bucket and box, until the engine begins to work. The valves are made of brass, and lined underneath with leather for covering the holes the more closely; and the bucket *G* is raised and depressed alternately by the handle *E* and rod *D d*, the bucket being supposed at *B* before the working begins.

Take hold of the handle *E*, and thereby draw up the bucket from *B* to *C*, which will make room for the air in the pump all the way below the bucket to dilate itself, by which its spring is weakened, and then its force is not equivalent to the weight or pressure of the outward air upon the water in the vessel *K*; and therefore, at the first stroke, the outward air will press up the water through the notched foot *A*, into the lower pipe, about as far as *e*: this will condense the rarified air in the pipe between *e* and *C* to the same state it was in before; and then, as its spring within the pipe is equal to the force or pressure of the outward air, the water will rise no higher by the first stroke; and the valve *b*, which was raised a little by the dilatation of the air in the pipe, will fall, and stop the hole in the box *H*; and the surface of the water will stand at *e*. Then, depress the piston or bucket from *C* to *B*, and as the air in the part *B* cannot get back again through the valve *b*, it will (as the bucket descends), raise the valve *a*, and so make its way through the upper part of the barrel *d* into the open air. But upon raising the bucket *G* a second time, the air between it and the water in the lower pipe at *a* will be again left at liberty to fill a larger space; and so its spring being again weakened, the pressure of the outward air on the water in the vessel *K* will force more water up into the lower pipe from *e* to *f*; and when the bucket is at its greatest height *C*, the lower valve *b* will fall, and stop the hole in the box *H* as before. At the next stroke of the bucket or piston, the water will rise through the box *H* toward *B*, and then the valve *b*, which was raised by it, will fall when the bucket *G* is at its greatest height. Upon depressing the bucket again, the water cannot be pushed back through the valve *b*, which keeps close upon the hole while the piston descends. And, upon raising the piston again, the outward pressure of the air will force the water up through *H*, where it will raise the valve, and follow the bucket to *C*. Upon the next depression of the bucket *G*, it will go down into the water in the barrel *B*; and as the water cannot be driven back through the new closed valve *b*, it will raise the valve *a* as the bucket descends, and will be lifted up by the bucket when it is next raised. And now, the whole space below the bucket being full, the water above it cannot sink when it is next depressed; but, upon its depression, the valve *a* will rise to let the bucket go down; and when it is quite down, the valve *a* will fall by its weight, and stop the hole in the bucket. When the bucket is next raised, all the water above it will be lifted up, and begin to run off by the pipe *F*. And thus, by raising and depressing the bucket alternately, there is still more water raised by it; which getting above the pipe *F*, into the wide top *I*, will supply the pipe, and make it run with a continued stream.

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[1] There are three kinds of valves, the *clack valve*, the *butterfly valve*, and the *button* or *tail valve*. The *clack valve*, which is represented by *b* in Figures 3 and 4, consists merely of a circular piece of leather covering the aperture of the pipe, and moving on a hinge sometimes made of metal. The *butterfly valve*, which is superior to the former, consists of two semicircular pieces of leather, moving round their diameters, which are fixed on a bar, placed across the opening in the piston. The *button valve* is composed of a plate of brass, with a conical edge, ground in such a manner as to fit the conical cavity in which it lies. A cylindrical tail rises at right angles from its under side, and passes through a bar which lies across the bottom of the box. A little knob is placed at the under part of the tail, to prevent the valve from rising too high. Sometimes valves are made in the form of pyramids. They consist of four triangular flaps, which represent the sides of the pyramid, and move upon hinges fixed on the circumference of the opening. Their vertices meet in the middle of the opening, and are supported by four bars which meet in the centre.—*Ed.*

So at every time the bucket is raised, the valve *b* rises, and the valve *a* falls; and at every time the bucket is depressed, the valve *b* falls, and *a* rises.

As it is the pressure of the air or atmosphere which causes the water to rise and follow the piston or bucket *G* as it is drawn up ; and since a column of water 33 feet high is of equal weight with as thick a column of the atmosphere, from the earth to the very top of the air ; therefore, the perpendicular height of the piston or bucket from the surface of the water in the well must always be less than 33 feet ; otherwise the water will never get above the bucket. But, when the height is less, the pressure of the atmosphere will be greater than the weight of the water in the pump, and will therefore raise it above the bucket ; and when the water has once got above the bucket, it may be lifted thereby to any height, if the rod *D* be made long enough, and a sufficient degree of strength be employed, to raise it with the weight of the water above the bucket.

The force required to work a pump will be as the height to which the water is raised, and as the square of the diameter of the pump-bore, in that part where the piston works. So that, if two pumps be of equal heights, and one of them be twice as wide in the bore as the other, the widest will raise four times as much water as the narrowest ; and will therefore require four times as much strength to work it.

The wideness or narrowness of the pump, in any other part beside that in which the piston works, does not make the pump either more or less difficult to work, except what difference may arise from the friction of the water in the bore ; which is always greater in a narrow bore than in a wide one, because of the greater velocity of the water.

The pump-rod is never raised directly by such a handle as *E* at the top, but by means of a lever, whose longer arm (at the end of which the power is applied) generally exceeds the length of the shorter arm five or six times ; and, by that means, it gives five or six times as much advantage to the power.[2] Upon these principles, it will be easy to find the dimensions of a pump that shall work with a given force, and draw water from any given depth. But, as these calculations have been generally neglected by pump-makers (either for want of skill or industry), the following table was calculated by the late ingenious Mr. Booth for their benefit.[3] In this calculation, he supposed the handle of the pump to be a lever increasing the power five times; and had often found, that a man can work a pump four inches diameter, and 30 feet high, and discharge 27½ gallons of water (English wine measure) in a minute. Now, if it be required to find the diameter of a pump, that shall raise water with the same ease from any other height above the surface of the *well;* look for that height in the first column, and over against it in the second you have the diameter or width of the pump; and, in the third, you find the quantity of water which a man of ordinary strength can discharge in a minute.

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[2] When it is necessary that more than one man should be employed, much power would be gained by fixing a handle at right angles to the lever, and at the extremity of its longer arm. Supposing three men to work at a lever, whose longer arm is 6 feet, and the shorter arm 1 foot, the outermost will work with a force which may be called 6 ; the second, who must be nearer the centre of motion, with a force not much greater than 5 ; and the third with a force nearly equal to 4 ; so that their united exertions will be equal to 15 ; whereas, by adopting the method proposed, each workman would exert a force equal to 6, and their united efforts would amount to 18. This cross handle might be so made as to take off and put on at pleasure.—*Ed.*

Height of the pump above the surface of the well.	Diameter of the bore where the bucket works.		Water discharged in a minute, English wine measure.	
Feet.	Inches.	100 parts.	Gallons.	Pints.
10	6	.93	81	6
15	5	.66	54	4
20	4	.90	40	7
25	4	.38	32	6
30	4	.00	27	2
35	3	.70	23	3
40	3	.46	20	3
45	3	.27	18	1
50	3	.10	16	3
55	2	.95	14	7
60	2	.84	13	5
65	2	.72	12	4
70	2	.62	11	5
75	2	.53	10	7
80	2	.45	10	2
85	2	.38	9	5
90	2	.31	9	1
95	2	.25	8	5
100	2	.19	8	1

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[3] I have taken the liberty to make a few alterations in Mr. Booth's numbers in the table, and to lengthen it out from 80 feet to 100.

The *forcing-pump.*

The *forcing pump* raises water through the box *H* (Fig. 4), in the same manner as the common pump does, when the plunger or piston *g* is lifted up by the rod *D d.* But this plunger has no hole through it, to let the water in the barrel *B C* get above it, when it is depressed to *B*, and the valve *b* (which arose by the ascent of the water through the box *H* when the plunger *g* was drawn up) falls down and stops the hole in *H*, the moment that the plunger is raised to its greatest height. Therefore, as the water between the plunger *g* and box *H* can neither get through the plunger upon its descent, nor back again into the lower part of the pump *L e*, but has a free passage by the cavity around *H* into the pipe *M M*, which opens into the air-vessel *K K* at *P*; the water is forced through the pipe *M M* by the descent of the plunger, and driven into the air-vessel; and in running up through the pipe at *P*, it opens the valve *a*; which shuts at the moment the plunger begins to be raised, because the action of the water against the under side of the valve then ceases.

The water being thus forced into the air-vessel *K K* by repeated strokes of the plunger, gets above the lower end of the pipe *G H I*, and then begins to condense the air in the vessel *K K.* For, as the pipe *G H* is fixed air-tight into the vessel below *F*, and the air has no way to get out of the vessel but through the mouth of the pipe at *I*, and cannot get out when the mouth *I* is covered with water, and is more and more condensed as the water rises upon the pipe, the air then begins to act forcibly by its spring against the surface of the water at *H*: and this action drives the water up through the pipe *I H G F*, from whence it spouts in a jet *S* to a great height; and is supplied by alternately raising and depressing of the plunger *g*, which constantly forces the water that it raises through the valve *H*, along the pipe *M M*, into the air-vessel *K K.*

The higher that the surface of the water *H* is raised in the air-vessel, the less space will the air be condensed into, which before filled that vessel; and therefore the force of its spring will be so much the stronger upon the water, and will drive it with the greater force through the pipe at *F*: and as the spring of the air continues while the plunger *g* is rising, the stream or jet *S* will be uniform, as long as the action of the plunger continues: and, when the valve *b* opens, to let the water follow the plunger upward, the valve *a* shuts, to hinder the water, which is forced into the air-vessel, from running back by the pipe *MM* into the barrel of the pump.

If there was no air-vessel to this engine, the pipe *GHI* would be joined to the pipe *MMN* at *P*; and then, the jet *S* would stop every time the plunger is raised, and run only when the plunger is depressed.

Mr. Newsham's water-engine, for extinguishing fire, consists of two forcing pumps, which alternately drive water into a close vessel of air, and, by forcing the water into that vessel, the air in it is thereby condensed, and compresses the water so strongly, that it rushes out with great impetuosity and force through a pipe that comes down into it, and makes a continued uniform stream by the condensation of the air upon its surface in the vessel.[4]

By means of forcing-pumps, water may be raised to any height above the level of a river or spring; and machines may be contrived to work these pumps, either by a running stream, a fall of water, or by horses. An instance in each sort will be sufficient to shew the method.

Plate V. Fig. 5.

First, by a running stream, or a fall of water. Let *AA* be a wheel, turned by the fall of water *BB*; and have any number of cranks (suppose six) as *C*, *D*, *E*, *F*, *G*, *H*, on its axis, according to the strength of the fall of water, and the height to which the water is intended to be raised by the engine. As the wheel turns round, these cranks move the levers, *c*, *d*, *e*, *f*, *g*, *h*, up and down by the iron rods *i*, *k*, *l*, *m*, *n*, *o*; which alternately raise and depress the pistons by the other iron rods *p*, *q*, *r*, *s*, *t*, *u*, *w*, *x*, *y*, in twelve pumps; nine whereof, as *L*, *M*, *N*, *O*, *P*, *Q*, *R*, *S*, *T*, appear in the plate, the other three being hid behind the work at *V*. And as pipes may go from all these pumps to convey the water drawn up by them to a small height, into a close cistern, from which the main pipe goes off, the water will be forced into this cistern by the descent of the pistons. And as each pipe, going from its respective pump into the cistern, has

A pump engine to go by water.

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[4] A curious machine for extinguishing fire was invented by one Greyl, in which a vessel full of water is dispersed in every direction, by the explosion of a quantity of gun-powder placed in its centre, and kindled by a match fitted for the purpose. Mr. Godfrey's idea of a water-bomb, for the same purpose, seems to be derived from the above machine.—*Ed.*

a valve at its end in the cistern, these valves will hinder the return of the water by the pipes; and, therefore, when the cistern is once full, each piston upon its descent will force the water (conveyed into the cistern by a former stroke) up the main pipe, to the height the engine was intended to raise it: which height depends upon the quantity raised, and the power that turns the wheel. When the power upon the wheel is lessened by any defect of the quantity of water turning it, a proportionable number of the pumps may be set aside, by disengaging their rods from the vibrating levers.

This figure is a representation of the engine erected at Blenheim for the duke of Marlborough, by the late ingenious Mr. Aldersea. The water-wheel is 7½ feet in diameter, according to Mr. Switzer's account in his Hydraulics.

When such a machine is placed in a stream that runs upon a small declivity, the motion of the levers and action of the pumps will be but slow, since the wheel must go once round for each stroke of the pumps. But, when there is a large body of slow running water, a cog or spur-wheel may be placed upon each side of the water-wheel *A A*, upon its axis, to turn a trundle upon each side; the cranks being upon the axis of the trundle. And, by proportioning the cog-wheels to the trundles, the motion of the pumps may be made quicker, according to the quantity and strength of the water upon the first wheel, which may be as great as the workman pleases, according to the length and breadth of the float-boards or wings of the wheel. In this manner, the engine for raising water at London-bridge is constructed; in which the water-wheel is 20 feet diameter, and the floats 14 feet long.

A pump engine to go by horses.

Where a stream or fall of water cannot be had, and gentlemen want to have water raised, and brought to their houses from a rivulet or spring, this may be effected by a horse-engine working three forcing pumps which stand in a reservoir filled by the spring or rivulet; the pistons being moved up and down in the pumps by means of a triple crank *A B C* (Fig. 6), which, as it is turned round by the trundle *G*, raises and depresses the rods *D*, *E*, *F*. The trundle may be turned by such a wheel as *F*, in Fig. 5 of Plate IV, having levers *y*, *y*, *y*, *y*, on its upright axle, to which horses may be joined for working the engine. And, if the wheel has three times as many cogs as the trundle has staves or rounds, the trundle and cranks will make three revolutions for every one of the wheel; and, as each crank will fetch a stroke in the time it goes round, the three cranks will make nine strokes for every turn of the great wheel.

The cranks should be made of cast iron, because that will not bend; and they should each make an angle of 120 with both of the others, as at *a*, *b*, *c* (Fig. 8), which is (as it were) a view of their radii, in looking endwise at the axis; and then there will be always one or other of them going downward, which will push the water forward with a continued stream into the main pipe. For, when *b* is almost at its lowest position, and is, therefore, just beginning to lose its action upon the piston which it moves, *c* is beginning to move downward, which will, by its piston, continue the propelling force upon the water; and, when *c* is come down to the position of *b*, *a* will be in the position of *c*.

The more perpendicularly the piston rods move up and down in the pumps, the freer and better will their strokes be: but a little deviation from the perpendicular will not be material. Therefore, when the pump-rods *D*, *E*, and *F* (Fig. 6), go down into a deep well, they may be moved directly by the cranks, as is done in a very good horse engine of this sort at the late Sir James Creed's at Greenwich, which forces up water about 64 feet from a well under ground, to a reservoir on the top of his house. But when the cranks are only at a small height above the pumps, the pistons must be moved by vibrating levers, as in the above engine at Blenheim; and the longer the levers are, the nearer will the strokes be to a perpendicular.

The quantity of water that may be raised by a horse engine.

Let us suppose, that in such an engine as Sir James Creed's, the great wheel is 12 feet diameter, the trundle 4 feet, and the radius or length of each crank 9 inches, working a piston in its pump. Let there be three pumps in all, and the bore of each pump be four inches diameter. Then, if the great wheel has three times as many cogs as the trundle has staves, the trundle and cranks will go three times round for each revolution of the horses and wheel, and the three cranks will make nine strokes of the pumps in that time, each stroke being 18 inches (or double the length of the crank) in a four-inch bore. Let the diameter of the horse-walk be 18 feet,[5] and the perpendicular height to which the water is raised above the surface of the well be 64 feet.

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[5] The diameter of the horse-walk should never be less than *forty* feet, when the nature of the place will permit; for when it is only 18 feet, the horse will pull with

If the horses go at the rate of two miles an hour (which is very moderate walking), they will turn the great wheel 187 times round in an hour.

In each turn of the wheel the pistons make 9 strokes in the pumps, which amount to 1683 in an hour.

Each stroke raises a column of water 18 inches long, and four inches thick, in the pump barrels; which column, upon the descent of the piston, is forced into the main pipe, whose perpendicular altitude above the surface of the well is 64 feet.

Now, since a column of water 18 inches long and 4 inches thick, contains 226.18 cubic inches, this number multiplied by 1683 (the strokes in an hour), gives 380,661 for the number of cubic inches of water raised in an hour.

A gallon, in wine measure, contains 231 cubic inches, by which divide 380,661, and it quotes 1468 in round numbers, for the number of gallons raised in an hour; which, divided by 63, gives $26\frac{1}{2}$ hogsheads. If the horses go faster, the quantity raised will be so much the greater.

In this calculation it is supposed that no water is wasted by the engine. But as no forcing engine can be supposed to lose less than a fifth part of the calculated quantity of water, between the pistons and barrels, and by the opening and shutting of the valves, the horses ought to walk almost $2\frac{1}{2}$ miles *per* hour, to fetch up this loss.

A column of water 4 inches thick, and 64 feet high, weighs $349\frac{9}{16}$ pounds avoirdupois, or $424\frac{5}{12}$ pounds troy; and this weight, together with the friction of the engine, is the resistance that must be overcome by the strength of the horses.

The horse-tackle should be so contrived, that the horses may rather push on, than drag the levers after them. For if they draw, in going round the walk, the outside leather straps will rub against their sides and hams, which will hinder them from drawing at right angles to the levers, and so make them pull at a disadvantage. But if they push the levers before their

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scarcely *one half* of his strength. A considerable portion of the animal's force is wasted in continually bending itself into the circular path; and the power thus lost, evidently decreases with the length of the lever. The most advantageous direction, too, in which any animal can exert its strength, is at right angles to the lever which it moves, or in the tangent to its circular walk; and it is sufficiently obvious, that as you lengthen the lever, or the radius of the horse's orbit, the line of traction approaches nearer and nearer the tangent.

breasts, instead of dragging them, they can always walk at right angles to these levers.[6]

It is nowise material what the diameter of the main or conduit pipe be; for the whole resistance of the water therein, against the horses, will be according to the height to which it is raised, and the diameter of that part of the pump in which the piston works, as we have already observed. So that by the same pump, an equal quantity of water may be raised in (and consequently made to run from) a pipe of a foot diameter, with the same ease as in a pipe of five or six inches; or rather with more ease, because its velocity in a large pipe will be less than in a small one; and therefore its friction against the sides of the pipe will be less also.

And the force required to raise water depends not upon the length of the pipe, but upon the perpendicular height to which it is raised therein above the level of the spring. So that the same force which would raise water to the height *A B*, in the upright pipe *A i k l m n o p q B*, will raise it to the same height or level *B I H* in the oblique pipe *A E F G H*. For the pressure of the water at the end *A* of the latter, is no more than its pressure against the end *A* of the former.

Plate V. Fig. 7.

The weight or pressure of water at the lower end of the pipe, is always as the sine of the angle to which the pipe is elevated above the level parallel to the horizon. For, although the water in the upright pipe *A B* would require a force applied immediately to the lower end *A* equal to the weight of all the water in it, to support the water, and a little more to drive it up, and out of the pipe; yet, if that pipe be inclined from its upright position to an angle of 80 degrees (as in *A* 80), the force required to support or raise the same cylinder of water, will then be as much less, as the sine 80 *h* is less than the radius *A B*; or as the sine of 80 degrees is less than the sine of 90. And so on, decreasing as the sine of the angle of elevation lessens, until it arrives at its level *A C* or place of rest, where the force of the water is nothing at either end of the pipe. For, although the absolute weight of the water is the same in all positions, yet its pressure at the lower end decreases, as the sine of the angle of elevation decreases, as will appear plainly by a farther consideration of the figure.

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[6] The advantages arising from the horse's walking at right angles to the levers, cannot compensate for the loss of power which will inevitably accompany the mode of action recommended by our author. It has been proved by experiment, that a horse will exert much more force in dragging a load, than in pushing it before him.—*Ed.*

Let two pipes, *A B* and *A C*, of equal length and bores, join each other at *A*; and let the pipe *A B* be divided into 100 equal parts, as the scale *S* is; whose length is equal to the length of the pipe. Upon this length, as a radius, describe the quadrant *B C D*, and divide it into 90 equal parts or degrees.

Let the pipe *A C* be elevated to 10 degrees upon the quadrant, and filled with water; then, part of the water that is in it will rise in the pipe *A B*, and if it be kept full of water, it will raise the water in the pipe *A B* from *A* to *i*; that is, to a level *i* 10 with the mouth of the pipe at 10: and the upright line *a* 10, equal to *A i*, will be the sine of 10 degrees elevation; which being measured upon the scale *S*, will be about 17.4 of such parts as the pipe contains 100 in length: and therefore, the force or pressure of the water at *A*, in the pipe *A* 10, will be to the force or pressure at *A* in the pipe *A B*, as 17.4 to 100.

Let the same pipe be elevated to 20 degrees in the quadrant, and if it be kept full of water, part of the water will run into the pipe *A B*, and rise therein to the height *A k*, which is equal to the length of the upright line *b* 20, or to the sine of 20 degrees elevation; which, being measured upon the scale *S*, will be 34.2 of such parts as the pipe contains 100 in length. And, therefore, the pressure of the water at *A*, in the full pipe *A* 20, will be to its pressure, if that pipe were raised to the perpendicular situation *A B*, as 34.2 to 100.

Elevate the pipe to the position *A* 30 on the quadrant, and if it be supplied with water, the water will rise from it into the pipe *A B*, to the height *A l*, or to the same level with the mouth of the pipe at 30. The sine of this elevation, or of the angle of 30 degrees, is *c* 30; which is just equal to half the length of the pipe, or to 50 of such parts of the scale, as the length of the pipe contains 100. Therefore, the pressure of the water at *A*, in a pipe elevated 30 degrees above the horizontal level, will be equal to one half of what it would be, if the same pipe stood upright in the situation *A B*.

And thus, by elevating the pipe to 40, 50, 60, 70, and 80 degrees on the quadrant, the sines of these elevations will be *d* 40, *e* 50, *f* 60, *g* 70, and *h* 80; which will be equal to the heights *Am*, *An*, *Ao*, *Ap*, and *Aq*: and these heights measured upon the scale *S* will be 63.3, 76.6, 86.6, 94.0, and 98.5; which express the pressures at *A* in all these elevations, considering the pressure in the upright pipe *A B*, as 100.

Sine of	Parts.	*Sine of*	Parts.	*Sine of*	Parts.
D. 1	17	D.31	515	D.61	875
2	35	32	530	62	883
3	52	33	545	63	891
4	70	34	559	64	899
5	87	35	573	65	906
6	104	36	588	66	913
7	122	37	602	67	920
8	139	38	616	68	927
9	156	39	629	69	934
10	174	40	643	70	940
11	191	41	656	71	945
12	208	42	669	72	951
13	225	43	682	73	956
14	242	44	695	74	961
15	259	45	707	75	966
16	276	46	719	76	970
17	292	47	731	77	974
18	309	48	743	78	978
19	325	49	755	79	982
20	342	50	766	80	985
21	358	51	777	81	988
22	375	52	788	82	990
23	391	53	799	83	992
24	407	54	809	84	994
25	423	55	819	85	996
26	438	56	829	86	997
27	454	57	839	87	998
28	469	58	848	88	999
29	485	59	857	89	1000
30	500	60	866	90	1000

Because it may be of use to have the lengths of all the sines of a quadrant from 0 degrees to 90, we have given the foregoing Table, shewing the length of the sine of every degree in such parts as the whole pipe (equal to the radius of the quadrant) contains 1000. Then the sines will be integral or whole parts in length. But if you suppose the length of the pipe to be divided only into 100 equal parts, the last figure of each part or sine must be cut off as a decimal; and then those which remain at the left hand of this separation will be integral or whole parts.

Thus, if the radius of the quadrant (supposed to be equal to the length of the pipe *A C*) be divided into 1000 equal parts, and the elevation be 45 degrees, the sine of that elevation will be equal to 707 of these parts: but if the radius be divided only into 100 equal parts, the same sine will be only 70.7, or 70$\frac{7}{10}$ of these parts. For, as 1000 is to 707, so is 100 to 70.7.

As it is of great importance to all engine-makers, to know what quantity and weight of water will be contained in an upright round pipe of a given diameter and height; so that by knowing what weight is to be raised, they may proportion their engines to the force which they can afford to work them, we shall subjoin tables shewing the number of cubic inches of water contained in an upright pipe of a round bore, of any diameter from one inch to six and a half; and of any height from one foot to two hundred; together with the weight of the said number of cubic inches, both in troy and avoirdupois ounces. The number of cubic inches divided by 231, will reduce the water to gallons in wine measure; and divided by 282, will reduce it to the measure of ale gallons. Also, the troy ounces divided by 12, will reduce the weight to troy pounds: and the avoirdupois ounces divided by 16, will reduce the weight to avoirdupois pounds.[7]

And here I must repeat it again, that the weight or pressure of the water acting against the power that works the engine, must always be estimated according to the perpendicular height to which it is to be raised, without any regard to the length of the conduit-pipe, when it has an oblique position, and as if the diameter of that pipe were just equal to the diameter of that part of the pump in which the piston works. Thus, by the following Tables, the pressure of the water, against an engine whose pump is of a 4½ inch bore, and the perpendicular height of the water in the conduit-pipe is 80 feet, will be equal to 8057.5 troy ounces, and to 8848.2 avoirdupois ounces; which makes 671.4 troy pounds, and 553 avoirdupois.

For any bore whose diameter exceeds 6½ inches, multiply the numbers on the following page, against any height (belonging to 1 inch diameter) by the square of the diameter of the given bore, and the products will be the number of cubic inches, troy ounces, and avoirdupois ounces of water, that the given bore will contain.

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[7] The contents of pipes of any size may be measured by the following short and easy method. Square the diameter of the pipe in inches, and the product will be the number of pounds of water, avoirdupois, contained in every yard's length of the pipe. If the last figure of this product be cut off, or considered as a decimal, the remaining figures will give the number of ale-gallons in a yard's length of the pipe; and if the product consist only of one figure, this figure will be tenths of an ale-gallon. The number of ale-gallons divided by 282, will give the number of cubic inches in every three feet of the pipe; and the contents of a pipe of a greater or less length may be found by proportion.—*Ed.*

1 Inch diameter.				1½ Inch diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupois ounces.	Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupois ounces.
1	9.42	4.97	5.46	1	21.21	11.19	12.29
2	18.85	9.95	10.92	2	42.41	22.38	24.58
3	28.27	14.92	16.38	3	63.62	33.57	36.87
4	37.70	19.89	21.85	4	84.82	44.76	49.16
5	47.12	24.87	27.31	5	106.03	55.95	61.45
6	56.55	29.84	32.77	6	127.23	67.15	73.73
7	65.97	34.82	38.23	7	147.44	78.34	86.02
8	75.40	39.79	43.69	8	169.65	89.53	98.31
9	84.82	44.76	49.16	9	190.85	100.72	110.60
10	94.25	49.74	54.62	10	212.06	111.91	122.89
20	188.49	99.48	109.24	20	424.12	223.82	245.78
30	282.74	149.21	163.86	30	636.17	335.73	368.68
40	376.99	198.95	218.47	40	848.23	447.64	491.57
50	471.24	248.69	273.09	50	1060.29	559 55	614.46
60	565.49	298.43	327.71	60	1272.35	671.46	737.35
70	659.73	348.17	382.33	70	1484.40	783.37	860.24
80	753.98	397.90	436.95	80	1696.46	895.28	983.14
90	848.23	447.64	491.57	90	1908.52	1007.19	1106.03
100	942.48	497.38	546.19	100	2120.58	1119.10	1228.92
200	1884.96	994.76	1092.38	200	4241.15	2238.20	2457.84

2 Inches diameter.				2½ Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupois ounces.	Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupois ounces.
1	37.70	19.89	21.85	1	58.90	31.08	34.14
2	75.40	39.79	43.69	2	117.81	62.17	68.27
3	113.10	59.68	65.54	3	176.71	93.26	102.41
4	150.80	79.58	87.39	4	235.62	124.34	136.55
5	188.50	99.47	109.24	5	294.52	155.43	170.68
6	226.19	119.37	131.08	6	353.43	186.52	204.82
7	263.89	139.26	152.93	7	412.33	217.60	238.96
8	301.59	159.16	174.78	8	471.24	248.69	273.09
9	339.29	179.06	196.63	9	530.14	279.77	307.23
10	376.99	198.95	218.47	10	589.05	310.86	341.37
20	753.98	397.90	436.95	20	1178.10	621.72	682.73
30	1130.97	596.85	655.42	30	1767.15	932.58	1024.10
40	1507.97	795.80	873.90	40	2356.20	1243.44	1365.47
50	1884.96	994.75	1092.37	50	2545.25	1554.30	1706.83
60	2261.95	1193.70	1310.85	60	3534.29	1865.16	2048.20
70	2638.94	1392.65	1529.32	70	4123.34	2176.02	2389.57
80	3015.93	1591.60	1747.80	80	4712.39	2486.88	2730.94
90	3392.92	1790.56	1966.27	90	5301.44	2797.74	3072.30
100	3769.91	1989.51	2184.75	100	5890.49	3108.60	3413.67
200	7539.82	3979.00	4369.50	200	11780.98	6217.20	4827.34

3 Inches diameter.				3½ Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupois ounces.	Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupois ounces.
1	84.8	44.76	49.16	1	115.4	60.9	66.9
2	169.6	89.53	98.31	2	230.9	121.8	133.8
3	254.5	134.29	147.47	3	346.4	182.8	200.7
4	239.3	179.06	196 63	4	461.8	243.7	267.6
5	424.1	223.82	245.78	5	577.3	304.6	334.5
6	508.9	268.58	294.94	6	692.7	365.6	401.4
7	593.7	313.35	344.10	7	808.2	426.5	468.4
8	698.6	358.11	393.25	8	923.6	487.4	535.3
9	763.4	402.87	442.41	9	1039.1	548.4	602.2
10	848.2	447.64	491.57	10	1154.5	609 3	669.1
20	1696.5	895.28	983.14	20	2309.1	1218.6	1338.2
30	2544 7	1342.92	1474.70	30	3463.6	1827.9	2007.2
40	3392.9	1790.56	1966.27	40	4618.1	2437.1	2676.3
50	4241.1	2238.19	2457.84	50	5772.7	3046.4	3345.4
60	5089.4	2685.83	2949.41	60	6927.2	3655.7	4014.5
70	5937.6	3133.47	3440.98	70	8081.8	4265.0	4683.6
80	6785.8	3581.11	3932.55	80	9236.3	4874.3	5352.6
90	7634.1	4028.75	4424.12	90	10390.8	5483.6	6021.7
100	8482.3	4476.39	4915.68	100	11545.4	6092.9	6690.8
200	16964.6	8952.78	9831.36	200	23099.7	12185.7	13381.5

4 Inches diameter.				4½ Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupois ounces.	Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupois ounces.
1	150.8	79.6	87.4	1	190.8	100.7	110.6
2	301.6	159.2	174.8	2	381.7	201.4	221.2
3	452.4	238.7	262.2	3	572.6	302.2	331.8
4	603.2	318.3	349.6	4	763.4	402.9	442.4
5	754.0	397.9	436.9	5	954.3	503.6	553.0
6	904.8	477.5	524.3	6	1145.1	604.3	663.6
7	1055.6	557.1	611.7	7	1338.0	705.0	774.2
8	1206.4	636.6	699.1	8	1526.8	805.7	884.8
9	1357.2	716.2	786.5	9	1717.7	906.5	995.4
10	1508.0	795.8	873.9	10	1908.5	1007.2	1106.0
20	3115.9	1591.6	1747.8	20	3817.0	2014.4	2212.1
30	4523.9	2387.4	2621.7	30	5725.6	3021.6	3818.1
40	6031.9	3183.2	3495.6	40	7634.1	4028.7	4424.1
50	7539.8	3997.0	4369.5	50	9542.6	5035.9	5520.1
60	9047.8	4774.8	5243.4	60	11451.1	6043.1	5636.2
10	10555.8	5570.6	6177.3	70	13359.6	7050.3	7742.2
80	12063.7	6366.4	6991.2	80	15268.2	8057.5	8848.2
90	13571.7	7162.1	7865.1	90	17176.7	9064.7	9954.3
100	15079.7	7958.0	8739.0	100	19085.2	10071.9	11060.3
200	30159.3	15916.0	17478.0	200	38170.4	20143.8	22120.6

5 Inches diameter.				5½ Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupois ounces.	Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupois ounces.
1	235.6	124.3	136.5	1	285.1	150.5	164.3
2	471.2	248.7	273.1	2	570.2	300.9	328.5
3	706.6	373.0	409.6	3	855.3	451.4	492.8
4	942.5	467.4	546.2	4	1140.4	601.8	657.1
5	1178.1	621.7	682.7	5	1425.5	752.3	821.3
6	1413.7	746.1	819.3	6	1710.6	902.7	985.6
7	1649.3	870.4	955.8	7	1995.7	1053.2	1149.9
8	1885.0	994.8	1092 4	8	2280.8	1203.6	1314.2
9	2120.6	1119.1	1228.9	9	2565.9	1354.1	1478.4
10	2356.2	1243.4	1365.5	10	2851.0	1504.6	1642.7
20	4712.4	2486.9	2730.9	20	5702.0	3009.1	3285.4
30	7068.6	3730.3	4096.4	30	8553.0	4513.7	4928.1
40	9424.8	4973.8	5461.9	40	11404.0	6018.2	6570.8
50	11780.0	6217.2	6827.3	50	14255.0	7522.8	8213.5
60	14137.2	7460.6	8192.8	60	17106.0	9027.4	9856.2
70	16493.4	8704.1	9558.3	70	19957.0	10531.9	11498.9
80	18849.6	9947.5	10923.7	80	22808.0	12036.5	13141.6
90	21205.8	11191.0	12289.2	90	25659.0	13541.1	14784.3
100	23562.0	12434.4	13654.7	100	28510.0	15045.6	16426.9
200	47124.0	24868.8	27309.3	200	57020.0	30091.2	32853.9

6 Inches diameter.				6½ Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupois ounces.	Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupois ounces.
1	339.3	179.1	196.6	1	398.2	210.1	230.7
2	678.6	358.1	393.3	2	797.4	420.2	461.4
3	1017.9	537.2	589.9	3	1195.6	630.4	692.1
4	1357.2	716.2	786.5	4	1593.8	840.6	922.8
5	1696.5	895.3	983.1	5	1991.9	1050.8	1153.6
6	2035.7	1074.3	1179.8	6	2390.1	1260.9	1384.3
7	2375.0	1253.4	1376.4	7	2788.3	1471.1	1615.0
8	2714.3	1432.4	1573.0	8	3186.5	1861.2	1845.7
9	3053.6	1611.5	1769.6	9	3854.7	1891.3	2076.4
10	3392.9	1790.6	1966.3	10	3982.9	2101.5	2307.1
20	6785.8	3581.1	3932.5	20	7965.8	4202.9	4614.3
30	10178.8	5371.7	5898.8	30	11948.8	6304.4	6921.4
40	13571.7	7162.2	7865.1	40	15931.7	8405.9	9228.6
50	16964.6	8952.8	9831.4	50	19914.6	10507.4	11535.7
60	20357.5	10743.3	11797.6	60	23897.9	12608.9	13842.9
70	23750.5	12533.9	13763.9	70	27880.5	14710.4	16150.0
80	24143.7	14324.4	15730.2	80	31863.4	16811.8	18457.2
90	30536.3	16115.0	17696.5	90	35746.3	18913.3	20764.3
100	33929.2	17905.6	19662.7	100	39829.3	21014.8	23071.5
200	67858.4	35811.2	39325.4	200	79658.6	42029.6	46143.0

EXAMPLE.—*Required the number of cubic inches, and the weight of the water, in an upright pipe 278 feet high, and 1½ inches diameter?*

Here the nearest single decimal figure is only taken into the account; and the whole being reduced by division, amounts to 25½ wine gallons in measure; to 259¼ pounds troy, and to 213½ pounds avoirdupois.

	Feet.	Cubic inches.	Troy ounces.	Avoirdup. ounces.
	200	4241.1	2238.2	2457.8
	70	1484.4	783.3	860.2
	8	169.6	89.5	98.3
Answer,	278	5895.1	3111.0	3416.3

These Tables were at first calculated to six decimal places, for the sake of exactness: but, in transcribing them, there are no more than two decimal figures taken into the account, and sometimes but one; because there is no necessity for computing to hundredth parts of an inch or of an ounce in practice. And, as they never appeared in print before, it may not be amiss to give the reader an account of the principles upon which they are constructed.

The solidity of cylinders is found by multiplying the areas of their bases by their altitudes. And Archimedes gives the following proportion for finding the area of a circle, and the solidity of a cylinder raised upon that circle:

As 1 is to 0.785399, so is the square of the diameter to the area of the circle. And as 1 is to 0.785399, so is the square of the diameter multiplied by the height to the solidity of the cylinder. By this analogy the solid inches and parts of an inch in the tables are calculated to a cylinder 200 feet high, of any diameter from 1 inch to 6½, and may be continued at pleasure.

The weight of running water.

And, as to the weight of a cubic foot of running water, it has been often found upon trial, by Dr. Wyberd and others, to be 76 pounds troy, which is equal to 62.5 pounds avoirdupois. Therefore, since there are 1,728 cubic inches in a cubic foot, a troy ounce of water contains 1.8949 cubic inches; and an avoirdupois ounce of water 1.72556 cubic inches.[8] Consequently, if the number of cubic inches contained in any given cylinder, be divided by 1.8949, it will give the weight in troy ounces; and divided by 1.72556 will give the weight in avoirdupois ounces. By this method, the weights shewn in the tables were calculated; and are near enough for any common practice.

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[8] It appears from the experiments of Sir George Clerk, Bart. and Dr. Wollaston, that an ounce avoirdupois of Thames water, whose apparent specific gravity was 1.00106, was equivalent to 1.731 cubic inches.—See *Edinburgh Encyclopædia*, Art. *Measures*, p. 491.—*Ed.*

The *fire-engine.*

The *Fire-engine* comes next in order to be explained; but as it would be difficult, even by the best plates, to give a particular description of its several parts, so as to make the whole intelligible, I shall only explain the principles upon which it is constructed.[9]

1. Whatever weight of water is to be raised, the pump rod must be loaded with weights sufficient for that purpose, if it be done by a forcing-pump, as is generally the case; and the power of the engine must be sufficient for the weight of the rod, in order to bring it up.

2. It is known that the atmosphere presses upon the surface of the earth with a force equal to 15 pounds upon every square inch.

3. When water is heated to a certain degree, the particles thereof repel one another, and constitute an elastic fluid, which is generally called *steam* or *vapour*.

4. Hot steam is very elastic: and when it is cooled by any means, particularly by its being mixed with cold water, its elasticity is destroyed immediately, and it is reduced to water again.

5. If a vessel be filled with hot steam, and then closed, so as to keep out the external air, and all other fluids; when that steam is by any means condensed, cooled, or reduced to water, *that* water will fall to the bottom of the vessel; and the cavity of the vessel will be almost a perfect vacuum.

6. Whenever a vacuum is made in any vessel, the air by its weight will endeavour to rush into the vessel, or to drive in any other body that will give way to its pressure: as may be easily seen by a common syringe. For, if you stop the bottom of a syringe, and then draw up the piston, if it be so tight as to drive out all the air before it, and leave a vacuum within the syringe, the piston being let go will be driven down with a great force.

7. The force with which the piston is driven down, when there is a vacuum under it, will be as the square of the diameter of the bore in the syringe: that is to say, it will be driven down with four times as much force in a syringe of a two-inch bore, as in a syringe of one inch; for the areas of circles are always as the squares of their diameters.

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[9] For a description and drawing of the *fire-engine*, or *steam-engine*, as it is now, with more propriety called, see the Appendix, vol. ii.—*Ed.*

8. The pressure of the atmosphere being equal to 15 pounds upon a square inch, it will be almost equal to 12 pounds upon a circular inch. So that if the bore of the syringe be round, and one inch in diameter, the piston will be pressed down into it by a force nearly equal to 12 pounds; but if the bore be two inches diameter, the piston will be pressed down with four times that force.

And hence it is easy to find with what force the atmosphere presses upon any given number, either of square or circular inches.

These being the principles upon which this engine is constructed, we shall next describe the chief working parts of it, which are, 1, A boiler; 2, A cylinder or piston; 3, A beam or lever.

The *boiler* is a large vessel made of iron or copper; and commonly so big as to contain about 2000 gallons.

The *cylinder* is about 40 inches diameter, bored so smooth, and its leathered piston fitting so close, that little or no water can get between the piston and sides of the cylinder.

Things being thus prepared, the cylinder is placed upright, and the shank of the piston is fixed to one end of the *beam*, which turns on a centre like a common balance.

The boiler is placed under the cylinder, with a communication between them, which can be opened and shut occasionally.

The boiler is filled about half full of water, and a strong fire is placed under it: then, if the communication between the boiler and the cylinder be opened, the cylinder will be filled with hot steam; which would drive the piston quite out at the top of it. But there is a contrivance by which the beam, when the piston is near the top of the cylinder, shuts the communication at the top of the boiler within.

This is no sooner shut, than another is opened, by which a little cold water is thrown upward in a jet into the cylinder, which mixing with the hot steam, condenses it immediately; by which means a vacuum is made in the cylinder, and the piston is pressed down by the weight of the atmosphere, and so lifts up the loaded pump-rod at the other end of the beam.

If the cylinder be 42 inches in diameter, the piston will be pressed down with a force greater than 20,000 pounds, and will consequently lift up that weight at the opposite end of the beam: and as the pump-rod with its plunger is fixed to that end, if the bore where the plunger works were 10 inches diameter, the water would be forced up through a pipe of 180 yards perpendicular height.

But, as the parts of this engine have a good deal of friction, and must work with a considerable velocity, and as there is no such thing as making a perfect vacuum in the cylinder, it is found that no more than 8 pounds of pressure must be allowed for on every circular inch of the piston in the cylinder, that it may make about 16 strokes in a minute, about 6 feet each.

Where the boiler is very large, the piston will make between 20 and 25 strokes in a minute, and each stroke 7 or 8 feet; which, in a pump of 9 inches bore, will raise upwards of 300 hogsheads of water in an hour.

It is found by experience that a cylinder, 40 inches diameter, will work a pump 10 inches diameter, and 100 yards long; and hence we can find the diameter and length of a pump, that can be worked by any other cylinder.

For the convenience of those who would make use of this engine for raising water, we shall subjoin part of a table, calculated by Mr. Beighton, shewing how any given quantity of water may be raised in an hour, from 48 to 400 hogsheads, at any given depth, from 15 to 100 yards; the machine working at the rate of 16 strokes *per* minute, and each stroke being 6 feet long.

One example of the use of this table will make the whole plain. Suppose it were required to draw 100 hogsheads *per* hour, at 90 yards depth: in the second column from the right hand, I find the nearest number, viz. 149 hogsheads 40 gallons, against which, on the right hand, I find the diameter of the bore of the pump must be 7 inches; and in the same collateral line, under the given depth 90, I find 27 inches the diameter of the cylinder fit for that purpose,—And so on for any other.

A TABLE SHEWING THE POWER OF THE ENGINE FOR RAISING WATER BY FIRE.

This table is calculated to the measure of ale gallons, at 282 cubic inches per gallon.

The depth to be drawn in yards.													In one hour.		Diam. of pump.
15	20	25	30	35	40	45	50	60	70	80	90	100	Hogs.	Galls.	Inches.
18½	21¾	24	26¼	29½	30½	32½	34¼	37¼	40				440		12
17	18¾	22	24	26¼	28	29¾	31¼	34¼	37	39½			569	33	11
15½	18	20	22	23¾	25¼	27	28¾	31¼	33¾	36	38¼	40	304	48	10
14	16¼	18	20	21½	23	24¼	25	28	30½	33	35	36½	247	7	9
13½	15¼	17¼	19	20¼	21¾	23	24	26¼	28½	31	32½	35¼	221	15	8½
12½	14½	16½	18½	19	20½	21½	23	25	27	29	30½	32½	195	22	8
12	14	15½	17½	18¾	19¾	21	22	24¼	26	28	29½	31½	182	13	7¾
11	13¾	15	16½	18	19	20	21¼	23¼	25	27	28½	30¼	172	30	7½
10¾	13	14	15½	16¾	18¾	19	20½	22	24	25½	27	28½	149	40	7
10	12	13	14	15½	16¼	18	19	20	22	23	24½	26¼	128	54	6½
9½	11	12	13	14	15½	16	17	19	20½	22	23	24½	110	1	6
	10	11	12	13	14	15	15¾	17	19	20	21	22¼	94	30	5½
		10	11	11¼	13	13¾	14	15½	16¾	18½	19¼	20¼	66	61	5
			10	10	11¼	12	13½	14	15	16	17	18¼	60	60	4½
				9	10	11	11½	12	13½	14	15	16	48	51	4

Diameter of the cylinder in inches.

Plate VI. Fig. I. The Persian wheel.

Water may be raised by means of a stream *A B* turning a wheel *C D E*, according to the order of the letters, with buckets *a*, *a*, *a*, *a*, &c. hung upon the wheel by strong pins *b*, *b*, *b*, *b*, &c. fixed in the side of the rim: but the wheel must be made as high as the water is intended to be raised above the level of that part of the stream in which the wheel is placed. As the wheel turns, the buckets on the right hand go down into the water, and are filled therewith, and go up full on the left hand, until they come to the top at *K*; where they strike against the end *n* of the fixed trough *M*, and are thereby overset, and empty the water into the trough; from which it may be conveyed in pipes to the place which it is designed for: and as each bucket gets over the trough, it falls into a perpendicular position again, and goes down empty, until it comes to the water at *A*, where it is filled as before. On each bucket is a spring *r*, which goes over the top or crown of the bar *m* (fixed to the trough *M*), raises the bottom of the bucket above the level of its mouth, and so causes it to empty all its water into the trough.

Sometimes this wheel is made to raise water no higher than its axle; and then, instead of buckets hung upon it, its spokes *c*, *d*, *e*, *f*, *g*, *h*, are made of a bent form, and hollow within; these hollows opening into the holes *C*, *D*, *E*, *F*, in the outside of the wheel, and also into those at *O* in the box *N* upon the axle. So that, as the holes *C*, *D*, &c. dip into the water, it runs into them; and, as the wheel turns, the water rises in the hollow spokes, *c*, *d*, &c. and runs out in a stream *P* from the holes at *O*, and falls into the trough *Q*, from whence it is conveyed by pipes. And this is a very easy way of raising water, because the engine requires no animal power to turn it.[1]

Of the specific gravities of bodies.

The art of weighing different bodies in water, and thereby finding their specific gravities,[2] or weights, bulk for bulk, was invented by ARCHIMEDES; of which we have the following account:—

Hiero, king of Syracuse, having employed a goldsmith to make a crown, and given him a mass of pure gold for that purpose, suspected that the workman had kept back part of the gold for his own use, and made up the weight by alloying the crown with copper. But the king, not knowing how to find out the truth of that matter, referred it to Archimedes; who, having studied a long time in vain, found it out at last by chance. For, going into a bathing-tub of water, and observing that he thereby raised the water higher in the tub than it was before, he concluded instantly that he had raised it just as high as any thing else could have done, that was exactly of his bulk: and, considering that any other body of equal weight, and of less bulk than himself, could not have raised the water so high as he did; he immediately told the king, that he had found a method by which he could discover whether there were any cheat in the crown. For, since gold is the heaviest of all known metals, it must be of less bulk, according to its weight, than any other metal. And, therefore, he desired that a mass of pure gold, equally heavy with the crown when weighed in air, should be weighed against it in water; and, if the crown was not alloyed, it would counterpoise the mass of gold when they were both immersed in water, as well as it did when they were weighed in air. But, upon making the trial, he found that the mass of gold weighed much heavier in water than the crown did. And not only so, but that, when the mass and crown were immersed separately in one vessel of water, the crown raised the water much higher than the mass did; which shewed it to be alloyed with some lighter metal that increased its bulk. And so, by making trials with different metals, all equally heavy with the crown when weighed in air, he found out the quantity of alloy in the crown.

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[1] For an account of other interesting hydraulic machines, see the Appendix, Vol. II.

[2] The *specific gravity*, or *density*, of any body, is the quantity of matter which it contains under a certain bulk. The specific gravity of *lead ore*, for example, is double that of *diamond*, because one cubic inch of the former is twice as heavy, or contains twice as much matter, as one cubic inch of the latter.—*Ed.*

The specific gravities of bodies are as their weights, bulk for bulk; thus a body is said to have two or three times the specific gravity of another, when it contains two or three times as much matter in the same space.

A body immersed in a fluid will sink to the bottom, if it be heavier than its bulk of the fluid. If it be suspended therein, it will lose as much of what is weighed in air, as its bulk of the fluid weighs. Hence, all bodies of equal bulks, which would sink in fluids, lose equal weights when suspended therein. And unequal bodies lose in proportion to their bulks.[3]

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[3] The losses of weight sustained by bodies suspended in different fluids, is in the

The *hydrostatic balance.*

The *hydrostatic balance* differs very little from a common balance that is nicely made; only it has a hook at the bottom of each scale, on which small weights may be hung by horse-hairs, or by silk threads. So that a body, suspended by the hair or thread, may be immersed in water without wetting the scale from which it hangs.

How to find the specific gravity of any body.

If the body thus suspended under the scale, at one end of the balance, be first counterpoised in air by weights in the opposite scale, and then immersed into water, the equilibrium will be immediately destroyed. Then, if as much weight be put into the scale from which the body hangs, as will restore the equilibrium (without altering the weights in the opposite scale), that weight, which restores the equilibrium, will be equal to the weight of a quantity of water as big as the immersed body. And if the weight of the body in the air be divided by what it loses in water, the quotient will shew how much that body is heavier than its bulk of water. Thus, if a guinea, suspended in air, be counterbalanced by 129 grains in the opposite scale of the balance; and then, upon its being immersed in water, it becomes so much lighter as to require $7\frac{1}{4}$ grains put into the scale over it, to restore the equilibrium, it shews that a quantity of water, of equal bulk with the guinea, weighs $7\frac{1}{4}$ grains, or 7.25; by which divide 129 (the weight of the guinea in air), and the quotient will be 17.793; which shews that the guinea is 17.793 times as heavy as its bulk of water.[4] And thus, any piece of gold may be tried by weighing it first in air, and then in water; and if, upon dividing the weight in air by the loss in water, the quotient comes out to be 17.793, the gold is good; if the quotient be 18, or between 18 and 19, the gold is very fine; but if it be less than 17, the gold is too much alloyed, by being mixed with some lighter metal.

If silver be tried in this manner, and found to be 11 times as heavy as water, it is very fine; if it be $10\frac{1}{2}$ times as heavy,

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compound ratio of their bulks, and the density of the fluids;—and when any body is suspended in a fluid, so as not to touch the bottom of the vessel, the fluid gains as much weight as is lost by the suspended body.—*Ed.*

[4] Since a quantity of water equal in bulk to a guinea weighs $7\frac{1}{4}$ grains, while a guinea weighs 129 grains, the specific gravities of these bodies must be in the ratio of $7\frac{1}{4}$ to 129. But $7\frac{1}{4} : 129 = 1 : 17.793$, the answer, or fourth term of this analogy, is therefore found by dividing the product of the second and third terms by the first; or by dividing 129 by $7\frac{1}{4}$.—*Ed.*

it is standard; but if it be of any less weight compared with water, it is mixed with some lighter metal, such as tin.

By this method, the specific gravities of all bodies that will sink in water, may be found. But as to those which are lighter than water, as most sorts of wood are, the following method may be taken, to shew how much lighter they are than their respective bulks of water.

Let an upright stud be fixed into a thick flat piece of brass, and in this stud let a small lever, whose arms are equally long, turn upon a fine pin as an axis. Let the thread which hangs from the scale of the balance be tied to one end of the lever, and a thread from the body to be weighed, tied to the other end. This done, put the brass and lever into a vessel, then pour water into the vessel, and the body will rise and float upon it, and draw down the end of the balance from which it hangs; then, put as much weight in the opposite scale as will raise that end of the balance, so as to pull the body down into the water by means of the lever; and this weight in the scale will shew how much the body is lighter than its bulk of water.

There are some things which cannot be weighed in this manner, such as quicksilver, fragments of diamonds, &c. because they cannot be suspended in threads; and must therefore be put into a glass bucket, hanging by a thread from the hook of one scale, and counterpoised by weights put into the opposite scale. Thus, suppose you want to know the specific gravity of quicksilver, with respect to that of water; let the empty bucket be first counterpoised in air, and then the quicksilver put into it and weighed. Write down the weight of the bucket, and also of the quicksilver; which done, empty the bucket, and let it be immersed in water as it hangs by the thread, and counterpoised therein by weights in the opposite scale: then, pour the quicksilver into the bucket in the water, which will cause it to preponderate; and put as much weight into the opposite scale as will restore the balance to an equipoise; and this weight will be the weight of a quantity of water equal in bulk to the quicksilver. Lastly, divide the weight of the quicksilver in air, by the weight of its bulk of water, and the quotient will shew how much the quicksilver is heavier than its bulk of water.

If a piece of brass, glass, lead, or silver, be immersed and suspended in different sorts of fluids, the different losses of weight therein will shew how much it is heavier than its bulk of the fluid; the fluid being lightest in which the immersed body loses least of its aërial weight. A solid bubble of glass is generally used for finding the specific gravities of fluids.[5]

Hence we have an easy method of finding the specific gravities both of solids and fluids, with regard to their specific bulks of common pump water, which is generally made a standard for comparing all others by.

In constructing tables of specific gravities with accuracy, the gravity of water must be represented by unity or 1.000, where three cyphers are added, to give room for expressing the ratios of other gravities in decimal parts, as in the following table.

N. B. Although guinea gold has been generally reckoned 17.798 times as heavy as its bulk of water, yet, by many repeated trials, I cannot say that I have found it to be more than 17.200 (or 17$\frac{2}{10}$) as heavy.

A Table of the specific Gravities of several solid and fluid Bodies.

A cubic inch of	Troy weight.			Avoirdupois.		Comparative weight.
	Oz.	Pwt.	Gr.	Oz.	Drams.	
Very fine gold -	10	7	3.83	11	5.80	19.637
Standard gold -	9	19	6.44	10	14.90	18.888
Guinea gold - -	9	7	17.48	10	4.76	17.793
Moidore gold -	9	0	19.84	9	14.71	17.140
Quicksilver - -	7	7	11.61	8	1.45	14.019
Lead - - - -	5	19	17.55	6	9.08	11.325
Fine silver - -	5	16	23.23	6	6.66	11.087
Standard silver -	5	11	3.36	6	1.54	10.535
Copper - - -	4	13	7.04	5	1.89	8.843
Plate Brass - -	4	4	9.60	4	10.09	8.000
Steel - - - -	4	2	20.12	4	8.70	7.852
Iron - - - -	4	0	15.20	4	6.77	7.645
Block tin - - -	3	17	5.68	4	3.79	7.321
Speltar - - -	3	14	12.86	4	1.42	7.065
Lead ore - - -	3	11	17.76	3	14.96	6.800
Glass of antimony	3	15	16.89	3	0.89	5.280
German antimony	2	2	4.80	2	5.04	4.000
Copper ore - -	2	1	11.83	2	4.43	3.775

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[5] A hydrometer, consisting of several small bubbles or beads of glass, was invented by Professor Wilson of Glasgow, for the purpose of measuring the strength of spiritous liquors. Several of these beads are put into a quantity of the fluid, some of which will sink, some will swim on the top, and others will remain suspended in the fluid. That which neither sinks nor swims, denotes by its mark the strength of the spirits. This hydrometer has been greatly improved by Mrs. Lovi, and is now much used.—*Ed.*

Table continued.

Diamond - - -	1	15	20.88	1	15.48	3,400
Clear glass - -	1	13	5.58	1	13.16	3.150
Lapis lazuli - -	1	12	5.27	1	12.27	3.054
Welsh asbestos -	1	10	17.57	1	10.97	2.913
White marble -	1	8	13.41	1	9.06	2.707
Black ditto - -	1	8	12.65	1	9.02	2.704
Rock crystal - -	1	8	1.00	1	8.61	2.658
Green glass - -	1	7	15.38	1	8.26	2.620
Cornelian stone -	1	7	1.21	1	7.73	2.568
Flint - - - -	1	6	19.63	1	7.53	2.542
Hard paving stone	1	5	22.87	1	6.77	2.460
Live sulphur - -	1	1	2.40	1	2.52	2.000
Nitre - - - -	1	0	1.08	1	1.59	1.900
Alabaster - - -	0	19	18.74	1	1.35	1.875
Dry ivory - -	0	19	6.09	1	0.89	1.825
Brimstone - -	0	18	23.76	1	0.66	1.800
Alum - - - -	0	17	21.92	0	15.72	1.714
Ebony - - -	0	11	18.82	0	10.3	1.117
Human blood -	0	11	2.89	0	9.74	1.054
Amber - - -	0	10	20.79	0	9.54	1.030
Cow's milk - -	0	10	20.79	0	9.54	1.030
Sea water - -	0	10	20.79	0	9.54	1.030
Pump water - -	0	10	13.30	0	9.26	1.000
Spring water -	0	10	12.94	0	9.25	0.999
Distilled water -	0	10	11.42	0	9.20	0.993
Red wine - - -	0	10	11.42	0	9.20	0.993
Oil of amber - -	0	10	7.63	0	9.06	0.978
Proof spirits - -	0	9	19.73	0	8.62	0.931
Dry oak - - -	0	9	18.00	0	8.56	0.925
Olive oil - - -	0	9	15.17	0	8.45	0.913
Pure spirits - -	0	9	3.27	0	8.02	0.866
Spirit of turpentine	0	9	2.76	0	7.99	0.864
Oil of turpentine	0	8	8.53	0	7.33	0.772
Dry crabtree - -	0	8	1.69	0	7.08	0.765
Sassafras wood -	0	5	2.04	0	4.46	0.482
Cork - - - -	0	2	12.77	0	2.21	0.240

Take away the decimal points from the numbers in the right-hand column, or (which is the same) multiply them by 1,000, and they will shew how many avoirdupois ounces are contained in a cubic foot of each body.

How to find out the quantity of adulteration in metals.

The use of the table of specific gravities will best appear by an example. Suppose a body to be compounded of gold and silver, and it is required to find the quantity of each metal in the compound.

First, find the specific gravity of the compound, by weighing it in air and in water, and dividing its aërial weight by what it loses thereof in water, the quotient will shew its specific gravity, or how many times it is heavier than its bulk of water. Then, subtract the specific gravity of silver (found in the table) from that of the compound, and the specific gravity of the compound from that of gold; the first remainder shews the bulk of gold, and the latter the bulk of silver, in the whole compound: and if these remainders be multiplied by the respective specific gravities, the products will shew the proportion of weights of each metal in the body. Example.—

Suppose the specific gravity of the compounded body be 13; that of standard silver (by the table) is 10.5, and that of gold 19.63: therefore 10.5 from 13, remains 2.5, the proportional bulk of the gold; and 13 from 19.63, remains 6.63, the proportional bulk of silver in the compound. Then, the first remainder 2.5, multiplied by 19.63, the specific gravity of gold, produces 49.075 for the proportional weight of gold; and the last remainder 6.63 multiplied by 10.5, the specific gravity of silver, produces 69.615 for the proportional weight of silver in the whole body. So that for every 49.07 ounces or pounds of gold, there are 69.6 pounds or ounces of silver in the body.

Hence it is easy to know whether any suspected metal be genuine, or alloyed, or counterfeit, by finding how much it is heavier than its bulk of water, and comparing the same with the table: if they agree, the metal is good; if they differ, it is alloyed or counterfeited.

How to try spirituous liquors.

A cubical inch of good brandy, rum, or other proof spirits, weighs 235.7 grains: therefore, if a true inch cube of any metal weighs 235.7 grains less in spirits than in air, it shews the spirits are proof. If it loses less of its aërial weight in spirits, they are above proof; if it loses more, they are under: for, the better the spirits are, they are the lighter; and the worse, the heavier. All bodies expand with heat, and contract with cold, but some more and some less than others. And therefore the specific gravities of bodies are not precisely the same in summer as in winter. It has been found, that a cubic inch of good brandy is ten grains heavier in winter than in summer; as much spirit of nitre, 20 grains; vinegar 6 grains, and spring water 3. Hence it is most profitable to buy spirits in winter, and sell them in summer, since they are always bought and sold by measure. It has been found, that 32 gallons of spirits in winter will make 33 in summer.

The expansion of all fluids is proportionable to the degree of heat; that is, with a double or triple heat a fluid will expand two or three times as much.

The *thermometer.*

Upon these principles depends the construction of the thermometer, in which the globe or bulb, and part of the tube, are filled with a fluid, which, when joined to the barometer, is spirits of wine tinged, that it may be more easily seen in the tube. But when thermometers are made by themselves, quicksilver is generally used.

In the thermometer, a scale is fitted to the tube, to shew the expansion of the quicksilver, and consequently the degree of heat. And, as Fahrenheit's scale is most in esteem at present, I shall explain the construction and graduation of thermometers according to that scale.[6]

First, let the globe or bulb, and part of the tube, be filled with a fluid; then immerse the bulb in water just freezing, or snow just thawing; and even with that part in the scale where the fluid then stands in the tube, place the number 32, to denote the freezing point; then put the bulb under your arm pit, when your body is of a moderate degree of heat, so that it may acquire the same degree of heat with your skin; and when the fluid has risen as far as it can by that heat, there place the number 97: then divide the space between these numbers into 65 equal parts, and continue those divisions both above 97 and below 32, and number them accordingly.

This may be done in any part of the world; for it is found that the freezing point is always the same in all places, and the heat of the human body differs but very little; so that the thermometers made in this manner will agree with one another; and the heat of several bodies will be shewn by them, and expressed by the numbers upon the scale thus:

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[6] Fahrenheit, whose thermometer is generally used in this country, places the freezing point at 32, and the boiling point at 212. In Reaumur's thermometer, which is chiefly used on the continent, the freezing point is placed at 0, and the boiling point at 80. In Celsius's thermometer, which is used in Sweden, and has lately been adopted in France, under the name of 'the *Centigrade Thermometer*, the freezpoint is at 0, and the boiling point at 100. In De La Hire's, the freezing point is at 28, and the boiling point at $199\frac{1}{3}$. In Amonton's, the freezing point is at $51\frac{1}{2}$, and the boiling point at 73. In Crucquius's, the freezing point is at 1070, and the boiling point at 1510; and in Sir Isaac Newton's, the freezing point is at 0, and the boiling point at 34.—For a larger list of the different methods by which thermometers have been graduated, see Cavallo's *Natural Philosophy*, vol. iii, p. 19-20. The most accurate method of constructing thermometers is to be found in the *Philosophical Transactions*, vol. lxvii, p. 816.—*Ed.*

Air, in severe cold weather, in our climate from 15 to 25. Air in winter, from 26 to 42. Air in spring and autumn, from 43 to 53. Air at midsummer, from 65 to 68. Extreme heat of the summer sun, from 86 to 100. Butter just melting, 95. Alcohol boils with 174 or 175. Brandy with 190. Water 212. Oil of turpentine 550. Tin melts with 408, and lead with 540. Milk freezes about 30, vinegar 38, and blood 27.[7]

A body specifically lighter than a fluid will swim upon its surface, in such a manner, that a quantity of the fluid, equal in bulk with the immersed part of the body, will be as heavy as the whole body. Hence, the lighter a fluid is, the deeper a body will sink in it; upon which depends the construction of the *hydrometer* or water-poise.[8]

How the weight of a ship may be estimated.

From this we can easily find the weight of a ship, or any other body that floats in water. For, if we multiply the number of cubic feet which are under the surface, by 62.5, the number of pounds in one cubic foot of fresh water; or by 64.4, the number of pounds in a cubic foot of salt water; the product will be the weight of the ship, and all that is in it. For since it is the weight of the ship that displaces the water, it must continue to sink until it has removed as much water as is equal to it in weight; and therefore the part immersed must be equal in bulk to such a portion of the water as is equal to the weight of the whole ship.

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[7] It is a remarkable fact, that when a thermometer is inclosed in a receiver, and the air condensed, the mercury suddenly rises a few degrees above the temperature of the ambient air; and, on the contrary, when the air in the receiver is rarified, the mercury falls suddenly below the temperature of the atmosphere. But in both cases, after some time, it resumes its former station. In an experiment by Mr. Dalton, the thermometer in the air stood at 36° 8′. When the air was exhausted it sunk to 34° 7′; and when the air was re-admitted and condensed, it rose to 38° 9′. See *Memoirs of the Philosophical Society of Manchester*, vol. v, p. 515.—*Ed.*

[8] The *hydrometer* generally consists of a concave spherical or elliptical bulb, attached to the lower extremity of a hollow cylindrical stem, with a scale engraven upon it. It is sometimes made of metal, but generally of glass, that it may not be corroded by immersion in acids, and is loaded at its lower end with a weight, or with a quantity of mercury in the hollow bulb. This apparatus, when immersed in a fluid, will float in a vertical position, at a certain depth, and from this depth, measured by the scale on the stem, the specific gravity of the liquor is determined. The most approved hydrometers are those constructed by Mr. Jones, Mr. Dicas of Liverpool, Mr. Quin, Mr. Nicholson, Mr. Sykes, and Mr. Adie; a full account of which may be seen in the *Edinburgh Encyclopædia*, Art. *Hydrodynamics*, vol. xi, p. 437.

To prove this by experiment, let a ball of some light wood, such as fir or pear-tree, be put into water contained in a glass vessel; and let the vessel be put into a scale at one end of a balance, and counterpoised by weights in the opposite scale: then, marking the height of the water in the vessel, take out the ball: and fill up the vessel with water to the same height that it stood at when the ball was in it; and the same weight will counterpoise it as before.

From the vessel's being filled up to the same height at which the water stood when the ball was in it, it is evident that the quantity poured in is equal in magnitude to the immersed part of the ball: and from the same weight counterpoising, it is plain that the water poured in, is equal in weight to the whole ball.

In troy weight 24 grains make a pennyweight, 20 pennyweights make one ounce, and 12 ounces a pound. In avoirdupois weight, 16 drams make an ounce, and 16 ounces a pound. The troy pound contains 5760 grains, and the avoirdupois pound 7000; and hence the avoirdupois dram weighs 27.34375 grains, and the avoirdupois ounce 437.5.

Because it is often of use to know how much any given quantity of goods in troy weight do make in avoirdupois weight, and the reverse, we shall here annex two tables for converting these weights into one another. Those from page 106 to page 108 are near enough for common hydraulic purposes; but the two following are better where accuracy is required in comparing the weights with one another: and I find by trial, that 175 troy ounces are precisely equal to 192 avoirdupois ounces, and 175 troy pounds are equal to 144 avoirdupois. And although there are several lesser integral numbers, which come very near to agree together, yet I have found none less than the above to agree exactly. Indeed 41 troy ounces are so nearly equal to 45 avoirdupois ounces, that the latter contains only $7\frac{1}{2}$ grains more than the former: and 45 troy pounds weigh only $7\frac{3}{10}$ drams more than 37 avoirdupois.

I have lately made a scale for comparing these weights with one another, and shewing the weight of pump-water, proof spirits, pure spirits, and guinea gold, taken in cubic inches to any quantity less than a pound, both in troy and avoirdupois;

only by sliding one side of a square along the scale, and the other side crossing it.

A Table for reducing Troy Weight into Avoirdupois Weight.

Troy weight.		Avoirdupois.			Troy weight.		Avoir.
		lb.	oz.	drams.			Drams.
Pounds	4000	3291	6	13.68	Pennywt.	19	16.67
	3000	2528	9	2.26		18	15.79
	2000	1645	11	6.84		17	14.92
	1000	822	13	11.42		16	14.04
	900	740	9	2.28		15	13.16
	800	658	4	9.14		14	12.29
	700	576	0	0.00		13	11.41
	600	493	11	6.85		12	10.53
	500	411	6	13.71		11	9.65
	400	329	2	4.57		10	8.78
	300	246	13	11.42		9	7.90
	200	164	9	2.28		8	7.02
	100	82	4	9.15		7	6.14
	90	74	0	13.62		6	5.27
	80	65	13	4.11		5	4.39
	70	57	9	9.60		4	3.51
	60	49	5	15.08		3	2.63
	50	41	2	4.57		2	1.75
	40	32	14	10.05		1	.88
	30	24	10	15.54	Grains	23	.84
	20	16	7	5.03		22	.80
	10	8	3	10.52		21	.77
	9	7	6	7.86		20	.73
	8	6	9	5.21		19	.69
	7	5	12	2.56		18	.66
	6	4	14	15.90		17	.62
	5	4	1	13.25		16	.58
	4	3	4	10.60		15	.55
	3	2	7	7.95		14	.51
	2	1	10	5.30		13	.47
	1		13	2.65		12	.44
Ounces	11		12	1.09		11	.40
	10		10	15.54		10	.36
	9		9	13.99		9	.33
	8		8	12.43		8	.29
	7		7	10.88		7	.26
	6		6	9.32		6	.22
	5		5	7.77		5	.18
	4		4	6.22		4	.15
	3		3	4.66		3	.11
	2		2	3.11		2	.07
	1		1	1.55		1	.04

A Table for reducing Avoirdupois Weight into Troy Weight.

Avoirdupois weight.	Troy weight.				Avoirdupois weight.	Troy weight.			
	lb.	oz.	pw.	gr.		lb.	oz.	pw.	gr.
Pounds 6000	7291	8	0	0	Ounces 15	1	1	13	10.50
5000	6076	4	13	8	14	1	0	15	5
4000	4861	1	6	16	13		11	16	23.50
3000	3645	10	0	0	12		10	18	18
2000	2430	6	13	8	11		10	0	12.50
1000	1215	3	6	16	10		9	2	7
900	1093	9	0	0	9		8	4	1.50
800	972	2	13	8	8		7	5	20
700	850	8	6	16	7		6	7	14.50
600	729	2	0	0	6		5	9	9
500	607	7	13	8	5		4	11	3.50
400	486	1	6	16	4		3	12	22
300	364	7	0	0	3		2	14	16.50
200	243	0	13	8	2		1	16	11
100	121	6	6	16	1			18	5.50
90	109	4	10	0	Drams 15			17	2.10
80	97	2	13	8	14			15	22.76
70	85	0	16	16	13			14	19.42
60	72	11	0	0	12			13	15.08
50	60	9	3	8	11			12	12.74
40	48	7	6	16	10			11	9.40
30	36	5	10	0	9			10	6.06
20	24	3	13	8	8			9	2.72
10	12	1	16	16	7			8	23.38
9	10	11	5	0	6			7	20.04
8	9	8	13	8	5			6	16.70
7	8	6	1	16	4			5	13.36
6	7	3	10	0	3			3	10.02
5	6	0	18	8	2			2	6.68
4	4	10	6	16	1			1	3.34
3	3	7	15	0	$\frac{3}{4}$				20.51
2	2	5	3	8	$\frac{1}{2}$				13.67
1	1	2	11	16	$\frac{1}{4}$				6.83

The two following examples will be sufficient to explain these two tables, and shew their agreement.

Ex. I. *In 6835 pounds 6 ounces 9 pennyweights 6 grains Troy,* Qu. *How much Avoirdupois weight?* (See page 124.)

	Troy.		Avoirdupois.		
			lb.	oz.	drams.
Pounds	4000	=	3291	6	13.68
	2000	=	1645	11	6.84
	800	=	658	4	9.14
	20	=	16	7	5.03
	10	=	8	3	10.52
	5	=	4	1	13.25
oz.	6	=		6	9.32
pw.	9	=			7.90
gr.	6	=			.22
Answer			5624	10	11.90

Ex. II. *In 5624 pounds* 10 *ounces* 12 *drams Avoirdupois,* Qu. *How much Troy weight?* (See page 125.)

	Avoird.		Troy.			
			lb.	oz.	pw.	gr.
Pounds	5000	=	6076	4	13	8
	600	=	729	2	0	0
	20	=	24	3	13	8
	4	=		10	6	16
oz.	10	=		9	2	7
dr.	12	=			13	15.08
Answer			6835	6	9	6.08

LECTURE VI.

OF PNEUMATICS.

THIS science treats of the nature, weight, pressure, and spring, of the air, and the effects arising therefrom.

The properties of air.

The air is that thin transparent fluid body in which we live and breathe. It encompasses the whole earth to a considerable height; and, together with the clouds and vapours that float therein, it is called the atmosphere. The air is justly reckoned among the number of fluids, because it has all the properties by which a fluid is distinguished; for, it yields to the least force impressed, its parts are easily moved among one another, it presses according to its perpendicular height, and its pressure is every way equal.

That the air is a fluid, consisting of such particles as have no cohesion between them, but easily glide over one another, and yield to the slightest impression, appears from that ease and freedom with which animals breathe in it, and move through it, without any difficulty or sensible resistance.

But it differs from all other fluids in the four following particulars :—1, It can be compressed into a much less space than what it naturally possesses, which no other fluid can: 2, It cannot be congealed or fixed, as other fluids may: 3, It is of a different density in every part, upward from the earth's surface, decreasing in its weight, bulk for bulk, the higher it rises; and therefore must also decrease in density: 4, It is of an elastic or springy nature, and the force of spring is equal to its weight.

That air is a body, is evident from its excluding all other bodies out of the space it possesses: for, if a glass jar be plunged with its mouth downward into a vessel of water, there will but very little water get into the jar, because the air of which it is full keeps the water out.[1]

As air is a body, it must needs have gravity or weight: and that it is weighty, is demonstrated by experiment: for, let the air be taken out of a vessel by means of the air-pump, then, having weighed the vessel, let in the air again, and upon weighing it when re-filled with air, it will be found considerably heavier. Thus, a bottle that holds a wine quart, being emptied of air and weighed, is found to be about 16 grains lighter than when the air is let into it again; which shews that a quart of air weighs 16 grains. But a quart of water weighs 14,621 grains; this divided by 16, quotes 914 in round numbers; which shews, that water is 914 times as heavy as air near the surface of the earth.

As the air rises above the earth's surface, it grows rarer, and consequently lighter, bulk for bulk: for, because it is of an elastic or springy nature, and its lowermost parts are pressed with the weight of all that is above them, it is plain that the air must be more dense or compact at the earth's surface than at any height above it, and gradually rarer the higher up; for the density of the air is always as the force that compresses it; and therefore, the air toward the upper parts of the atmosphere being less pressed than that which is near the earth, it will expand itself, and thereby become thinner than at the earth's surface.

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[1] On this principle the diving-bell is constructed. A large bell of glass or metal is immersed in the sea by means of weights suspended to it; and however great be the depths to which it is sunk, the elasticity of the interior air prevents the water from reaching the diver seated within the bell. The air indeed is condensed by the pressure of the water, but not to such a degree as to prevent respiration, unless the depth be very great. Since the diving-bell has come into use for the purposes of submarine architecture, it is always supplied with air by means of a flexible pipe from above, communicating with a forcing-pump. See the *Edinburgh Philosophical Journal*, vol. v, p. 8.—*Ed.*

Dr. Cotes has demonstrated, that if altitudes in the air be taken in arithmetical proportion, the rarity of the air will be in geometrical proportion. For instance,

At the altitude of	miles above the surface of the earth, the air is	times thinner and lighter than at the earth's surface.
7		4
14		16
21		64
28		256
35		1024
42		4096
49		16384
56		65536
63		262144
70		1048576
77		4194304
84		16777216
91		67108864
98		268435456
105		1073741824
112		4294967296
119		17179869184
126		68719476736
133		274877906944
140		1099511627776

And hence it is easy to prove by calculation, that a cubic inch of such air as we breathe, would be so much rarefied at the altitude of 500 miles, that it would fill a hollow sphere equal in diameter to the orbit of Saturn.

The weight or pressure of the air is exactly determined by the following experiment:—

The *Toricellian* experiment.

Take a glass tube about three feet long, and open at one end; fill it with quicksilver, and putting your finger upon the open end, turn that end downward, and immerse it into a small vessel of quicksilver, without letting in any air; then take away your finger, and the quicksilver will remain suspended in the tube 29½ inches above its surface in the vessel, sometimes more, and at other times less, as the weight of the air is varied by winds and other causes. That the quicksilver is kept up in the tube by the pressure of the atmosphere upon that in the bason, is evident; for, if the bason and tube be put under a glass, and the air be then taken out of the glass, all the quicksilver in the tube will fall down into the bason; and if the air be let in again, the quicksilver will rise to the same height as before. Therefore the air's pressure on the surface of the earth is equal to the weight of 29½ inches depth of quicksilver all over the earth's surface, at a mean rate.

A square column of quicksilver, 29½ inches high, and one inch thick, weighs just 15 pounds, which is equal to the pressure of air upon every square inch of the earth's surface; and 144 times as much, or 2160 pounds, upon every square foot; because a square foot contains 144 square inches. At this rate, a middle sized man, whose surface may be about 14 square feet, sustains a pressure of 30,240 pounds, when the air is of a mean gravity; a pressure which would be insupportable, and even fatal to us, were it not equal on every part, and counterbalanced by the spring of the air within us, which is diffused through the whole body, and reacts with an equal force against the outward pressure.

Now, since the earth's surface contains (in round numbers) 200,000,000 square miles, and every square mile 27,878,400 square feet, there must be 5,575,680,000,000,000 square feet on the earth's surface, which, multiplied by 2160 pounds (the pressure on each square foot, gives 12,043,468,800,000,000,000 pounds for the pressure or weight of the whole atmosphere.

When the end of a pipe is immersed in water, and the air is taken out of the pipe, the water will rise in it to the height of 33 feet above the surface of the water in which it is immersed, but will go no higher; for it is found that a common pump will draw water no higher than 33 feet above the surface of the well: and unless the bucket goes within that distance from the well, the water will never get above it. Now, as it is the pressure of the atmosphere on the surface of the water in the well, that causes the water to ascend in the pump, and follow the piston or bucket, when the air above it is lifted up; it is evident, that a column of water 33 feet high is equal in weight to a column of quicksilver of the same diameter, 29½ inches high; and to as thick a column of air, reaching from the earth's surface to the top of the atmosphere.

The *barometer.*

In serene calm weather, the air has weight enough to support a column of quicksilver 31 inches high; but in tempestuous stormy weather, not above 28 inches. The quicksilver, thus supported in a glass tube, is found to be a nice counterbalance to the weight or pressure of the air, and to shew its alterations at different times. And being now generally used to denote the changes in the weight of the air, and of the weather consequent upon them, it is called the *barometer*, or weather-glass.

The pressure of the air being equal on all sides of a body exposed to it, the softest bodies sustain this pressure without suffering any change in their figure; and so do the most brittle bodies without being broken.

The cause of *winds.*

The air is rarefied, or made to swell with heat; and of this property, *wind* is a necessary consequence: for, when any part of the air is heated by the sun, or otherwise, it will swell, and thereby affect the adjacent air: and so by various degrees of heat in different places, there will arise various winds.[2]

When the air is much heated, it will ascend toward the upper part of the atmosphere, and the adjacent air will rush in to supply its place; and therefore, there will be a stream or current of air from all parts towards the place where the heat is. And hence we see the reason why the air rushes with such force into a glass-house, or toward any place where a great fire is made. And also, why smoke is carried up a chimney, and why the air rushes in at the key-hole of the door, or any small chink, when there is a fire in the room. So we may take it in general, that the air will press toward that part of the world where it is most heated.[3]

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[2] The pressure of the air, and its rarefaction by heat, are excellently illustrated by the following simple experiment:—Take hold of a wine glass with your right hand, and with your left put into it a small piece of burning paper. When the paper has burned for a few seconds, strike the mouth of the glass against the palm of your left hand, and it will remain firmly fixed to it for a considerable time. The cause of this is, that the internal air is so rarefied by the burning paper, that the pressure upon the inside of the glass is greatly diminished. The equilibrium, therefore, of the pressures upon the outside and inside of the glass being destroyed, the glass must adhere to the hand till that equilibrium is restored.—*Ed.*

The trade-winds.

Upon this principle, we can easily account for the *trade-winds*, which blow constantly from east to west about the equator. For when the sun shines perpendicularly on any part of the earth, it will heat the air very much in that part, which air will therefore rise upward, and when the sun withdraws, the adjacent air will rush in to fill its place; and consequently will cause a stream or current of air from all parts towards that which is most heated by the sun. But as the sun, with respect to the earth, moves from east to west, the common course of the air will be that way too; continually pressing after the sun: and therefore, at the equator, where the sun shines strongly, there will be a continual wind from the east; but, on the north side, it will incline a little to the north, and on the south side, to the south.

This general course of the wind about the equator, is changed in several places, and upon several accounts: as, 1, By exhalations that rise out of the earth at certain times, and from certain places; in earthquakes, and from volcanoes; 2, By the falling of great quantities of rain, causing thereby a sudden condensation or contraction of the air; 3, By burning sands, that often retain the solar heat to a degree incredible to those who have not felt it, causing a more than ordinary rarefaction of the air contiguous to them; 4, By high mountains, which alter the direction of the winds in striking against them; 5, By the declination of the sun toward the north or south, heating the air on the north or south side of the equator.[4]

The monsoons.

To these and such like causes is owing, 1, The irregularity and uncertainty of winds in climates distant from the equator, as in most parts of Europe; 2, Those periodical winds called *monsoons*, which, in the Indian seas, blow half a year one way, and the other half another; 3, Those winds which, on the coast of Guinea, and on the western coasts of America, blow always from west to east; 4, The sea-breezes, which, in hot countries, blow generally from sea to land, in the day-time; and the land-breezes, which blow in the night; and, in short, all those storms, hurricanes, whirlwinds, and irregularities, which happen at different times and places.[5]

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[3] The following experiment illustrates, in a very beautiful manner, this property of heated air. Roll up a piece of paper in a conical form, and prevent it from unrolling either by putting in a pin into its vertex, or by joining its sides with paste. Take off one of the scales of a balance, and fasten this cone by its vertex, at the extremity of the arm in place of the scale, adding such a weight as will keep the cone *in equilibrio* with the other scale. Put a lighted candle below the paper cone, and it will immediately rise, and require the addition of a very considerable weight to restore its equilibrium with the remaining scale.—*Ed.*

[4] The following curious table, containing the force and velocity of winds, was constructed by Mr. Rouse, from a great number of facts and experiments:—

The *vivifying spirit* in air.

All common air is impregnated with a certain kind of *vivifying spirit* or quality, which is necessary to continue the lives of animals: and this, in a gallon of air, is sufficient for one man during the space of a minute, and not much longer.[6]

Velocity of the wind in miles per hour.	Perpendicular force on one foot area in pounds avoirdupois.	Common appellations of the force of winds.
1	.005	Hardly perceptible.
2	.020	Just perceptible.
3	.044	
4	.079	Gentle pleasant wind.
5	.123	
10	.492	Pleasant brisk gale.
15	1.107	
20	1.968	Very brisk.
25	3.075	
30	4.429	High winds.
35	6.027	
40	7.873	Very high.
45	9.963	
50	12.300	A storm or tempest.
60	12.715	A great storm.
80	31.490	A hurricane.
100	49.200	A hurricane that tears up trees, carries buildings before it, &c.

Mr. Smeaton, by whom the above table was first published, observes, that the evidence for those numbers, where the velocity of the wind exceeds 50 miles an hour, do not seem of equal authority with those of 50 miles an hour and under. A very ingenious method of finding the velocity of the wind, from the motion of the sails of windmills, may be seen in Mr. Smeaton's Essay on the construction and effects of Wind-mill Sails. *Phil. Trans.* vol. li, 1759.—*Ed.*

[5] The winds called *solanos*, which are of a scorching and suffocating nature, are those which blow over a great tract of country greatly heated by the sun. They occur most commonly in the deserts of Arabia, and the interior of Africa. *Tornadoes* are winds which shift from one point of the horizon to another, and return to the same again. The *siroc*, or *sirocco*, is a warm south-easterly wind, which is felt in the southern parts of Italy. It occasions such a degree of lassitude, that both natives and strangers are incapable of performing their usual functions.—*Ed.*

[6] The air of our atmosphere is a mixture, or more properly a combination, of two different gases, oxygenous gas, and azotic or nitrogen gas, in the proportion of

This spirit in air is destroyed by passing through the lungs of animals: and hence it is, that an animal dies soon after being put under a vessel which admits no fresh air to come to it. This spirit is also in the air which is in water; for fish die when they are excluded from fresh air, as in a pond that is closely frozen over. And the little eggs of insects, stopped up in a glass, do not produce their young, though assisted by a kindly warmth. The seed also of plants mixed with good earth, and inclosed in a glass, will not grow.

This enlivening quality in air is also destroyed by the air's passing through fire; particularly charcoal fire, or the flame of sulphur. Hence, smoking chimneys must be very unwholesome, especially if the rooms they are in be small and close.

Air is also vitiated, by remaining closely pent up in any place for a considerable time; or, perhaps, by being mixed with malignant steams and particles flowing from the neighbouring bodies: or lastly, by the corruption of the vivifying spirit; as in the holds of ships, in oil-cisterns, or wine-cellars, which have been shut for a considerable time. The air in any of them is sometimes so much vitiated, as to be immediate death to any animal that comes into it.

Damps.

Air that has lost its vivifying spirit, is called *damp*, not only because it is filled with humid or moist vapours, but because it deadens fire, extinguishes flame, and destroys life. The dreadful effects of damps are sufficiently known to such as work in mines.

If part of the vivifying spirit of air in any country begins to putrefy, the inhabitants of that country will be subject to an epidemical disease, which will continue until the putrefaction is over. And as the putrefying spirit occasions the disease, so if the diseased body contributes toward the putrefying of the air, then the disease will not only be epidemical, but pestilential and contagious.

The atmosphere is the common receptacle of all the effluvia or vapours arising from different bodies; of the steams and

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one part of the former to three of the latter. This oxygenous gas is what our author calls the *vivifying spirit* of air, and the azotic gas is what he improperly denominates *damp*. That the constitution of the air in the superior regions of the atmosphere is similar to its constitution near the earth's surface, appears from the late æronautic experiments of M. Guy Lusac. This was determined by filling a vessel with air, at a great height from the earth, and afterwards subjecting it to chemical examination.—*Ed.*

smoke of things burnt or melted; the fogs or vapours proceeding from damp watery places; and of the effluvia from sulphureous, nitrous, acid, and alkaline bodies. In short, whatever may be called volatile, rises in the air to greater or less heights, according to its specific gravity.

Fermentation. When the effluvia, which arise from acid and alkaline bodies, meet each other in the air, there will be a strong conflict or *fermentation* between them; which will sometimes be so great, as to produce a fire; then if the effluvia be combustible, the fire will run from one part to another, just as the inflammable matter happens to lie.

Any one may be convinced of this, by mixing an acid and an alkaline fluid together, as the spirit of nitre and oil of cloves: upon the doing of which, a sudden ferment, with a fine flame, will arise; and if the ingredients be very pure and strong, there will be a sudden explosion.

Thunder and lightning. Whoever considers the effects of fermentation, cannot be at a loss to account for the dreadful effects of *thunder* and *lightning*: for the effluvia of sulphureous and nitrous bodies, and others that may rise into the atmosphere, will ferment with each other, and take fire very often of themselves; sometimes by the assistance of the sun's heat.

If the inflammable matter be thin and light, it will rise to the upper part of the atmosphere, where it will flash without doing any harm: but if it be dense, it will lie near the surface of the earth, where, taking fire, it will explode with a surprising force; and by its heat rarefy and drive away the air, kill men and cattle, split trees, walls, rocks, &c. and be accompanied with terrible claps of thunder.

The heat of lightning appears to be quite different from that of other fires; for it has been known to run through wood, leather, cloth, &c. without hurting them, while it has broken and melted iron, steel, silver, gold, and other hard bodies. Thus it has melted or burnt asunder a sword, without hurting the scabbard; and money in a man's pocket, without hurting his clothes: the reason of this seems to be, that the particles of *that* fire are so fine, as to pass through soft loose bodies without dissolving them, while they spend their whole force upon the hard ones.

It is remarkable, that knives and forks which have been struck with lightning have a very strong magnetical virtue for several years after: and I have heard that lightning striking upon the mariner's compass, will sometimes turn it round; and often make it stand the contrary way, the north pole toward the south.

Fire-damps.

Much of the same kind with lightning, are those explosions called *fulminating* or *fire-damps,* which sometimes happen in mines; and are occasioned by sulphureous and nitrous, or rather oleaginous particles, rising from the mine, and mixing with the air, where they will take fire by the lights which the workmen are obliged to make use of.[7] The fire being kindled, will run from one part of the mine to another, like a train of gunpowder, as the combustible matter happens to lie. And as the elasticity of the air is increased by heat, *that* in the mine will consequently swell very much, and so, for want of room, will explode with a greater or less degree of force, according to the density of the combustible vapours. It is sometimes so strong as to blow up the mine; and at other times so weak, that when it has taken fire at the flame of a candle, it is easily blown out.

Air that will take fire at the flame of a candle may be produced thus: having exhausted a receiver of the air-pump, let the air run into it through the flame of the oil of turpentine; then remove the cover of the receiver, and holding a candle to that air, it will take fire, and burn quicker or slower, according to the density of the oleaginous vapour.[8]

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[7] The formation of *fire-damp,* or hydrogen gas, and of *choak-damp,* or carbonic acid gas, may be accounted for from the decomposition of water when in contact with coal, or carbonaceous matter, and from the affinity which the oxygen of the water has for carbon. As the specific gravity of hydrogen gas is $\frac{1}{15}$ of that of atmospheric air, it will rise to the roof of the mine, and the carbonic acid gas, having greater specific gravity than common air, will fall to the bottom, while the atmospheric air will occupy the intermediate place. When the hydrogen gas, mixed with atmospheric air, is by any accident set on fire, water is formed in consequence of the combustion; hence the miners who are scorched by the fire damp, always appear as if they were drenched with water. For a full account of these kinds of damp, and the methods of ventilating mines, see Mr. Bald's article on *Coal Mines* in the *Edinburgh Encyclopœdia,* vol. xiv, p. 361. An account of Sir Humphry Davy's lamp will be found in the Appendix, Vol. II.—*Ed.*

[8] This species of air, or *oxygenous gas,* may be procured in greater purity by heating the oxyd of manganese to redness in an earthen retort; or by heating it in a glass retort, over the flame of a lamp, when mixed with half its weight of strong sulphuric acid. This gas can be obtained from a variety of other oxyds, and by different methods, which are to be found in every system of chemistry. See particularly an excellent Epitome of Chemistry by Dr. Henry of Manchester.—*Ed.*

Earthquakes. When such combustible matter, as is above mentioned, kindles in the bowels of the earth, where there is little or no vent, it produces *earthquakes,* and violent storms or hurricanes of wind when it breaks forth into the air.

An artificial earthquake may be made thus: Take 10 or 15 pounds of sulphur, and as much of the filings of iron, and knead them with common water into the consistency of a paste: this being buried in the ground, will, in eight or ten hours time, burst out in flames, and cause the earth to tremble all around to a considerable distance.

From this experiment we have a very natural account of the fires of mount Ætna, Vesuvius, and other volcanoes, they being probably set on fire at first by the mixture of such metalline and sulphureous particles.

The air-pump. The *air-pump* being constructed the same way as the water-pump, whoever understands the one, will be at no loss to understand the other.

Plate XIV. Fig. 2. Having put a wet leather on the plate *L L* of the air-pump, place the glass receiver *M* upon the leather, so that the hole *i* in the plate may be within the glass.[9] Then, turning the handle *F* backward and forward, the air will be pumped out of the receiver; which will then be held down to the plate by the pressure of the external air, or atmosphere. For as the handle *F* (Fig. 3), is turned backward, it raises the piston *d e*, in the barrel *B K*, by means of the wheel *E* and rack *D d*:[1] and, as the piston is leathered so tight as to fit the barrel exactly, no air can get between the piston and barrel; and, therefore, all the air above *d* in the barrel is lifted up towards *B*, and a vacuum is made in the barrel from

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[9] When leathers, soaked in water or oil, are employed, an elastic vapour sometimes arises, which influences the gage, and prevents it from shewing to what degree the air is rarefied. On this account the edge of the receiver, and the plate *L L*, should be ground perfectly flat, and rubbed with hogs' lard or soft pomatum, which will keep out the air without generating moisture. When this cannot be conveniently done, the leather should be soaked in oil, from which the air has been previously expelled by boiling; or it may be rubbed with hogs' lard, or bees' wax, which communicate a clamminess to the leather. By these means the production of elastic vapour may be prevented. See Mr. Edward Nairne's curious experiments with the air-pump, in the *Phil. Trans.* 1777, vol. lxvii, p. 614.

[1] In such a delicate instrument as the air-pump, it is of great importance that the pistons be raised and depressed with an uniform force and velocity. This can be effected only by giving a proper curvature to the teeth of the wheel and rack. For this purpos see the article in the Appendix, Vol. II, *On the Formation of the Teeth of Rack-work, &c.—Ed.*

b to *c*; upon which, part of the air in the receiver *M* (Fig. 2), by its spring, rushes through the hole *i*, in the brass plate *L L*, along the pipe *G G*, which communicates with both barrels by the hollow trunk *I H K* (Fig. 3), and pushing up the valve *b*, enters into the vacant place *b e* of the barrel *B K*. For, wherever the resistance or pressure is taken off, the air will run to that place, if it can find a passage. Then, if the handle *F* be turned forward from *c* to its former position *F*, the piston *d e* will be depressed in the barrel; and, as the air which had got into the barrel cannot be pushed back through the valve *b*, it will ascend through a hole in the piston, and escape through a valve at *d*; and be hindered by that valve from returning into the barrel, when the piston is again raised. At the next raising of the piston a vacuum is again made in the same manner as before, between *b* and *e*; upon which, more of the air that was left in the receiver *M*, gets out thence by its spring, and runs into the barrel *B K*, through the valve *B*. The same thing is to be understood with regard to the other barrel *A I*; and as the handle *F* is turned backward and forward, it alternately raises and depresses the pistons in their barrels; always raising one while it depresses the other. And, as there is a vacuum made in each barrel when its piston is raised, the particles of air in the receiver *M* push out one another by their spring or elasticity, through the hole *i*, and pipe *G G* into the barrels; until at last the air in the receiver comes to be so much dilated, and its spring so far weakened, that it can no longer get through the valves; and then no more can be taken out. Hence, there is no such thing as making a perfect vacuum in the receiver; for the quantity of air taken out at any one stroke, will always be as the density thereof in the receiver:[2] and, therefore, it is impossible to take it all out, because supposing the receiver and barrels of equal capacity, there will be always as much left as was taken out at the last turn of the handle.

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[2] The density of the air in the receiver decreases in a geometrical progression. If d be the density of the air, n the number of reciprocations of the piston, r the capacity of the receiver, and a that of the barrels, we shall have $d = \left(\frac{r}{a + r}\right)^n$, and since the rarefaction of the air is inversely as its density, $\left(\frac{a + r}{r}\right)^n$ will represent the rarefaction. Hence, if we wished the air in the receiver to have a certain density, we can find n the number of reciprocations of the piston necessary to produce such a density, for $n = \frac{\text{Log. } d.}{\text{Log. } r. - \text{Log. } \overline{a + r}}$. *Ed.*

There is a cock *k* below the pump-plate, which being turned, lets the air into the receiver again; and then the receiver becomes loose, and may be taken off the plate. The barrels are fixed to the frame *E e e* by two screw-nuts *f f*, which press down the top-piece *E* upon the barrels: and the hollow trunk *H* (in Fig. 3), is covered by a box, as *G H* in Fig. 2.

There is a glass tube *l m m m n* open at both ends, and about 34 inches long; the upper end communicating with the hole in the pump-plate, and the lower end immersed in quicksilver at *n* in the vessel *N*. To this tube is fitted a wooden ruler *m m*, called the *gage*, which is divided into inches and parts of an inch, from the bottom at *n* (where it is even with the surface of the quicksilver), and continued up to the top, a little below *l*, to 30 or 31 inches.

As the air is pumped out of the receiver M, it is likewise pumped out of the glass tube *l m n*, because that tube opens into the receiver through the pump-plate; and as the tube is gradually emptied of air, the quicksilver in the vessel *N* is forced up into the tube by the pressure of the atmosphere. And if the receiver could be perfectly exhausted of air, the quicksilver would stand as high in the tube as it does at that time in the barometer: for it is supported by the same power or weight of the atmosphere in both.

The quantity of air exhausted out of the receiver on each turn of the handle, is always proportional to the ascent of the quicksilver on that turn; and the quantity of air remaining in the receiver is proportionable to the defect of the height of the quicksilver in the gage, from what it is at that time in the barometer.[3]

I shall now give an account of the experiments made with the air-pump in my lectures; shewing the resistance, weight, and elasticity, of the air.

I. *To shew the resistance of the air.*

Resistance of the air.

1. There is a little machine, consisting of two mills, *a* and *b* (Fig. 4), which are of equal weights, independent of each other, and turn equally free on their axis in the frame. Each mill has four thin arms or sails, fixed into

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[3] If vapour has been produced in the course of any experiment, the receiver, the tubes, and the barrels, of the pump must be carefully freed from it before a new experiment is performed. This is done by exhausting a large receiver, into which the vapour diffuses itself; and if not taken away, is at least greatly rarefied, according to the capacity of the receiver.—*Ed.*

the axis: those of the mill *a* have their planes at right angles to its axis, and those of *b* have their planes parallel to it. Therefore, as the mill *a* turns round in common air, it is but little resisted thereby, because its sails cut the air with their thin edges: but the mill *b* is much resisted, because the broad sides of its sails move against the air when it turns round. In each axle is a pin near the middle of the frame, which goes quite through the axle, and stands out a little on each side of it: upon these pins the slider *d* may be made to bear, and so hinder the mills from going, when the strong spring *c* is set on bend against the opposite ends of the pins.

Having set this machine upon the pump-plate *L L* (Fig. 2), draw up the slider *d* to the pins on one side, and set the spring *c* at bend upon the opposite ends of the pins: then push down the slider *d*, and the spring acting equally strong upon each mill, will set them both agoing with equal forces and velocities: but the mill *a* will run much longer than the mill *b*, because the air makes much less resistance against the edges of its sails, than against the sides of the sails of *b*.

Draw up the slider again, and set the spring upon the pins as before; then cover the machine with the receiver *M* (Fig. 2) upon the pump-plate, and having exhausted the receiver of air, push down the wire *P P* (through the collar of leathers in the neck *q*) upon the slider; which will disengage it from the pins, and allow the mills to turn round by the impulse of the spring: and as there is no air in the receiver to make any sensible resistance against them, they will both move a considerable time longer than they did in the open air; and the moment that one stops, the other will do so too. This shews that air resists bodies in motion, and that equal bodies meet with different degrees of resistance, according as they present greater or less surfaces to the air, in the planes of their motions.

2. Take off the receiver *M* and the mills; and having put the guinea *a* (Fig. 5), and feather *b* upon the brass flap *c*, turn up the flap, and shut it into the notch *d*. Then, putting a wet leather over the top of the tall receiver *A B* (it being open both at top and bottom) cover it with the plate *C*, from which the guinea and feather tongs *e d* will then hang within the receiver. This done, pump the air out of the receiver; and then draw up the wire *f* a little, which by a square piece on its lower end will open the tongs *e d*; and the flap falling down as at *c*, the guinea and feather will descend with equal veloci-

ties in the receiver; and both will fall upon the pump-plate at the same instant.[4]

N. B. In this experiment, the observers ought not to look at the top, but at the bottom, of the receiver; in order to see the guinea and feather fall upon the plate; otherwise on account of the quickness of their motion, they will escape the sight of the beholders.

II. *To shew the weight of the air.*

Weight of the air.

1. Having fitted a brass cap, with a valve tied over it, to the mouth of a thin bottle or Florence flask, whose contents are exactly known, screw the neck of this cap into the hole *i* of the pump-plate: then, having exhausted the air out of the flask, and taken it off from the pump, let it be suspended at one end of a balance, and nicely counterpoised by weights in the scale at the other end: this done, raise up the valve with a pin, and the air will rush into the flask with an audible noise: during which time, the flask will descend, and pull down that end of the beam. When the noise is over, put as many grains into the scale at the other end as will restore the equilibrium; and they will shew exactly the weight of the quantity of air which has got into the flask, and filled it. If the flask holds an exact quart, it will be found, that 16 grains will restore the equipoise of the balance, when the quicksilver stands at 29½ inches in the barometer: which shews, that when the air is at a mean rate of density, a quart of it weighs 16 grains: it weighs more when the quicksilver stands higher; and less when it stands lower.

2. Place the small receiver *O* (Fig. 2) over the hole *i* in the pump-plate, and upon exhausting the air, the receiver will be fixed down to the plate by the pressure of the air on its outside, which is left to act alone, without any air in the receiver to act against it: and this pressure will be equal to as many times 15 pounds, as there are square inches in that part of the plate which the receiver covers; which will hold down the receiver

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[4] This experiment may be made without an air-pump. Place a piece of thin paper on the outside of the bottom of a small box, of such a weight that in falling the bottom will always keep lowermost, and having let fall the box and the paper from the height of two or three yards above a cushion, they will both reach it at the same time; while a piece of paper of the same size, let fall at the same instant, will flutter slowly and obliquely to the ground. The experiment will succeed if the paper is placed on a crown or half-crown piece, without using a box.—*Ed.*

so fast, that it cannot be got off, until the air be let into it by turning the cock *k*; and then it becomes loose.

3. Set the little glass *A B* (Fig. 6), which is open at both ends, over the hole *i* upon the pump plate *L L*, and put your hand close upon the top of it at *B*: then, upon exhausting the air out of the glass, you will find your hand pressed down with a great weight upon it: so that you can hardly release it, until the air be re-admitted into the glass by turning the cock *k*; which air, by acting as strongly upward against the hand as the external air acted in pressing it downward, will release the hand from its confinement.

4. Having tied a piece of wet bladder *b* (Fig. 7), over the open top of the glass *A* (which is also open at bottom) set it to dry, and then the bladder will be tight like a drum. Then place the open end *A* upon the pump-plate, over the hole *i*, and begin to exhaust the air out of the glass. As the air is exhausting, its spring in the glass will be weakened, and give way to the pressure of the outward air on the bladder, which, as it is pressed down, will put on a spherical concave figure, which will grow deeper and deeper, until the strength of the bladder be overcome by the weight of the air; and then it will burst with a report as loud as that of a gun. If a flat piece of glass be laid upon the open top of this receiver, and joined to it by a flat ring of wet leather between them; upon pumping the air out of the receiver, the pressure of the outward air upon the flat glass will break it to pieces.

5. Immerse the neck *c d* (Fig. 8), of the hollow glass ball *e b* in water, contained in the phial *a a*; then set it upon the pump-plate, and cover it and the hole *i* with the close receiver *A*; and then begin to pump out the air. As the air goes out of the receiver by its spring, it will also by the same means go out of the hollow ball *e b*, through the neck *d c*, and rise up in bubbles to the surface of the water in the phial; from whence it will make its way, with the rest of the air in the receiver, through the air-pipe *G G* and valves *a* and *b*, into the open air. When it has done bubbling in the phial, the ball is sufficiently exhausted; and then, upon turning the cock *k*, the air will get into the receiver, and press so much upon the surface of the water in the phial, as to force the water up into the ball in a jet, through the neck *c d*; and will fill the ball almost full of water. The reason why the ball is not quite filled, is because

all the air could not be taken out of it; and the small quantity that was left in, and had expanded itself so as to fill the whole ball, is now condensed into the same state as the outward air, and remains in a small bubble at the top of the ball; and so keeps the water from filling that part of the ball.

6. Pour some quicksilver into the jar *D* (Fig. 9), and set it on the pump-plate near the hole *i*; then set on the tall open receiver *A B*, so as to be over the jar and hole; and cover the receiver with the brass plate *C*. Screw the open glass tube *f g* (which has a brass top on it at *h*) into the syringe *H*, and putting the tube through a hole in the middle of the plate, so as to immerse the lower end of the tube *e* in the quicksilver at *D*, screw the end *h* of the syringe into the plate. This done, draw up the piston in the syringe by the ring *I*, which will make a vacuum in the syringe below the piston; and as the upper end of the tube opens into the syringe, the air will be dilated in the tube, because part of it, by its spring, gets up into the syringe; and the spring of the undulated air in the receiver acting upon the surface of tne quicksilver in the jar, will force part of it up into the tube: for the quicksilver will follow the piston in the syringe, in the same way, and for the same reason, that water follows the piston of a common pump when it is raised in the pump barrel; and this, according to some, is done by suction. But to refute that erroneous notion, let the air be pumped out of the receiver *A B*, and then all the quicksilver in the tube will fall down by its own weight into the jar; and cannot be again raised one hair's breadth in the tube by working the syringe: which shews that suction had no hand in raising the quicksilver; and, to prove that it is done by pressure, let the air into the receiver by the cock *k* (Fig. 2), and its action upon the surface of the quicksilver in the jar will raise it up into the tube, although the piston of the syringe continues motionless. If the tube be about 32 or 33 inches high, the quicksilver will rise in it very near as high as it stands at that time in the barometer. And, if the syringe has a small hole, as *m*, near the top of it, and the piston be drawn up above that hole, the air will rush through the hole into the syringe and tube, and the quicksilver will immediately fall down into the jar. If this part of the apparatus be air-tight, the quicksilver may be pumped up into the tube to the same height that it stands in the barometer; but it will go no

higher, because then the weight of the column of quicksilver in the tube is the same as the weight of a column of air of the same thickness with the quicksilver, reaching from the earth to the top of the atmosphere.

7. Having placed the jar *A* (Fig. 10), with some quicksilver in it, on the pump-plate, as in the last experiment, cover it with the receiver *B;* then push the open end of the glass tube *d e* through the collar of leathers in the brass neck *C* (which it fits so as to be air-tight) almost down to the quicksilver in the jar. Then exhaust the air out of the receiver, and it will also come out of the tube, because the tube is close at top. When the gauge *m m* shews that the receiver is well exhausted, push down the tube, so as to immerse its lower end into the quicksilver in the jar. Now, although the tube be exhausted of air, none of the quicksilver will rise into it, because there is no air left in the receiver to press upon its surface in the jar. But let the air into the receiver by the cock *k*, and the quicksilver will immediately rise in the tube; and stand as high in it, as it was pumped up in the last experiment.

Both these experiments shew, that the quicksilver is supported in the barometer by the pressure of the air on its surface in the box, in which the open end of the tube is placed. And that the more dense and heavy the air is, the higher does the quicksilver rise; and, on the contrary, the thinner and lighter the air is, the more will the quicksilver fall. For if the handle *F* be turned ever so little, it takes some air out of the receiver, by raising one or other of the pistons in its barrel; and consequently, that which remains in the receiver is so much rarer, and has so much the less spring and weight; and thereupon, the quicksilver falls a little in the tube; but upon turning the cock, and re-admitting the air into the receiver, it becomes as weighty as before, and the quicksilver rises again to the same height. Thus we see the reason why the quicksilver in the barometer falls before rain or snow, and rises before fair weather; for, in the former case, the air is too thin and light to bear up the vapours, and in the latter, too dense and heavy to let them fall.

N. B. In all mercurial experiments with the air-pump, a short pipe must be screwed into the hole *i*, so as to rise about an inch above the plate, to prevent the quicksilver from getting into the air-pipe and barrels, in case any of it should be accidentally spilt over the jar: for if it once gets into the pipes or barrels, it spoils them, by loosening the solder, and corroding the brass.

8. Take the tube out of the receiver, and put one end of a bit of dry hazel branch, about an inch long, tight into the hole, and the other end tight into a hole quite through the bottom of a small wooden cup: then pour some quicksilver into the cup, and exhaust the receiver of air, and the pressure of the outward air, on the surface of the quicksilver, will force it through the pores of the hazel, from whence it will descend in a beautiful shower into a glass cup placed under the receiver to catch it.

9. Put a wire through the collar of leathers in the top of the receiver, and fix a bit of dry wood on the end of the wire within the receiver; then exhaust the air, and push the wire down, so as to immerse the wood into a jar of quicksilver on the pump-plate; this done, let in the air, and upon taking the wood out of the jar, and splitting it, its pores will be found full of quicksilver, which the force of the air, upon being let into the receiver, drove into the wood.

Magdeburg hemispheres.

10. Join the two brass hemispherical cups *A* and *B* together (Fig. 11), with a wet leather between them, having a hole in the middle of it; then screw the end *D* of the pipe *C D* into the plate of the pump at *i*, and turn the cock *E*, so that the pipe may be open all the way into the cavity of the hemispheres: then exhaust the air out of them, and turn the cock a quarter round, which will shut the pipe *C D*, and keep out the air. This done, unscrew the pipe at *D* from the pump; and screw the piece *F h* upon it at *D*; and let two strong men try to pull the hemispheres asunder by the rings *g* and *h*, which they will find hard to do: for if the diameter of the hemispheres be four inches, they will be pressed together by the external air with a force equal to 190 pounds. And to shew that it is the pressure of the air that keeps them together, hang them by either of the rings upon the hook *P* of the wire in the receiver *M* (Fig. 2), and upon exhausting the air out of the receiver, they will fall asunder of themselves.

11. Place a small receiver *O* (Fig. 2) near the hole *i* on the pump-plate, and cover both it and the hole with the receiver *M*; and turn the wire so by the top *P*, that its hook may take hold of the little receiver by a ring at its top, allowing that re-

ceiver to stand with its own weight on the plate. Then, upon working the pump, the air will come out of both receivers; but the large one *M* will be forcibly held down to the pump by the pressure of the external air; while the small one *O*, having no air to press upon it, will continue loose, and may be drawn up and let down at pleasure by the wire *P P*. But, upon letting it quite down to the plate, and admitting the air into the receiver *M*, by the cock *k*, the air will press so strongly upon the small receiver *O*, as to fix it down to the plate; and at the same time, by counterbalancing the outward pressure on the large receiver *M*, it will become loose. This experiment evidently shews, that the receivers are held down by pressure, and not by suction, for the internal receiver continued loose while the operator was pumping, and the external one was held down; but the former became fast immediately by letting in the air upon it.

12. Screw the end *A* of the brass pipe *A B F* (Fig. 12) into the hole of the pump-plate, and turn the cock *e* until the pipe be open; then put a wet leather upon the plate *c d*, which is fixed on the pipe, and cover it with the tall receiver *G H*, which is close at top: then exhaust the air out of the receiver, and turn the cock *e* to keep it out; which being done, unscrew the pipe from the pump, and set its end *A* into a bason of water, and turn the cock *e* to open the pipe; on which, as there is no air in the receiver, the pressure of the atmosphere on the water in the bason will drive the water forcibly through the pipe, and make it play up in a jet to the top of the receiver.

13. Set the square phial *A* (Fig. 15) upon the pump-plate, and having covered it with the wire cage *B*, put a close receiver over it, and exhaust the air out of the receiver; in doing which, the air will also make its way out of the phial through a small hole in its neck under the valve *b*. When the air is exhausted, turn the cock below the plate, to re-admit the air into the receiver: and as it cannot get into the phial again, because of the valve, the phial will be broke into a thousand pieces by the pressure of the air upon it. Had the phial been of a round form, it would have sustained this pressure like an arch, without breaking: but as its sides are flat, it cannot.

To shew the elasticity or spring of the air.

Elasticity of the air.

14. Tie up a very small quantity of air in a bladder, and put it under a receiver; then exhaust the air out of the receiver; and the small quantity which is confined in the bladder (having nothing to act against it) will expand itself so by the force of its spring, as to fill the bladder as full as it could be blown of common air. But upon letting the air into the receiver again, it will overpower the air in the bladder, and press its sides almost close together.

15. If the bladder so tied up be put into a wooden box, and have 20 or 30 pound weight of lead put upon it in the box, and the box be covered with a close receiver, upon exhausting the air out of the receiver, that air which is confined in the bladder will expand itself so as to raise up all the lead by the force of its spring.

16. Take the glass ball mentioned in the fifth experiment (Fig. 7), which was left full of water all but a small bubble of air at top, and having set it with its neck downward into the empty phial *a a*, and covered it with a close receiver, exhaust the air out of the receiver, and the small bubble of air in the top of the ball will expand itself, so as to force all the water out of the ball into the phial.

17. Screw the pipe *A B* (Fig. 12) into the pump-plate; place the tall receiver *G H* upon the plate *c d*, as in the twelfth experiment, and exhaust the air out of the receiver; then turn the cock *e* to keep out the air, unscrew the pipe from the pump, and screw it into the mouth of the copper vessel *C C* (Fig. 16), the vessel having first been about half filled with water. Then open the cock *e* (Fig. 12), and the spring of the air which is confined in the copper vessel will force the water up through the pipe *A B* in a jet into the exhausted receiver, as strongly as it did by its pressure on the surface of the water in a bason, in the twelfth experiment.

18. If a fowl, a cat, rat, mouse, or bird, be put under a receiver, and the air be exhausted, the animal will be at first oppressed as with a great weight, then grow convulsed, and at last expire in all the agonies of a most bitter and cruel death. But as this experiment is too shocking to every spectator who has the least degree of humanity, we substitute a machine called the lungs-glass in place of the animal.

19. If a butterfly be suspended in a receiver, by a fine thread

tied to one of its horns, it will fly about in the receiver, as long as the receiver continues full of air; but if the air be exhausted, though the animal will not die, and will continue to flutter its wings, it cannot remove itself from the place where it hangs in the middle of the receiver, until the air be let in again, and then the animal will fly about as before.

20. Pour some quicksilver into the small bottle *A* (Fig. 13), and screw the brass collar *c* of the tube *B C* into the brass neck *b* of the bottle, and the lower end of the tube will be immersed in the quicksilver, so that the air above the quicksilver in the bottle will be confined there, because it cannot get out about the joinings, nor can it be drawn out through the quicksilver into the tube. This tube is also open at top, and is to be covered with the receiver *G* and large tube *E F*, which tube is fixed by brass collars to the receiver, and is close at the top. This preparation being made, exhaust the air both out of the receiver and its tube; and the air will, by the same means, be exhausted out of the inner tube *B C*, through its open top at *C:* and as the receiver and tubes are exhausting, the air that is confined in the glass bottle *A* will press so by its spring upon the surface of the quicksilver, as to force it up in the inner tube as high as it was raised in the ninth experiment by the pressure of the atmosphere, which demonstrates that the spring of the air is equivalent to its weight.

21. Screw the end *C* of the pipe *C D* (Fig. 14) into the hole of the pump-plate, and turn all the three cocks *d*, *G*, and *H*, so as to open the communications between all the three pipes *E*, *F*, *D*, *C*, and the hollow trunk *A B*. Then cover the plates *g* and *h* with wet leathers, which have holes in their middle where the pipes open into the plates; and place the close receiver *I* upon the plate *g:* this done, shut the pipe *F* by turning the cock *H*, and exhaust the air out of the receiver *I*. Then, turn the cock *d* to shut out the air, unscrew the machine from the pump, and having screwed it to the wooden foot *L*, put the receiver *K* upon the plate *h:* this receiver will continue loose on the plate as long as it keeps full of air; which it will do until the cock *H* be turned to open the communication between the pipes *F* and *E*, through the trunk *A B;* and then the air in the receiver *K*, having nothing to act against its spring, will run from *K* into *I*, until it be so divided between these receivers, as to be of equal density in both; and then

they will be held down with equal forces to their plates by the pressure of the atmosphere; though each receiver will then be kept down but with one half the pressure upon it, that the receiver *I* had, when it was exhausted of air; because it has now one half of the common air in it which filled the receiver *K* when it was set upon the plate; and therefore a force equal to half the force of the spring of common air, will act within the receivers against the whole pressure of the common air upon their outsides. This is called transferring the air out of one vessel into another.

22. Put a cork into the square phial *A* (Fig. 15), and fix it in with wax or cement; put the phial upon the pump-plate with the wire cage *B* over it, and cover the cage with a close receiver. Then, exhaust the air out of the receiver, and the air that was corked up in the phial will break the phial by the force of its spring, because there is no air left on the outside of the phial to act against the air within it.

23. Put a shrivelled apple under a close receiver, and exhaust the air; then the spring of the air within the apple will plump it out, so as to cause all the wrinkles to disappear; but upon letting the air into the receiver again, to press upon the apple, it will instantly return to its former decayed and shrivelled state.

24. Take a fresh egg, and cut off a little of the shell and film from its smallest end, then put the egg under a receiver, and pump out the air; upon which, all the contents in the egg will be forced out into the receiver, by the expansion of a small bubble of air contained in the great end, between the shell and film.

25. Put some warm beer into a glass, and having set it on the pump, cover it with a close receiver, and then exhaust the air. While this is doing, and thereby the pressure more and more taken off from the beer in the glass, the air therein will expand itself, and rise up in innumerable bubbles to the surface of the beer; and from thence it will be taken away with the other air in the receiver. When the receiver is nearly exhausted, the air in the beer, which could not disentangle itself quick enough to get off with the rest, will now expand itself so as to cause the beer to have all the appearance of boiling; and the greatest part of it will go over the glass.

26. Put some warm water into a glass, and put a bit of dry wainscot or other wood into the water. Then, cover the glass

with a close receiver, and exhaust the air; upon which, the air in the wood having liberty to expand itself, will come out plentifully, and make all the water to bubble about the wood, especially about the ends, because the pores lie lengthways. A cubic inch of dry wainscot has so much air in it, that it will continue bubbling for near half an hour together.

Miscellaneous Experiments.

27. Screw the syringe *H* (Fig. 9) to a piece of lead that weighs one pound at least; and, holding the lead in one hand, pull up the piston in the syringe with the other; then quitting hold of the lead, the air will push it upward, and drive back the syringe upon the piston. The reason of this is, that the drawing up of the piston makes a vacuum in the syringe, and the air, which presses every way equally, having nothing to resist its pressure upward, the lead is thereby pressed upward, contrary to its natural tendency by gravity. If the syringe, so loaded, be hung in a receiver, and the air be exhausted, the syringe and lead will descend upon the piston-rod by their natural gravity; and upon admitting the air into the receiver, they will be driven upward again, until the piston be at the very bottom of the syringe.

28. Let a large piece of cork be suspended by a thread at one end of a balance, and counterpoised by a leaden weight, suspended in the same manner, at the other. Let this balance be hung to the inside of the top of a large receiver; which being set on the pump, and the air exhausted, the cork will preponderate, and shew itself to be heavier than the lead; but upon letting in the air again, the equilibrium will be restored. The reason of this is, that since the air is a fluid, and all bodies lose as much of their absolute weight in it as is equal to the weight of their bulk of the fluid, the cork, being the larger body, loses more of its real weight than the lead does; and therefore must in fact be heavier, to balance it under the disadvantage of losing some of its weight: which disadvantage being taken off by removing the air, the bodies then gravitate according to their real quantities of matter, and the cork, which balanced the lead in air, shews itself to be heavier when *in vacuo.*[5]

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[5] In this experiment the cork must be covered all over with a piece of thin wet bladder glued to it, and not used until it be thoroughly dry.—*Ed.*

29. Set a lighted candle upon the pump, and cover it with a tall receiver. If the receiver holds a gallon, the candle will burn a minute; and then, after having gradually decayed from the first instant, it will go out: which shews, that a constant supply of fresh air is necessary to feed flame; and so it also is for animal life. For a bird kept under a close receiver will soon die, although no air be pumped out; and it is found, that, in the diving-bell, a gallon of air is sufficient only for one minute for a man to breathe in.

The moment when the candle goes out, the smoke will be seen to ascend at the top of the receiver, and there it will form a sort of cloud; but, upon exhausting the air, the smoke will fall down to the bottom of the receiver, and leave it as clear at the top as it was before it was set upon the pump. This shews, that smoke does not ascend on account of its being positively light, but because it is lighter than air; and its falling to the bottom when the air is taken away, shews that it is not destitute of weight. So most sorts of wood ascend or swim in water; and yet there are none who doubt of the wood's having gravity or weight.

30. Set a receiver, which is open at top, upon the air-pump, and cover it with a brass plate, and wet leather; and having exhausted it of air, let the air in again at top through an iron pipe, making it pass through a charcoal flame at the end of the pipe; and, when the receiver is full of that air, lift up the cover, and let down a mouse or bird into the receiver, and the burnt air will immediately kill it. If a candle be let down into that air, it will go out directly; but, by letting it down gently, it will purify the air so far as it goes; and so, by letting it down more and more, the flame will drive out the bad air, and good air will get in.

31. Set a bell upon a cushion on the pump-plate, and cover it with a receiver; then shake the pump to make the clapper strike against the bell, and the sound will be very well heard; but exhaust the receiver of air, and then if the clapper be made to strike ever so hard against the bell, it will make no sound at all; which shews, that air is absolutely necessary for the propagation of sound.

32. Let a candle be placed on one side of a receiver, and viewed through the receiver at some distance; then, as soon as the air begins to be exhausted, the receiver will be filled with

vapours which rise from the wet leather, by the spring of the air in it; and the light of the candle being refracted through that medium of vapours, will have the appearance of circles of various colours, of a faint resemblance to those in the rain-bow.

The air-pump was invented by Otho Guerick of Magdeburg, but was much improved by Mr. Boyle, to whom we are indebted for our greatest part of the knowledge of the wonderful properties of the air, demonstrated in the above experiments.[6]

The elastic air which is contained in many bodies, and is kept in them by the weight of the atmosphere, may be got out of them either by boiling, or by the air-pump, as shewn in the 25th experiment: but the fixed air, which is by much the greater quantity, cannot be got out but by distillation, fermentation, or putrefaction.

If fixed air did not come out of bodies with difficulty, and spend some time in extricating itself from them, it would tear them to pieces. Trees would be rent by the change of air from a fixed to an elastic state, and animals would be burst in pieces by the explosion of air in their food.

Dr. Hales found, by experiment, that the air in apples is so much condensed, that if it were let out into the common air, it would fill a space 48 times as great as the bulk of the apples themselves; so that its pressure was equal to 11,776 lbs. and in a cubic inch of oak, to 19860 lbs. against their sides. So that if the air was let loose at once in these substances, they would tear every thing to pieces about them with a force superior to that of gunpowder. Hence, in eating apples, it is well that they part with the air by degrees, as they are chewed, and ferment in the stomach, otherwise an apple would be immediate death to him who eats it.

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[6] The air-pump was invented in 1654, and afterwards much improved by Papin, Hooke, Hawksbee, and S'Gravesande. The greatest improvement, however, upon this instrument, we owe to the ingenious Mr. Smeaton (see *Phil. Trans.* vol. xlvii, p. 416). Smeaton's air-pump has lately been much improved by Mr. Haas, who has likewise invented another of his own. The first is described in the *Phil. Trans.* vol. lxxiii, and the last in Cavallo's *Nat. Phil.* vol. ii, p. 472. New air-pumps have also been constructed by Mr. Prince, an American gentleman, and Mr. Cuthbertson, philosophical instrument-maker, London; both of which are described in the *Encyclopædia Britannica*, article *Pneumatics*. Mr. Smeaton's air-pump, and that which was first constructed by Mr. Haas, may be employed as a condensing engine, merely by changing the communication between the plate of the pump and the cylinders. About a year ago, a mercurial air-pump was invented by Sir A. N. Edelcrantz, counsellor of the chancery, and private secretary to the king of Sweden. Its exhausting power is unlimited; a property which no other air-pump possesses. It operates by means of mercury, raised and lowered in a wooden cylinder by a plug, and the upper communication with the receiver is made through an iron cock. Excepting this cock, all the parts of the machine may be executed by common workmen. A description of this air-pump, by its inventor, may be seen in *Nicholson's Journal* for March 1804, No. 27. Mr. Mendelsshon, a mathematical instrument-maker in London, has simplified the air-pump considerably. He makes the cylinders of glass, and the pistons of tin, so as to be air-tight without the interposition of leathers. See the *Journal* already quoted, for March 1805, No. 39, p. 204.—*Ed.*

The mixing of some substances with others will release the air from them all of a sudden, which may be attended with very great danger. Of this we have a remarkable instance in an experiment made by Dr. Slare; who having put half a dram of oil of caraway-seed into one glass, and a dram of compound spirit of nitre in another, covered them both on the air-pump with a receiver six inches wide, and eight inches deep, and then exhausted the air, and continued pumping until all that could possibly be got both out of the receiver, and out of the two fluids, was extricated: then, by a particular contrivance from the top of the receiver, he mixed the fluids together; upon which they produced such a prodigious quantity of air, as instantly blew up the receiver, although it was pressed down by the atmosphere with upwards of 400 pounds weight.[7]

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[7] The *Air-gun* is an instrument of sufficient importance to deserve notice in a treatise on experimental philosophy. It consists of a barrel, to which is attached a concave ball, furnished with an air-valve. When this ball is filled with condensed air, by means of a condensing syringe, it is screwed below the barrel in such a manner, that the valve may be opened by pulling the trigger of the lock. The condensed air being thus allowed to escape, impels the bullet in the barrel with a very considerable momentum. Twelve pennyweights of air, injected into a ball 3¾ inches in diameter, has been found to drive out fifteen balls with great velocity. Sometimes the air-gun consists of two barrels, the space between which is the receptacle for the condensed air. In this case, the condenser is fixed to the barrel, and comes through the butt-end of the gun.

The *magazine air-gun* is furnished with a winding or serpentine barrel, which holds about a dozen balls. These are introduced one after another into the straight barrel by means of a lever, and may be discharged with such rapidity as to answer the purpose of several guns.

When the concave ball is filling with condensed air, by means of the syringe, a flash of light is often perceived, capable of kindling a piece of match. This curious fact must have the same origin as the rise of mercury in the thermometer when placed in condensed air, as mentioned in a preceding note.

A philosophical instrument, in the form of a small syringe with a piston, is made for the purpose of setting fire to a piece of amadou, by a rapid condensation of the included air.—*Ed.*

LECTURE VII.

OF OPTICS.

Light consists of an inconceivably great number of particles flowing from a luminous body in all manner of directions; and these particles are so small, as to surpass all human comprehension.

That the number of particles of light is inconceivably great, appears from the light of a candle; which, if there be no obstacle in the way to obstruct the passage of its rays, will fill all the space within two miles of the candle, every way, with luminous particles, before it has lost the least sensible part of its substance.

The amazing smallness of the particles light.

A ray of light is a continued stream of these particles, flowing from any visible body in a straight line: and that the particles themselves are incomprehensiby small, is manifest from the following experiment. Make a small pin hole in a piece of black paper, and hold the paper upright on a table facing a row of candles standing by one another; then place a sheet of pasteboard at a little distance behind the paper, and some of the rays which flow from all the candles through the hole in the paper will form as many specks of light on the pasteboard as there are candles on the table before the plate, each speck being as distinct and clear as if there was only one speck from one single candle: which shews that the particles of light are exceedingly small, otherwise they could not pass through the hole from so many different candles without confusion. Dr. Niewentyt has computed that there flows more than 6,000,000,000,000 times as many particles of light from a candle in one second of time, as there are grains of sand in the whole earth, supposing each cubic inch of it to contain 1,000,000.

These particles, by falling directly upon our eyes, excite in our minds the idea of light. And, when they fall upon bodies, and are thereby reflected to our eyes, they excite in us the ideas of these bodies. And as every point of a visible body reflects the rays of light in all manner of directions, every point will be visible in every part to which the light is reflected from it.

Plate VII. Fig. 1. Reflected light.

Thus the object *A C B* is visible to an eye in any part where the rays *A a*, *A b*, *A c*, *A d*, *A e*, *B a*, *B b*, *B c*, *B d*, *B e*, and *C a*, *C b*, *C c*, *C d*, *C e*, come. Here we have shewn the rays as if they were only reflected from the ends *A* and *B*, and from the middle point *C* of the object; every other point being supposed to reflect rays in the same manner. So that, wherever a spectator is placed with regard to the body, every point of that part of the surface which is toward him will be visible, when no intervening object stops the passage of the light.

As no object can be seen through the bore of a bended pipe, it is evident that the rays of light move in straight lines, while there is nothing to refract or turn them out of their rectilineal course.

While the rays of light continue in any[1] medium of an uniform density, they are straight; but when they pass obliquely out of one medium into another, which is either more dense or more rare, they are refracted toward the denser medium: and this refraction is more or less, as the rays fall more or less obliquely on the refracting surface which divides the mediums.

To prove this by experiment, set the empty vessel *A B C D* (Fig. 2) into any place where the sun shines obliquely, and observe the part where the shadow of the edge *B C* falls on the bottom of the vessel at *E;* then fill the vessel with water, and the shadow will reach no farther than *e;* which shews that the ray *a B E*, which came straight in the open air, just over the edge of the vessel at *B* to its bottom at *E*, is refracted by falling obliquely on the surface of the water at *B;* and, instead of going on in the rectilineal direction *a B E*, it is bent downward in the water from *B* to *e;* the whole bend being at the surface of the water: and so of all the other rays *a b c*.

Refracted light.

If a stick be laid over the vessel, and the sun's rays be reflected from a glass perpendicularly into the vessel, the shadow of the stick will fall upon the same part of the bottom, whether the vessel be empty or full, which shews, that the rays of light are not refracted when they fall perpendicularly on the surface of any medium.

The rays of light are as much refracted by passing out of water into air, as by passing out of air into water. Thus, if a ray of light flows from the point *e*, under water, in the direction *e B*, when it comes to the surface of the water at *B*, it will not go on thence in the rectilineal course *B d*, but will be refracted into the line *B a*. Therefore, to an eye at *e* looking through a plane glass in the bottom of the empty vessel, the point *a* cannot be seen, because the side *B c* of the vessel interposes; and the point *d* will just be seen over the edge of the vessel at *B*. But if the vessel be filled with water, the point *a* will be seen from *e*; and will appear as at *d*, elevated in the direction of the ray *e B*.[2]

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[1] Any thing through which the rays of light can pass, is called a medium; as *Air*, *Water*, *Glass*, *Diamond*, or even a *Vacuum*.

The days are made longer by the refraction of the sun's rays.

The time of sun-rising or setting, supposing its rays suffered no refraction, is easily found by calculation. But observation proves, that the sun rises sooner, and sets later every day than the calculated time; the reason of which is plain, from what was said immediately above. For, though the sun's rays do not come part of the way to us through water, yet they do through the air or atmosphere, which being a grosser medium than the free space between the sun and the top of the atmosphere, the rays, by entering obliquely into the atmosphere, are there refracted, and thence bent down to the earth. And although there are many places of the earth to which the sun is vertical at noon, and, consequently, his rays can suffer no refraction at that time, because they come perpendicularly through the atmosphere: yet there is no place to which the sun's rays do not fall obliquely on the top of the atmosphere, at his rising and setting; and, consequently, no clear day in which the sun will not be visible before he rises in the horizon, and after he sets in it: and the longer or shorter, as the atmosphere is more or less replete with vapours. For, let *A B C* (Fig. 3) be part of the earth's surface, *D E F* the atmosphere that covers it, and *E B G H* the sensible horizon of an observer at *B*. As every point of the sun's surface sends out rays of light in all manner of directions, some of his rays will constantly fall upon and enlighten some half of the atmosphere; and, therefore, when the sun is at *I*, below the horizon *H*, those rays which go on in the free space *I k K* preserve a rectilineal course until they fall upon the top of the atmosphere, and those which fall so about *K*, are refracted at their entrance into the atmosphere, and bent down in the line *K m B*, to the observer's place at *B*: and, therefore, to him the sun will appear at *L*, in the direction of the ray *B m K*, above the horizon *B G H*, when he is really below it at *I*.

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[2] Hence a piece of money lying at *e*, in the bottom of an empty vessel, cannot be seen by an eye at *a*, because the edge of the vessel intervenes; but let the vessel be filled with water, and the ray *e a* being then refracted at *B*, will strike the eye at *a*, and so render the money visible, which will appear as if it were raised up to *f* in the line *a B f*.—*A.*

If the piece of money, mentioned in the preceding note, be a sixpence or a shilling, and if it be placed at the bottom of a wine glass, a curious optical deception will take place. The real piece of money will be seen near the surface of the water, while another piece, similar to a half-crown or crown, will appear at the bottom of the glass. The former is seen, by refraction, of its natural size, through the upper surface of the water, while the latter is seen magnified through the curvilineal sides of the glass.—*Ed.*

Angles of incidence and refraction.

The angle contained between a ray of light, and a perpendicular to the refracting surface, is called *the angle of incidence*; and the angle contained between the same perpendicular, and the same ray after refraction, is called *the angle of refraction*. Thus, let *L B M* (Fig. 4) be the refracting surface of a medium (suppose water), and *A B C* a perpendicular to that surface: let *D B* be a ray of light going out of air into water at *B*, and therein refracted in the line *B H*; the angle *A B D* is the angle of incidence, of which *D F* is the sine; and the angle *K B H* is the angle of refraction, whose sine is *K I*.

When the refracting medium is water, the sine of the angle of incidence is to the sine of the angle of refraction as 4 to 3, or as 1.336 to 1; which is confirmed by the following experiment, taken from Dr. Smith's Optics.

Describe the circle *D A E C* on a plain square board, and cross it at right angles with the straight lines *A B C*, and *L B M*; then, from the intersection *A*, with any opening of the compasses, set off the equal arcs *A D* and *A E*, and draw the right line *D F E*: then, taking *F a*, which is three quarters of the length *F E*, from the point *a*, draw *a I* parallel to *A B K*, and join *K I* parallel to *B M*: so *K I* will be equal to three quarters of *F E* or of *D F*. This done, fix the board upright upon the leaden pedestal *O*, and stick three pins perpendicularly into the board, at the points *D*, *B*, and *I*: then set the board upright into the vessel *V U T*, and fill up the vessel with water to the line *L B M*. When the water has settled, look along the line *D B*, so as you may see the head of the pin *B* over the head of the pin *D*; and the pin *I* will appear in the same right line produced to *G*, for its head will be seen just over the head of the pin at *B*: which shews that the ray *I B*, coming from the pin at *I*, is so refracted at *B*, as to proceed from thence in the line *B D* to the eye of the observer; the same as it would do from any point *G* in the right line *D B G*, if there were no water in the vessel: and also shews that *K I*, the sine of refraction in water, is to *D F*, the sine of incidence in the air, as 3 to 4.[3]

Hence, if *D B H* were a crooked stick put obliquely into the water, it would appear a straight one, as *D B G*. Therefore, as the line *B H* appears at *B G*, so the line *B G* will appear at *B g*; and, consequently, a straight stick *D B G* put obliquely into water, will seem bent at the surface of the water, and crooked, as *D B g*.

When a ray of light passes out of air into *glass*, the sine of incidence is to the sine of refraction, as 3 to 2, or as 1.5 to 1; and when out of air into a *diamond*, as 5 to 2, or as 2.5 to 1.[4]

Glass may be ground into eight different shapes at least (Fig. 5), for optical purposes, viz.

Lenses.

1, A *plane glass*, which is flat on both sides, and of equal thickness in all its parts, as *A*.

2, A *plano convex*, which is flat on one side, and convex on the other, as *B*.

3, A *double convex*, which is convex on both sides, as *C*.

4, A *plano concave*, which is flat on one side, and concave on the other, as *D*.

5, A *double concave*, which is concave on both sides, as *E*.

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[3] This is strictly true of the *Yellow* rays only, for the other coloured rays are differently refracted; but the difference is so small, that it need not be considered in this place.

[4] When 1 is taken to represent the sine of the refractive ray, the other number, 2.5 for example, is called the *Index of Refraction*. The following are the refractive powers of some of the leading substances which I have determined:—

	Index of Ref.		Index of Ref.
Chromate of lead, greatest	2.974	Garnet,	1.815
Ditto least,	2.503	Topaz,	1.636
Ruby silver,	2.564	Flint glass,	1.600
Octohedrite, least ref.	2.500	Crown glass,	1.540
Diamond,	2.480	Fluor spar,	1.436
Blende,	2.260	Alcohol,	1.374
Phosphorus,	2.224	Water,	1.336
Sulphur,	2.148	Tabasheer,	1.111

An acquaintance with this property of light is useful to *Divers*, and those who are acustomed to shoot fish in the water. For if they should aim directly at the object, the diver and the bullet would arrive at a point considerably beyond it; whereas, by having some idea of the depth of the water, and the angle which a line drawn from the eye to the object makes with its surface, the point at the bottom of the water, between the eye and the object, at which the aim is to be taken, may be easily determined.—*Ed.*

6, A *meniscus*, which is concave on one side, and convex on the other, as *F*.

7, A *flat plano convex*, or *multiplying glass*, whose convex side is ground into several little flat surfaces, as *G*.

8, A *prism*, which has three flat sides, and when viewed endwise, appears like an equilateral triangle, as *H*.[5]

Glasses ground into any of the shapes, *B*, *C*, *D*, *E*, *F*, are generally called *lenses*.

A right line *L I K*, going perpendicularly through the middle of a lens, is called *the axis of the lens*.

A ray of light *G h* falling perpendicularly on a plane glass *E F* (Fig. 6), will pass through the glass in the same direction *h i*, and go out of it into the air in the same right course *i H*.

A ray of light *A B*, falling obliquely on a plane glass, will go out of the glass in the same direction, but not in the same right line; for, in touching the glass, it will be refracted in the line *B C*, and, in leaving the glass, it will be refracted in the line *C D*.

A ray of light *C D* (Fig. 7), falling obliquely on the middle of a convex glass, will go forward in the same direction *D E*, as if it had fallen with the same degree of obliquity on a plane glass; and will go out of the glass in the same direction with which it entered: for it will be equally refracted at the points *D* and *E*, as if it had passed through a plane surface. But the rays *C G* and *C I* will be so refracted, as to meet again at the point *F*. Therefore, all the rays which flow from the point *C*, so as to go through the glass, will meet again at *F*: and if they go farther onward, as to *L*, they cross at *F*, and go forward on the opposite sides of the middle ray *C D E F*, to what they were in approaching it in the directions *H F* and *K F*.

The properties of different lenses.

When parallel rays, as *A B C* (Fig. 8), fall directly upon a plano-convex glass *D E*, and pass through it, they will be so refracted, as to unite in a point *f* behind it; and this point is called the *principal focus*: the distance of which, from the middle of the glass, is called the *focal distance*; which is equal to twice the radius of the sphere of the glass's convexity. And,

When parallel rays, as *A B C* (Fig. 9), fall directly upon a glass *D E*, which is equally convex on both sides, and pass through it, they will be so refracted, as to meet in a point or principal focus *f*, whose distance is equal to the radius or semi-diameter of the sphere of the glass's convexity. But if a glass be more convex on one side than on the other, the rule for finding the focal distance is this: as the sum of the semi-diameters of both convexities is to the semi-diameter of either, so is double the semi-diameter of the other to the distance of the focus. Or, divide the double product of the radii by their sum, and the quotient will be the distance sought.

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[5] Another kind of lens has been lately proposed, consisting of two plano-cylindrical lenses placed transversely. It has however no valuable properties.—*Ed.*

Since all those rays of the sun which pass through a convex glass are collected together in its focus, the force of all their heat is collected into that part; and is in proportion to the common heat of the sun, as the area of the glass is to the area of the focus.[6] Hence we see the reason why a convex glass causes the sun's rays to burn after passing through it.

All these rays cross the middle ray in the focus *f*, and then diverge from it, to the contrary sides, in the same manner *F f G*, as they converged in the space *D f E* in coming to it.

If another glass *F G*, of the same convexity as *D E*, be placed in the rays at the same distance from the focus, it will refract them so, as that after going out of it, they will be all parallel, as *a b c*; and go on in the same manner as they came to the first glass *D E*, through the space *A B C*; but on the contrary sides of the middle ray *B f b*: for the ray *A d f* will go on from *f* in the direction *f G a*, and the ray *C E f* in the direction *f F c*; and so of the rest.

The rays diverge from any radiant point, as from a principal focus; therefore, if a candle be placed at *f*, in the focus of the convex glass *F G*, the diverging rays in the space *F f G* will be so refracted by the glass, as, that after going out of it, they will become parallel, as shewn in the space *c b a*.

If the candle be placed nearer the glass than its focal distance, the rays will diverge after passing through the glass, more or less, as the candle is more or less distant from the focus.

If the candle be placed farther from the glass than its focal distance, the rays will converge after passing through the glass, and meet in a point which will be more or less distant from the glass, as the candle is nearer to, or farther from, its focus, and where the rays meet, they will form an inverted image of the flame of the candle; which may be seen on a paper placed in the meeting of the rays.[7]

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[6] The focus of a lens, properly speaking, is a mathematical point, and therefore can have no area. By the *area of the focus*, the author means the *area of the sun's image in the focus*.—*Ed.*

Plate VII. Fig. 10. Hence, if any object *A B C* be placed beyond the focus *F* of the convex glass *d e f*, some of the rays which flow from every point of the object, on the side next the glass, will fall upon it, and after passing through it, they will be converged into as many points on the opposite side of the glass, where the image of every point will be formed: and consequently, the image of the whole object, which will be inverted. Thus, the rays *A d*, *A e*, *A f*, flowing from the point *A*, will converge in the space *d a f*, and by meeting at *a*, will there form the image of the point *A*. The rays *B d*, *B e*, *B f*, flowing from the point *B*, will be united at *b* by the refraction of the glass, and will there form the image of the point *B*. And the rays *C d*, *C e*, *C f*, flowing from the point *C*, will be united at *c*, where they will form the image of the point *C*. And so of all the other intermediate points between *A* and *C*. The rays which flow from every particular point of the object, and are united again by the glass, are called *pencils of rays*.

If the object *A B C* be brought nearer to the glass, the picture *a b c* will be removed to a greater distance. For then, the rays flowing from every single point will fall more diverging upon the glass; and therefore cannot be so soon collected into the corresponding points behind it. Consequently, if the distance of the object *A B C* (Fig. 11) be equal to the distance *e B* of the focus of the glass, the rays of each pencil will be so refracted by passing through the glass, that they will go out of it parallel to each other; as *d I*, *e H*, *f h*, from the point *C*; *d G*, *e K*, *f D*, from the point *B*; and *d K*, *e E*, *f L*, from the point *A*: and therefore there will be no picture formed behind the glass.

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[7] The radiant point, or the point where the candle is placed, and the point where the image is formed, are called *conjugate foci*; so that, though every lens has only one *principal focus*, yet its conjugate foci are innumerable, varying with the distance of the radiant point. If the distance of the radiant point from the centre of the lens be four inches, and the distance of the image six inches, then, on the contrary, the distance of the image will be four, when the distance of the radiant point is six inches. Thus (Plate VII, Fig. 10), if an object *A B C*, placed before the lens *d f*, forms behind it the image *c b a*, at the distance *e b*, then the same object placed at *b* would form an image at the distance *e B*. If a be the distance of the object, or radiant point, f the focal length of the lens for parallel rays, or the distance of the image from the lens when the distance of the object is infinitely great, and x the distance of the image, then we shall have $x = \frac{a f}{a - f}$, when the rays fall diverging upon the lens, and $x = \frac{a f}{a + f}$ when they are convergent; that is, the distance of the conjugate focus is equal to the product of the distance of the radiant point, and the principal focal length of the lens, divided by the difference of the same quantities in the first case, and their sum in the second.—*Ed.*

If the focal distance of the glass, and the distance of the object from the glass, be known, the distance of the picture from the glass may be found by this rule, viz. multiply the distance of the focus by the distance of the object, and divide the product by their difference: the quotient will be the distance of the picture.

The picture (Fig. 10), will be as much bigger or less than the object, as its distance from the glass is greater or less than the distance of the object. For, as *B e* is to *e b*, so is *A C* to *c a*. So that if *A B C* be the object, *c b a* will be the picture; or, if *c b a* be the object, *A B C* will be the picture.

The manner of vision.

Having described how the rays of light, flowing from objects, and passing through convex glasses, are collected into points, and form the images of the objects; it will be easy to understand how the rays are affected by passing through the humours of the eye, and are thereby collected into innumerable points on the bottom of the eye, and thereon form the images of the objects which they flow from. For, the different humours of the eye, and particularly the crystalline humour, are to be considered as a convex glass, and the rays in passing through them to be affected in the same manner as in passing through a convex glass.

The eye described.

The eye is nearly globular. It consists of three coats and three humours. The part *D H H G* (Fig. 12) of the outer coat, is called the *sclerotica*, the rest *D E F G* the *cornea*. Next within this coat is that called the *choroides*, which serves as it were for a lining to the other, and joins with the *iris m n*, *m n*. The *iris* is composed of two sets of muscular fibres; the one of a circular form, which contracts the hole in the middle called the *pupil*, when the light would otherwise be too strong for the eye; and the other of radial fibres, tending everywhere from the circumference of the iris toward the middle of the pupil; which fibres by their contraction, dilate and enlarge the pupil when the light is weak, in order to let in the more of its rays. The third coat is only a fine expansion of the optic nerve *L*, which spreads like net-work all over the inside of the choroides, and is therefore called the *retina;* upon which are painted (as it were) the images of all visible objects, by the rays of light which either flow or are reflected from them.[1]

Under the *cornea* is a fine transparent fluid, like water, which is therefore called the *aqueous humour.* It gives a protuberant figure to the cornea, fills the two cavities *m m* and *n n*, which communicate by the pupil *P*, and has the same limpidity, specific gravity, and refractive power, as water. At the back of this lies the *crystalline humour I I*, which is shaped like a double convex glass; and is a little more convex on the back than the fore part. It converges the rays which pass through it from every visible object to its focus at the bottom of the eye. This humour is transparent like crystal, is much of the consistence of hard jelly, and exceeds the specific gravity of water in the proportion of 11 to 10. It is inclosed in a fine transparent membrane, from which proceed radial fibres *o o*, called the *ligamentum ciliare*, all round its edge, and join to the circumference of the iris. These fibres have a power of contracting and dilating occasionally, by which means they alter the shape or convexity of the crystalline humour, and also shift it a little backward or forward in the eye, so as to adapt its focal distance at the bottom of the eye to the different distances of objects; without which provision, we could only see those objects distinctly that were all at one distance from the eye.[2]

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[1] It has been much disputed among philosophers, whether vision is produced by the impression of light upon the *retina*, or upon the *choroid coat;* and many plausible arguments have been advanced in support of each opinion. M. Mariotte and M. de la Hire have taken opposite sides in this question, the former maintaining that the choroides is the seat of vision. The truth, however, seems to be, that both the retina and the choroides are concerned in producing the effect. See Priestley's *History of Vision*, § 189, 792.—*Ed.*

[2] Philosophers have been at much pains to determine in what way the eye adapts itself to different distances. Huygens and many others have imagined, along with our author, that the crystalline lens approaches to, or recedes from, the retina, by the contraction or dilatation of the ciliary processes. Dr. Pemberton and Dr. Young (*Phil. Trans.* 1793, part ii, p. 175) have maintained, that the fibres of the crystalline are muscular, and diminish its convexity by their contraction. M. de la Hire supposes that the eye accommodates itself to different distances, by the contraction and dilatation of the iris; and Dr. Monro imagines, that this accommodation is caused by the pressure of the orbicular muscles upon the upper and under parts of the cornea, or by the action of the recti-muscles, which, as they press chiefly upon the sides of the eye-balls, will elongate the axis of the eye. (See Monro *On the Eye*

At the back of the crystalline lies the *vitreous humour K K*, which is transparent like glass, and is largest of all in quantity, filling the whole orb of the eye, and giving it a globular shape. It is much of a consistence with the white of an egg, and very little exceeds the specific gravity and refractive power of water.[3]

As every point of an object *A B C* (Fig. 13) sends out rays in all directions, some rays, from every point on the side next the eye, will fall upon the cornea between *E* and *F*; and by passing on through the humours and pupil of the eye, they will be converged to as many points on the retina or bottom of the eye, and will thereon form a distinct inverted picture *c b a* of the object. Thus, the pencil of rays *q r s*, that flows from the point *A* of the object, will be converged to the point *a* on the retina; those from the point *B* will be converged to the point *b*; those from the point *C* will be converged to the point *c*; and so of all the intermediate points; by which means the whole image *a b c* is formed, and the object made visible; although it must be owned, that the method by which this sensation is carried from the eye by the optic nerve to the common sensorium in the

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Ear, and Brain, p. 132–139. Smith's *Optics*, vol. ii, *Remarks*, p. 2; and *Mem. de l'Acad.* 1728, p. 206, 4to.) The most accurate inquiries into this subject have lately been made by Mr. Ramsden and Sir Everard Home. From a variety of experiments these philosophers conclude, that the organ of vision has a power of accommodating itself to different distances when deprived of the crystalline lens; that the laminated structure of the crystalline is not intended to alter its form, but to prevent reflection, and to correct spherical aberration; that the cornea is composed of laminæ, elastic, and capable of being elongated $\frac{1}{31}$ of its diameter; that the tendons of the four straight muscles of the eye are continued to the edge of the cornea, and are inserted in its external laminæ; that there is a visible alteration on the figure of the cornea in adapting the eye to different distances; that, in short, the adjustment of the eye is produced by three changes in the organ, an increase of curvature in the cornea, an elongation of the axis of vision, and a motion of the crystalline lens. The *first* of these changes, according to Mr. Ramsden's computation, will produce $\frac{1}{3}$, and the other *two* $\frac{2}{3}$ of the quantity of adjustment. See *Phil. Trans.* 1795, p. 1, and 1796, p. 1.

[3] The following dimensions of the eye were determined by Mr. Petit, and will be interesting to many readers, as they are not frequently to be met with. The diameter of the eye, from outside to inside, taken at a mean rate from six adult eyes, 9.4 tenths of an inch. The radius of the convexity of the cornea, 3.3294 tenths of an inch. The radius of convexity of the anterior surface of the crystalline lens, from twenty-six eyes, 3.3081 tenths of an inch. The radius of convexity of the posterior surface, from the same eyes, 2.5056 tenths. The thickness of the crystalline, from the same eyes, 1.8525 tenths. The thickness of the cornea and aqueous humour together, 1.0358 tenths of an inch. See the *Edinburgh Philosophical Journal*, vol i, p. 42, &c. for various observations on the structure of the human eye.—*Ed.*

brain, and there discerned, is above the reach of our comprehension.

But that vision is effected in this manner, may be demonstrated experimentally. Take a bullock's eye while it is fresh, and having cut off the three coats from the back part, quite to the vitreous humour, put a piece of white paper over that part, and hold the eye toward any bright object, and you will see an inverted picture of the object upon the paper.[4]

As the image is inverted, many have wondered why the object appears upright. But we are to consider, 1, That *inverted* is only a relative term; and, 2, That there is a very great difference between the real object and the means or image by which we perceive it. When all the parts of a distant prospect are painted upon the retina, they are all right with respect to one another, as well as the parts of the prospect itself; and we can only judge of an object's being inverted, when it is turned reverse to its natural position, with respect to other objects which we see and compare it with. If we lay hold of an upright stick in the dark, we can tell which is the upper or lower part of it, by moving our hand upward or downward; and know very well that we cannot feel the upper end by moving our hand downward. Just so we find by experience, that upon directing our eyes toward a tall object, we cannot see its top by turning our eyes downward, nor its foot by turning our eyes upward; but must trace the object the same way by the eye to see it from head to foot, as we do by the hand to feel it; and as the judgment is informed by the motion of the hand in one case, so it is also by the motion of the eye in the other.

In Fig. 13 is exhibited the manner of seeing the same object *A B C*, by both the eyes *D* and *E* at once.

When any part of the image *c b a* falls upon the optic nerve *L*, the corresponding part of the object becomes invisible. On which account nature has wisely placed the optic nerve of each eye, not in the middle of the bottom of the eye, but toward the side next the nose; so that whatever part of the image falls upon the optic nerve of one eye, may not fall upon the optic nerve of the other. Thus the point *a* of the image *c b a* falls upon the optic nerve of the eye *D*, but not of the eye *E*; and the point *c* falls upon the optic nerve of the eye *E*, but not of the eye *D*: and therefore to both eyes taken together, the whole object *A B C* is visible.

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[4] The best way to perform this experiment is to hold the bullock's eye with its cornea directed to the window or any other object, and to pare the coats on the back part so thin that the image of the window or object is visible upon the retina; for if the three coats are cut completely away, as our author directs, the vitreous humour will be in danger of running out; and though this could be prevented, the white paper is not sufficiently transparent. If the coats be pared thin, the picture of the object is seen upon the *real retina*, whereas, if they be completely cut away, the image is seen merely upon paper, which is substituted instead of the retina.—*Ed.*

Plate VIII. Fig. 1.

The nearer that any object is to the eye, the larger is the angle under which it is seen, and the magnitude under which it appears. Thus to the eye *D*, the object *A B C* is seen under the angle *A P C*; and its image *c b a* is very large upon the retina: but to the eye *E*, at a double distance, the same object is seen under the angle *A p C*, which is equal only to half the angle *A p C*, as is evident by the figure. The image *c b a* is likewise twice as large in the eye *D*, as the other image *c b a* is in the eye *E*. In both these representations, a part of the image falls on the optic nerve, and the object in the corresponding part is invisible.

As the sense of seeing is allowed to be occasioned by the impulse of the rays from the visible object upon the retina of the eye, and forming the image of the object thereon, and that the retina is only the expansion of the optic nerve all over the choroides, it should seem surprising that the part of the image which falls on the optic nerve should render the like part of the object invisible; especially as that nerve is allowed to be the instrument by which the impulse and image are conveyed to the common sensorium in the brain. But this difficulty vanishes, when we consider that there is an artery within the trunk of the optic nerve, which entirely obscures the image in that part, and conveys no sensation to the brain.

That the part of the image which falls upon the middle of the optic nerve is lost, and consequently the corresponding part of the object is rendered invisible, is plain by experiment. For, if a person fixes three patches, *A*, *B*, *C* (Fig. 2), horizontally, upon a white wall, at the height of the eye, and the distance of about a foot from each other, and places himself before them, shutting the right eye, and directing the left toward the patch *C*, he will see the patches *A* and *C*, but the middle patch *B* will disappear. Or, if he shuts his left eye, and directs the right toward *A*, he will see both *A* and *C*, but *B* will disappear; and if he directs his eye toward *B*, he will see both *B* and *A*, but not *C*. For whatever patch is directly opposite to the optic nerve *N* vanishes. This requires a little practice, after which he will find it easy to direct his eye, so as to lose the sight of whichever patch he pleases.

We are not commonly sensible of this disappearance, because the motions of the eye are so quick and instantaneous, that we no sooner lose the sight of any part of an object, than we recover it again; much the same as in the twinkling of our eyes, for at each twinkling we are blinded; but it is so soon over, that we are scarcely ever sensible of it.[5]

Why some eyes require spectacles.

Some eyes require the assistance of convex glasses to make them see objects distinctly, and others of concave. If either the cornea *a b c* (Fig. 4), or crystalline humour *e*, or both of them, be too flat, as in the eye *A*, their focus will not be on the retina, as at *d*, where it ought to be, in order to render vision distinct; but beyond the eye, as at *f*. Consequently those rays which flow from the object *C*, and pass through the humours of the eye, are not converged enough to unite at *d*; and therefore the observer can have but a very indistinct view of the object. This is remedied by placing a convex glass *g h* before the eye, which makes the rays converge sooner, and imprints the image duly on the retina at *d*.[6]

If either the cornea or crystalline humour, or both of them, be too convex, as in the eye *B*, the rays that enter in from the object *C*, will be converged to a focus in the vitreous humour, as at *f*; and by diverging from thence to the retina, will form a very confused image thereon: and so, of course, the observer will have as confused a view of the object, as if his eye had been too flat. This inconvenience is remedied by placing a concave glass *g h* before the eye; which glass, by causing the rays to diverge between it and the eye, lengthens the focal distance so, that if the glass be properly chosen, the rays will unite at the retina, and form a distinct picture of the object upon it.[7]

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[5] The reason why vision is not disturbed by the twinkling of our eyes is not, as our author imagines, "that we no sooner lose the sight of any part of an object than we recover it again," but because the impression of light continues for eight thirds of time on the retina. For if we fix our eyes steadily upon a very minute object, and move our hand even slowly between the object and our eyes, vision will not be in the smallest degree disturbed.—*Ed.*

[6] From what is said above, the reader will see the reason why a person below water does not see objects distinctly. For as the aqueous humour has the same refractive power as water, the rays of light from any object under water will suffer no refraction in passing through the cornea and aqueous humour, and will therefore meet in a point far behind the retina. But if a diver, or any person who is accustomed to go below water, should use a pair of spectacles, consisting of two convex lenses, the radius of whose surfaces is *three tenths* of an inch, which is nearly the radius of the convexity of the cornea, he will see objects as distinctly below water as above it. If the lenses are plano-convex, the radius of their curvature must be three twentieths of an inch.—*Ed.*

Such eyes as have their humours of a due convexity, cannot see any object distinctly at a less distance than six inches; and there are numberless objects too small to be seen at that distance, because they cannot appear under any sensible angle. The method of viewing such minute objects is by a *microscope*, of which there are three sorts, viz. the *single*, the *double*, and the *solar*.

The *single microscope.*

The *single microscope* is only a small convex glass, as *c d* (Fig. 5), having the object *a b* placed in its focus, and the eye at the same distance on the other side; so that the rays of each pencil, flowing from every point of the object on the side next the glass, may go on parallel in the space between the eye and the glass; and then, by entering the eye at *C*, they will be converged to as many different points on the retina, and form a large inverted picture *A B* upon it, as in the figure.[8]

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[7] In order to find the focal distance of a lens, with which a short or long sighted person will see objects distinctly at a given distance, multiply the given distance by the distance at which the person sees distinctly, and divide this product by the difference of these distances; the quotient will be the focal distance of the lens required.—*Ed.*

[8] When single microscopes of large magnifying powers are required, it will be necessary to use glass globules, which may be made in the following manner. Having broken a bit of crown glass into small pieces by means of a hammer, take up several of these pieces upon the wet point of a piece of wire, and hold them in the flame of a candle till they melt into a globule and fall down. If this globule is round and clear, it may be prepared for use by placing it between two folds of thin lead or brass, perforated with a hole a little less than the globule. These globules may be made with greater ease, and equal accuracy, by cutting a piece of glass with a diamond into small slips, about one tenth of an inch broad, and three or four inches long. Hold one of these slips with both hands in the flame of a candle and draw it out into a thread when melted. Break this thread at the middle, and hold the two extremities in the flame of the candle till a small globule be formed upon each. When these globules are broken from the glass thread, they may be prepared for use in the way already mentioned, care being taken that the part of the

To find how much this glass magnifies, divide the least distance[9] (which is about six inches) at which an object can be seen distinctly with the bare eye, by the focal distance of the glass, and the quotient will shew how much the glass magnifies the diameter of the object.

The *double microscope.*

The *double* or *compound microscope* consists of an object-glass *c d* (Fig. 6), and an eye-glass *e f.* The small object *a b* is placed at a little greater distance from the glass *c d* than its principal focus, so that the pencils of rays flowing from the different points of the object, and passing through the glass, may be made to converge and unite in as many points between *g* and *h*, where the image of the object will be formed; which image is viewed by the eye through the eye-glass *e f.* For the eye-glass being so placed, that the image *g h* may be in its focus,[1] and the eye much about the same distance on the other side, the rays of each pencil will be parallel, after going out of the eye-glass, as at *e* and *f*, till they come to

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globule from which the thread is broken be concealed between the two plates of lead or brass. Single microscopes may also be made by taking up a drop of water on the point of a needle, and placing it in a hole made in a card or piece of lead (see the Appendix, Vol. II); or by placing drops of transparent varnish on one or both sides of a plate of glass.—The following method of making single microscopes has been executed by Mr. Sivright. Take a piece of platinum leaf, and put little pieces of glass into holes made in it, from $\frac{1}{20}$th to $\frac{1}{16}$th of an inch in diameter; direct the flame of a blow-pipe against the glass, and it will melt into a lens, which adheres strongly to the platinum leaf. The lens is therefore formed and set at the same time. An eye or loop made by bending the extremity of a platinum wire may be used instead of the platinum leaf. See the *Edinburgh Philosophical Journal*, vol. i, p. 82.—*Ed.*

[9] This distance varies in different individuals. Several optical writers make it eight inches; but *seven inches*, which is a medium between both, seems to be the distance at which good eyes examine common objects with the greatest distinctness and satisfaction. When the object, however, is microscopic, and exceedingly minute, as parallel lines at the distance of $\frac{1}{300}$ of an inch, the distance at which the eye examines them will be between *four and a half* and *five inches*; so that the magnifying power of microscopes should be computed from this distance. Upon this principle we have calculated a table for single microscopes. See Appendix.—*Ed.*

[1] At whatever distance the two glasses are placed, the image *g h* may be kept in the focus of the eye-glass, by changing the distance of the object from the object-glass. As the magnifying power of the microscope increases in proportion to the distance between the lenses, the object will not be very distinctly seen when this distance exceeds *eight* inches. It may be proper also to remark, for the sake of those who may wish to construct microscopes of this kind, that the aperture of the object-glass should, in general, not exceed *one tenth* of an inch, and must be lessened when the focal distance of the object-glass is small. See Appendix, Vol. II.—*Ed.*

the eye at *k*, where they will begin to converge by the refractive power of the humours; and after having crossed each other in the pupil, and passed through the crystalline and vitreous humours, they will be collected into points on the retina, and form the large inverted image *A B* thereon.

The magnifying power of this microscope is as follows:—Suppose the image *g h* to be six times the distance of the object *a b* from the object-glass *c d*; then will the image be six times the length of the object: but since the image could not be seen distinctly by the bare eye at a less distance than six inches, if it be viewed by an eye-glass *e f*, of one inch focus, it will thereby be brought six times nearer the eye; and consequently viewed under an angle six times as large as before; so that it will be again magnified six times: that is, six times by the object-glass, and six times by the eye-glass, which multiplied into one another, makes 36 times; and so much is the object magnified in diameter more than what it appears to the bare eye; and consequently 36 times 36, or 1,296 times in surface.

But because the extent or field of view is very small in this microscope, there are generally two eye-glasses placed sometimes close together, and sometimes an inch asunder; by which means, although the object appears less magnified, yet the visible area is much enlarged by the interposition of a second eye-glass; and consequently a much pleasanter view is obtained.[2]

The *solar microscope.* Fig. 7.

The *solar microscope*, invented by Dr. Lieberkhun, is constructed in the following manner. Having procured a very dark room, let a round hole be made in the window-shutter, about three inches diameter, through which the sun may cast a cylinder of rays *A A* into the room. In this hole place the end of a tube, containing two convex glasses and an object, viz. 1, A convex glass *a a*, of about two inches diameter, and three inches focal distance, is to be placed in that end of the tube which is put into the hole. 2, The object *b b*, being put between two glasses (which must be concave to hold it at liberty), is placed about two inches and a half from the glass *a a*. 3, A little more than a quarter of an inch from the object is placed the small convex glass *c c*, whose focal distance is a quarter of an inch.[3]

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[2] This additional lens is called the *Amplifying-glass*, and is generally about an inch and a half in diameter, and two and a half inches in focal length.—*Ed.*

The tube may be so placed, when the sun is low, that his rays *A A* may enter directly into it: but when he is high, his rays *B B* must be reflected into the tube by the plain mirror or looking-glass *C C*.

Things being thus prepared, the rays that enter the tube will be conveyed by the glass *a a* toward the object *b b*, by which means it will be strongly illuminated; and the rays *d* which flow from it, through the magnifying-glass *c c* will make a large inverted picture of the object at *D D*, which, being received on a white paper, will represent the object magnified in length, in the proportion of the distance of the picture from the glass *c c* to the distance of the object from the same glass. Thus, suppose the distance of the object from the glass to be $\frac{3}{10}$ parts of an inch, and the distance of the distant picture to be 12 feet, or 144 inches, in which there are 1440 tenths of an inch; and this number divided by 3 tenths, gives 480; which is the number of times the picture is longer or broader than the object;[4]

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[3] A considerable improvement may be made upon the solar microscope by substituting Ramsden's achromatic eye-piece instead of the convex lens *c c*. The image formed by this combination of lenses, is, according to Dr. Robison, so accurate and well defined, that it is infinitely superior to that which is formed by microscopes of the common description.—*Ed.*

[4] The *Magic Lantern* invented by Kircher is the same as the solar microscope, with this difference only, that the former, instead of natural objects, exhibits magnified pictures of transparent paintings on glass, which are strongly illuminated by the flame of a lamp. The diameters and focal distances of the two lenses also must be larger in the magic lantern than in the solar microscope, as a considerable quantity of light is necessary to shew the transparent figures to the greatest advantage. The images in the *phantasmagoria* were produced by a magic lantern fitted up in a peculiar manner. In this exhibition, a luminous figure, representing sometimes a skeleton, and sometimes the head of some eminent person, appeared before the spectators, who were seated in a dark chamber. It grew less and less, and seemed to retire to a great distance. It again advanced, and consequently increased in size, and having retired a second time, it appeared to vanish in a luminous cloud, from which another figure gradually arose, and advanced and retreated as before. The glass sliders on which the figures were drawn, had been first covered with opaque varnish, or some black pigment, and the figures had been scratched out on this dark ground by the point of a needle. By this means the figures were luminous. A screen made of gauze or thin silk was placed between the spectators and the lantern, to receive the image, and when the figures appeared to retire and grow less, the lantern was brought nearer the screen, till the front glass was within its focal distance of the screen. The figure then vanished in the luminous cloud, which was nothing more than the image of the flame which illuminated the figures. When the luminous cloud appeared on

and the length multiplied by the breadth shews how much the whole surface is magnified.[5]

Telescope.

Before we enter upon the description of telescopes, it will be proper to shew how the rays of light are affected by passing through concave glasses, and also by falling upon concave mirrors.

Plate VIII. Fig. 8.

When parallel rays, as *a b c d e f g h*, pass directly through a glass *A B*, which is equally concave on both sides, they will diverge after passing through the glass, as if they had come from a radiant point *C*, in the centre of the glass's concavity; which point is called the negative or virtual focus of the glass. Thus the ray *a*, after passing through the glass *A B*, will go on in the direction *k l*, as if it had proceeded from the point *C*, and no glass been in the way. The ray *b* will go on in the direction *m n;* the ray *c* in the direction *o p*, &c. The ray *C*, that falls directly upon the middle of the glass, suffers no refraction in passing through it; but goes on in the same rectilineal direction, as if no glass had been in its way.

If the glass had been concave only on one side, and the other side quite plane, the rays would have diverged, after passing through it, as if they had come from a radiant point at double the distance of *C* from the glass; that is, as if the radiant point had been at the distance of a whole diameter of the glass's concavity.

If rays come more converging to such a glass, than parallel rays diverge after passing through it, they will continue to converge after passing through it; but will not meet so soon as if no glass had been in the way; and will incline toward the same side to which they would have diverged, if they had come parallel to the glass. Thus the rays *f* and *h*, going in a converging state toward the edge of the glass at *B*, and converging more

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the screen, the figure was taken out, and another substituted, which in its turn was made to advance and appear larger, by withdrawing the lantern from the screen.—*Ed.*

[5] In addition to the microscopes mentioned by our author, it may be proper to take notice of two reflecting microscopes, one of which was invented by Sir Isaac Newton, and the other by Dr. Smith of Cambridge. Sir Isaac's is composed of a concave mirror and a convex eye-glass, with the object placed between them; and Dr. Smith's consists of two mirrors, the one concave and the other convex, with a convex eye-glass, the object being placed between the specula. See *Phil. Trans.* p. 3080; Smith's *Optics*, vol. ii, and *Remarks*, pp. 87, 97; and the *Edinburgh Philosophical Journal*, vol. ii, p. 135, for an account of Amici's improved Catoptrical Microscope.—*Ed.*

in their way to it than the parallel rays diverge after passing through it, they will go on converging after they pass through it, though in a less degree than they did before, and will meet at I: but if no glass had been in their way, they would have met at i.

Mirrors. Fig. 9. When the parallel rays, as $d f a$, $C m b$, $e l c$, fall upon a concave mirror $A B$ (which is not transparent, but has only the surface $A b B$ of a clear polish), they will be reflected back from that mirror, and meet in a point m, at half the distance of the surface of the mirror from C, the centre of its concavity: for they will be reflected at as great an angle from the perpendicular to the surface of the mirror, as they fell upon it, with regard to that perpendicular, but on the other side thereof. Thus, let C be the centre of concavity of the mirror $A b B$, and let the parallel rays $d f a$, $C m b$, and $e l c$, fall upon it at the points a, b, and c. Draw the lines $C i a$, $C m b$, and $C h c$, from the centre C to these points; and all these lines will be perpendicular to the surface of the mirror, because they proceed thereto like so many *radii* or spokes from its centre. Make the angle $C a h$ equal to the angle $d a C$, and draw the line $a m h$, which will be the direction of the ray $d f a$, after it is reflected from the point a of the mirror: so that the angle of incidence $d a C$, is equal to the angle of reflection $C a h$; the rays making equal angles with the perpendicular $C i a$ on its opposite sides.

Draw also the perpendicular $C h c$ to the point c, where the ray $e l c$ touches the mirror; and, having made the angle $C c i$ equal to the angle $C c e$, draw the line $c m i$, which will be the course of the ray $e l c$, after it is reflected from the mirror.

The ray $C m b$ passes through the centre of concavity of the mirror, and falls upon it at b, the perpendicular to it; and is therefore reflected from it in the same line $b m C$.

All these reflected rays meet in the point m; and in that point the image of the body which emits the parallel rays $d a$, $C b$, and $e c$, will be formed: which point is distant from the mirror equal to half the radius $b m C$ of its concavity.[6]

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[6] Unless the mirror $A B$ be exactly parabolic, the rays will not meet in the point m. On this account the mirrors of good reflecting telescopes are portions of a paraboloid, which is a figure generated by the revolution of a parabola about its axis. Let $A a b c B$ (Plate VIII, Fig. 9) be part of a parabola, $C b$ its axis, m its focus, $d a$ a diameter, and let $C a$ be perpendicular to a line touching the parabola in the point a. Then, from Simson's *Con. Sect.* B. i, Prop. 5, the angles $d a C$, $C a m$

The rays which proceed from any celestial object may be esteemed parallel at the earth; and therefore the images of that object will be formed at *m*, when the reflecting surface of the concave mirror is turned directly toward the object. Hence, the focus *m* of parallel rays is not in the centre of the mirror's concavity, but half way between the mirror and that centre.

The rays which proceed from any remote terrestrial object, are nearly parallel at the mirror; not strictly so, but come diverging to it, in separate pencils, or, as it were, bundles of rays, from each point of the side of the object next the mirror: and, therefore, they will not be converged to a point, at the distance of half the radius of the mirror's concavity from its reflecting surface; but into separate points at a little greater distance from the mirror. And the nearer the object is to the mirror, the farther these points will be from it; and an inverted image of the object will be formed in them, which will seem to hang pendant in the air; and will be seen by an eye placed beyond it (with regard to the mirror), in all respects like the object, and as distinct as the object itself.

Let *A c B* (Fig. 10) be the reflecting surface of a mirror, whose centre of concavity is at *C*; and let the upright object *D E* be placed beyond the centre *C*, and send out a conical pencil of diverging rays from its upper extremity *D*, to every point of the concave surface of the mirror *A c B*. But to avoid confusion, we only draw three rays of that pencil, as *DA*, *D c*, *D B*.

From the centre of concavity *C*, draw the three right lines *C A*, *C c*, *C B*, touching the mirror in the same points where the foresaid rays touch it; and all these lines will be perpendicular to the surface of the mirror. Make the angle *C A d* equal to the angle *D A C*, and draw the right line *A d* for the

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will be equal. But if *d a* be a ray of light parallel to the axis *C b*, then since the angle of reflection *m a C* is equal to the angle of incidence *d a C*, *a m* will be the course of the reflected ray which passes through the focus *m*. In the same way it may be shewn, that every other ray parallel to *d a* will pass through *m*. The parabolic speculum *A B*, therefore, will converge parallel rays accurately to its focus. When rays are incident upon a concave mirror, which is a large portion of a sphere, instead of being collected in its focus, they cross each other in different points; and thus form what are denominated *Tschirnhausenian*, or more frequently, *Caustic curves*. These luminous curves may be seen at the bottom of a circular vessel of any kind, with a plane bottom, that is polished in the inside, and exposed either to the light of a candle or to that of the sun.—*Ed.*

course of the reflected ray *d A*: make the angle *C c d* equal to the angle *D c C*, and draw the right line *c d* for the course of the reflected ray *c d:* make also the angle *C B d* equal to the angle *D B C*, and draw the right line *B d* for the course of the reflected ray *B d.* All these reflected rays will meet in the point *d*, where they will form the extremity *d* of the inverted image *e d,* similar to the extremity *D* of the upright object *D E.*

If the pencils of rays *E f, E g, E h,* be also continued to the mirror, and their angles of reflection from it be made equal to their angles of incidence upon it, as in the former pencil from *D*, they will all meet at the point *e*, by reflection, and form the extremity *e* of the image *e d*, similar to the extremity *E* of the object *D E.*

And, as each intermediate point of the object, between *D* and *E*, sends out a pencil of rays in like manner to every part of the mirror, the rays of each pencil will be reflected back from it, and meet in all the intermediate points between the extremities *e* and *d* of the image; and so the whole image will be formed, not at *i*, half the distance of the mirror from its centre of concavity *C*; but at a greater distance, between *i* and the object *D E*; and the image will be inverted with respect to the object.[7]

This being well understood, the reader will easily see how the image is formed by the large concave mirror of the reflecting telescope, when he comes to the description of that instrument.

When the object is more remote from the mirror than its centre of concavity *C*, the image will be less than the object, and between the object and mirror: when the object is nearer than the centre of concavity, the image will be more remote and bigger than the object: thus, if *D E* be the object, *e d* will be its image; for, as the object recedes from the mirror, the image approaches nearer to it; and as the object approaches nearer to the mirror, the image recedes farther from it, on account of the lesser or greater divergency of the pencils of rays which proceed from the object; for, the less they diverge, the sooner they are converged to points by reflection;

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[7] It is a curious fact, that the image of a straight line, formed by a mirror or lens of a spherical form, is a conic section. The reader may see the demonstration of this in Smith's *Optics*, Remarks, v. ii.—*Ed.*

and the more they diverge, the farther they must be reflected before they meet.

If the radius of the mirror's concavity, and the distance of the object from it be known, the distance of the image from the mirror is found by this rule: divide the product of the distance and radius by double the distance made less by the radius, and the quotient is the distance required.

If the object be in the centre of the mirror's concavity, the image and object will be coincident and equal in bulk.

If a man places himself directly before a large concave mirror, but farther from it than its centre of concavity, he will see an inverted image of himself in the air, between him and the mirror, of a less size than himself. And if he holds out his hand toward the mirror, the hand of the image will come out toward his hand, and coincide with it, of an equal bulk, when his hand is in the centre of concavity; and he will imagine he may shake hands with his image. If he reaches his hand farther, the hand of the image will pass by his hand, and come between his hand and his body: and if he moves his hand toward either side, the hand of the image will move toward the other; so that whatever way the object moves, the image will move the contrary way.

All the while a by-stander will see nothing of the image, because none of the reflected rays that form it enter his eyes.

If a fire be made in a large room, and a smooth mahogany table be placed at a good distance near the wall, before a large concave mirror, so placed that the light of the fire may be reflected from the mirror to its focus upon the table; if a person stands by the table, he will see nothing upon it, but a longish beam of light: but if he stands at a distance toward the fire, not directly between the fire and mirror, he will see an image of the fire upon the table, large and erect. And if another person who knows nothing of this matter before-hand, should chance to come into the room, and should look from the fire toward the table, he would be startled at the appearance; for the table would seem to be on fire, and, by being near the wainscot, to endanger the whole house. In this experiment there should be no light in the room, but what proceeds from the fire; and the mirror ought to be at least fifteen inches in diameter.

If the fire be darkened by a screen, and a large candle be placed at the back of the screen, a person standing by the candle will see the appearance of a fine large star, or rather planet, upon the table, as bright as Venus or Jupiter. And if a small wax taper (whose flame is much less than the flame of the candle) be placed near the candle, a satellite to the planet will appear on the table: and if the taper be moved round the candle, the satellite will go round the planet.

For these two pleasing experiments I am indebted to the late reverend Dr. Long, Lowndes's professor of astronomy at Cambridge, who favoured me with the sight of them, and many more of his curious inventions.

Refracting telescopes.

In a *refracting telescope*, the glass which is nearest the object in viewing it, is called the *object-glass;* and that which is nearest the eye, is called the *eye-glass.* The object-glass must be convex, but the eye-glass may be either convex or concave;[8] and generally, in looking through a telescope, the eye is in the focus of the eye-glass; though that is not very material: for the distance of the eye, as to distinct vision, is indifferent, provided the rays of the pencils fall upon it parallel: only, the nearer the eye is to the end of the telescope, the larger is the scope or area of the field of view.

Galilean telescope.

Let *c d* (Fig. 11), be a convex glass fixed in a long tube, and let it have its focus at *E.* Then, a pencil of rays *g h i*, flowing from the upper extremity *A* of the remote object *A B*, will be so refracted by passing through the glass, as to converge and meet in the point *f;* while the pencil of rays *k l m* flowing from the lower extremity *B*, of the same object *A B*, and passing through the glass, will converge and meet in the point *e:* and the images of the points *A* and *B* will be formed in the points *f* and *e.* And as all the intermediate points of the object, between *A* and *B*, send out pencils of rays in the same manner, a sufficient number of these pencils will pass through the object-glass *c d*, and converge to as many intermediate points between *e* and *f;* and so will form the whole inverted image *e E f*, of the distinct object. But because this image is small, a concave glass *n o* is so placed in the end of the tube next the eye, that its virtual focus may be at *F*. And as the rays of the pencils pass converging through the concave glass, but converge less after passing through it than before, they go on further, as to *b* and *a*, before they meet; and the pencils themselves being made to diverge by passing through the concave glass, they enter the eye, and form the large picture *a b* upon the retina, whereon it is magnified under the angle *b F a*.

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[8] When the eye-glass is concave, it is called the *Galilean telescope*, and when convex it is called the *Astronomical telescope.* Although the Galilean telescope has a very small field of view, and is, on that account, unfit for viewing terrestrial objects, yet it shews the satellites of Jupiter, the ring of Saturn, and other celestial appearances, with greater distinctness than the astronomical telescope.—*Ed.*

But this telescope has one inconveniency, which renders it unfit for most purposes, which is, that the pencils of rays being made to diverge by passing through the concave glass *n o*, very few of them can enter the pupil of the eye; and therefore the field of view is but very small, as is evident by the figure. For none of the pencils which flow either from the top or bottom of the object *A B* can enter the pupil of the eye at *C*, but are all stopt by falling upon the iris above and below the pupil: and, therefore, only the middle part of the object can be seen when the telescope lies directly toward it, by means of those rays which proceed from the middle of the object. So that, to see the whole of it, the telescope must be moved upward and downward, unless the object be very remote; and then it is never seen distinctly.

Astronomical telescope.

This inconvenience is remedied by substituting a convex eye-glass, as *g h* (Fig. 12), in place of the concave one; and fixing it so in the tube, that its focus may be coincident with the focus of the object-glass *c d*, as at *E*.[9] For then, the rays of the pencils flowing from the object *A B*, and passing through the object-glass *c d*, will meet in its focus, and form the inverted image *m E p*: and as the image is formed in the focus of the eye-glass *g h*, the rays of each pencil will be parallel, after passing through that glass; but the pencils themselves will cross in its focus, on the other side, as at *e*: and the pupil of the eye being in this focus, the image will be viewed through the glass, under the angle *g e h*; and being at *E*, it will appear magnified, so as to fill the whole space *C m e p D*.

But, as this telescope inverts the image with respect to the object, it gives an unpleasant view of terrestrial objects; and is only fit for viewing the heavenly bodies, in which we regard not their position, because their being inverted does not appear on account of their being round. But whatever way the object seems to move, this telescope must be moved the contrary way, in order to keep sight of it; for, since the object is inverted, its motion will be so too.

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[9] Sometimes another convex glass is placed between the object-glass and its principal focus, in order to enlarge the field of view, and diminish the aberration of the lateral rays. This eye-piece, consisting of two convex lenses, whose distance is equal to the sum of their focal distances, is called the *Huygenian eye-piece.*—*Ed.*

Method of finding its magnifying power.

The magnifying power of this telescope is as the focal distance of the object-glass to the focal distance of the eye-glass. Therefore, if the former be divided by the latter, the quotient will express the magnifying power.

When we speak of magnifying by a telescope or microscope, it is only meant with regard to the diameter, not to the area or solidity of the object. But as the instrument magnifies the vertical diameter, as much as it does the horizontal, it is easy to find how much the whole visible area or surface is magnified: for, if the diameters be multiplied into one another, the product will express the amplification of the whole visible area. Thus, suppose the focal distance of the object-glass be ten times as great as the focal distance of the eye-glass; then, the object will be magnified ten times, both in length and breadth: and 10 multiplied by 10 produces 100; which shews, that the area of the object will appear 100 times as big when seen through such a telescope as it does to the bare eye.

Hence it appears, that if the focal distance of the eye-glass were equal to the focal distance of the object-glass, the magnifying power of the telescope would be nothing.

This telescope may be made to magnify in any given degree, provided it be of a sufficient length: for, the greater the focal distance of the object-glass, the less may be the focal distance of the eye-glass, though not directly in proportion. Thus, an object-glass of 10 feet focal distance, will admit of an eye-glass whose focal distance is little more than $2\frac{1}{2}$ inches; which will magnify near 48 times: but an object-glass, of 100 feet focus, will require an eye-glass somewhat more than 6 inches; and will therefore magnify almost 200 times.

Telescope for terrestrial objects.

A telescope for viewing terrestrial objects should be so constructed as to shew them in their natural posture. And this is done by one object-glass *c d* (Fig. 13), and three eye-glasses *e f*, *g h*, *i k*, so placed, that the distance between any two, which are nearest to each other, may be equal to the sum of their focal distances; as in the figure, where the focus of the glasses *c d* and *e f* meet at *F*, those of the glasses *e f* and *g h* meet at *l*, and of *g h* and *i k* at *m*; the eye being at *n*, in or near the focus of the eye-glass *i k*, on the other side. Then, it is plain that these pencils of rays, which flow from the object *A B*, and pass through the object-glass *c d*, will meet and form an inverted image *C F D* in the focus of that glass: and the image being also in the focus of the glass *e f*, the rays of the pencils will become parallel, after passing through that glass, and cross at *l*, in the focus of the glass *e f*; from whence they pass on to the next glass *g h*, and by going through it they are converged to points in its other focus, where they form an erect image *E m F*, of the object *A B*: and as this image is also in the focus of the eye-glass *i k*, and the eye on the opposite side of the same glass, the image is viewed through the eye-glass in this telescope, in the same manner as through the eye-glass in the former one; only in a contrary position, that is, in the same position with the object.

The three glasses next the eye have all their focal distances equal: and the magnifying power of this telescope is found the same way as that of the last; viz. by dividing the focal distance of the object-glass *c d*, by the focal distance of the eye-glass *i k*, or *g h*, or *e f*, since all these three are equal.

Why the object appears coloured, when seen through a telescope.

When the rays of light are separated by refraction, they become coloured, and if they be united again, they will be a perfect white. But those rays which pass through a convex glass, near its edges, are more unequally refracted than those which are nearer the middle of the glass. And when the rays of any pencil are unequally refracted by the glass, they do not at all meet again in one and the same point, but in separate points; which makes the image indistinct and coloured about its edges.[1] The remedy is, to have a plate with a small round hole in its middle, fixed in the tube at *m*, parallel to the glasses. For, the wandering rays about the edges of the glasses will be stopt, by the plate, from coming to the eye; and none admitted but those which come through the middle of the glass, or at least at a good distance from its edges, and pass through the hole in the middle of the plate. But this circumscribes the image, and lessens the field of view, which would be much larger if the plate could be dispensed with.

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[1] For an account of achromatic telescopes and eye-pieces, by which the colour, arising from the different refrangibility of the rays of light, is removed, see Appendix, Vol. II.—*Ed.*

The *Reflecting telescope.* The great inconvenience attending the management of long telescopes of this kind has brought them much into disuse ever since the *reflecting telescope* was invented: for one of this sort, six feet in length, magnifies as much as one of the other an hundred feet. It was invented by Sir Isaac Newton, but has received considerable improvements since his time; and is now generally constructed in the following manner, which was first proposed by Dr. Gregory.[2]

Gregorian telescope. At the bottom of the great tube *T T T T* (Fig. 14) is placed the large concave mirror *D U V F,* whose principal focus is at *m;* and in its middle is a round hole *P*, opposite to which is placed the small mirror *L*, concave toward the great one;[3] and so fixed to a strong wire *M*, that it may be moved farther from the great mirror, or nearer to it by means of a long screw on the outside of the tube, keeping its axis still in the same line *P m n* with that of the great one. Now, since in viewing a very remote object, we can scarcely see a point of it but what is at least as broad as the great mirror, we may consider the rays of each pencil, which flow from every point of the object, to be parallel to each other, and to cover the whole reflecting surface *D U V F*. But, to avoid confusion in the figure, we shall only draw two rays of a pencil flowing from each extremity of the object into the great tube, and trace their progress, through all their reflections and refractions, to the eye *f*, at the end of the small tube *t t*, which is joined to the great one.

Let us then suppose the object *A B* to be at such a distance, that the rays *C* may flow from its lower extremity *B*, and the rays *E* from its upper extremity *A*. Then the rays *C* falling parallel upon the great mirror at *D*, will be thence reflected, converging in the direction *D G;* and by crossing at *I* in the principal focus of the mirror, they will form the upper extremity *I* of the inverted image *I K*, similar to the lower extremity *B* of the object *A B*: and passing on to the concave mirror *L* (whose focus is at *n*), they will fall upon it at *g*, and be thence reflected, converging in the direction *g N*, because *g m* is longer than *g n*; and, passing through the hole *P* in the large mirror, they would meet somewhere about *r*, and form the lower extremity *b* of the erect image *a b*, similar to the lower extremity *B* of the object *A B*. But, by passing through the plano-convex glass *R* in their way, they form that extremity of the image at *b*. In like manner, the rays *E*, which come from the top of the object *A B*, and fall parallel upon the great mirror at *F*, are thence reflected converging to its focus, where they form the lower extremity *K* of the inverted image *I K*, similar to the upper extremity *A* of the object *A B*; and thence passing on to the small mirror *L*, and falling upon it at *h*, they are thence reflected in the converging state *h O*; and, going on through the hole *P* of the great mirror, they will meet somewhere about *q*, and form there the upper extremity *a* of the erect image *a b*, similar to the upper extremity *A* of the object *A B*: but, by passing through the convex glass *R* in their way, they meet and cross sooner, as at *a*, where that point of the erect image is formed. The like being understood of all those rays which flow from the intermediate points of the object, between *A* and *B*, and enter the tube *T T*, all the intermediate points of the image between *a* and *b* will be formed: and the rays passing on from the image through the eye-glass *S*, and through a small hole *e* in the end of the lesser tube *t t*, they enter the eye *f*, which sees the image *a b* (by means of the eye-glass) under the large angle *c e d*, and magnified in length, under that angle from *c* to *d*.

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[2] A fuller description of Newtonian and Gregorian telescopes, with tables of their apertures and magnifying powers, &c. is given in the Appendix, Vol. II.—*Ed.*

[3] When the small mirror is convex, it is then called the *Cassegrainian* telescope. As the small mirror is in this case placed between the large speculum and its focus, a Cassegrainian telescope will be shorter than a Gregorian telescope of the same magnifying power, by twice the focal distance of the small mirror. The object, however, in the former appears inverted. See Appendix, Vol. II, for a farther account of the Cassegrainian telescope.—*Ed.*

In the best reflecting telescopes, the focus of the small mirror is never coincident with the focus *m* of the great one, where the first image *I K* is formed, but a little beyond it (with respect to the eye), as at *n*: the consequence of which is, that the rays of the pencils will not be parallel, after reflection from the small mirror, but converge so as to meet in points about *q*, *c*, *r*, where they will form a larger upright image than *a b*, if the glass *R* was not in their way; and this image might be viewed by means of a single eye-glass properly placed between the image and the eye: but then the field of view would be less, and, consequently, not so pleasant; for which reason, the glass *R* is still retained, to enlarge the scope or area of the field.[4]

To find the magnifying power of this telescope, multiply the focal distance of the great mirror by the distance of the small mirror from the image next the eye, and multiply the focal distance of the small mirror by the focal distance of the eye-glass; then divide the product of the former multiplication by the product of the latter, and the quotient will express the magnifying power.[5]

I shall here set down the dimensions of one of Mr. Short's reflecting telescopes, as described in Dr. Smith's Optics.

The focal distance of the great mirror is 9.6 inches, its breadth 2.3; the focal distance of the small mirror 1.5, its breadth 0.6: the breadth of the hole in the great mirror 0.5; the distance between the small mirror and the next eye-glass 14.2; the distance between the two eye-glasses 2.4; the focal distance of the eye-glass next the metals 3.8; and the focal distance of the eye-glass next the eye 1.1.

One great advantage of the reflecting telescope is, that it will admit of an eye-glass of a much shorter focal distance than a refracting telescope will; and, consequently, it will magnify so much the more: for the rays are not coloured by reflection from a concave mirror, if it be ground to a true figure, as they are by passing through a convex-glass, let it be ground ever so true.

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[4] Although our author, along with other optical writers, seems to prefer the Gregorian to the Newtonian telescope, yet it can scarcely be doubted, that the latter is best fitted for celestial observations. As a proof of this assertion, it might be sufficient to mention the telescopes constructed by Dr. Herschel, which are of the Newtonian form, and certainly superior, not merely in size, but in distinctness, to any that have hitherto been made. If the plain mirror be taken away from the Newtonian telescope, and the large speculum inclined by means of a screw to the incident rays, (as recommended by *Le Maire*, in the *Machines approuvées par l'Acad. T.* 6, *p.* 61), the observer, with his back to the object, may view the image formed by the large speculum, with a convex glass. This improvement, which was made by *Le Maire* in 1728, seems to have suggested to Dr. Herschel a similar practice, of throwing away the small mirror; though I believe the great speculum in the Doctor's largest telescopes is scarcely inclined to the incident rays.—*Ed.*

[5] The following method of finding the magnifying power of telescopes is the most correct that can be given, and answers for refracting and reflecting ones of every kind. Having put up a small circle of paper, an inch or two in diameter, at the distance of about a hundred yards, draw upon a card two black parallel lines, whose distance from each other is equal to the diameter of the paper circle. Then view through the telescope the paper circle with one eye, and the parallel lines with the other; and let the parallel lines be moved nearer to, or farther from, the eye, till they seem exactly to cover the small circle viewed through the telescope. The quotient, obtained by dividing the distance of the paper circle by the distance of the parallel lines from the eye will be the magnifying power of the telescope. A little practice is necessary before this experiment can be made with accuracy. See Appendix, Vol. II.—*Ed.*

The adjusting screw on the outside of the great tube fits this telescope to all sorts of eyes, by bringing the small mirror either nearer to the eye, or removing it farther from it; by which means, the rays are made to diverge a little for short-sighted eyes, or to converge for those of a long sight.

The nearer an object is to the telescope the more its pencils of rays will diverge before they fall upon the great mirror, and therefore they will be the longer of meeting in points after reflection; so that the first image *I K* will be formed at a greater distance from the larger mirror, when the object is near the telescope, than when it is very remote. But as this image must be formed farther from the small mirror than its principal focus *n*, this mirror must be always set at a greater distance from the large one, in viewing near objects, than in viewing remote ones. And this is done by turning the screw on the outside of the tube, until the small mirror be so adjusted that the object (or rather its image) appears perfect.

In looking through any telescope toward an object, we never see the object itself, but only that image of it which is formed next the eye in the telescope. For, if a man holds his finger or a stick between his bare eye and an object, it will hide part, if not the whole, of the object from his view. But if he ties a stick across the mouth of a telescope, before the object-glass, it will hide no part of the imaginary object he saw through the telescope before, unless it covers the whole mouth of the tube: for, all the effect will be, to make the object appear dimmer, because it intercepts part of the rays. Whereas, if he puts only a piece of wire across the inside of the tube, between the eye-glass and his eye, it will hide part of the object which he thinks he sees: which proves that he sees not the real object, but its image. This is also confirmed by means of the small mirror *L*, in the reflecting telescope, which is made of opaque metal, and stands directly between the eye and the object toward which the telescope is turned, and will hide the whole object from the eye at *e*, if the two glasses *R* and *S* are taken out of the tube.

Plate IX. Fig. 1. The *Multiplying glass.*

The multiplying glass is made by grinding down the round side *h i k* of a convex glass *A B* into several flat surfaces, as *h b*, *b l d*, *d k*. An object *C* will not appear magnified, when seen through this glass, by the eye at *H*; but it will appear multiplied into as many different objects as the glass contains plane surfaces. For, since rays will flow from the object *C* to all parts of the glass, and each plane surface will refract these rays to the eye, the same object will appear to the eye in the direction of the rays which enter it through each surface. Thus, a ray *g i H*, falling perpendicularly on the middle surface, will go through the glass to the eye without suffering any refraction: and will therefore shew the object in its true place at *C*: while a ray *a b* flowing from the same object, and falling obliquely on the plane surface *b h*, will be refracted in the direction *b e*, by passing through the glass; and, upon leaving it, will go on to the eye in the direction *e H*; which will cause the same object *C* to appear also at *E*, in the direction of the ray *H e*, produced in the right line *H e n*. And the ray *c d* flowing from the object *C*, and falling obliquely on the same surface *d k*, will be refracted (by passing through the glass and leaving it at *f*) to the eye at *H*; which will cause the same object to appear at *D*, in the direction *H f m*. If the glass be turned round the line *g l H*, as an axis, the object *C* will keep its place, because the surface *b l d* is not removed; but all the other objects will seem to go round *C*, because the oblique planes, on which the rays *a b*, *c d*, fall, will go round by the turning of the glass.

The *Camera obscura.*

The *Camera-obscura* is made by a convex-glass *C D* (Fig. 2), placed in the hole of a window-shutter. Then, if the room be darkened so that no light can enter but what comes through the glass, the pictures of all the objects (as fields, trees, buildings, men, cattle, &c.) on the outside, will be shewn in an inverted order,[6] on a white paper placed at *G H* in the focus of the glass, and will afford a most beautiful and perfect piece of perspective or landscape of whatever is before the glass; especially if the sun shines upon the objects.

If the convex glass *C D* be placed in a tube in the side of a square box, within which is the plane mirror *E F*, reclining backward in an angle of 45 degrees from the perpendicular *k q*, the pencils of rays flowing from the outward objects, and passing through the convex glass to the plane mirror, will be reflected upward from it, and meet in points, as *I* and *K* (at the same distance that they would have met at *H* and *G*, if the mirror had not been in the way), and will form the aforesaid images on an oiled paper stretched horizontally in the direction *I K;* on which paper, the outlines of the images may be easily drawn with a black-lead pencil, and then copied on a clean sheet, and coloured by art, as the objects themselves are by nature. In this machine, it is usual to place a plane glass, unpolished, in the horizontal situation *I K*, which glass receives the images of the outward objects; and their outlines may be traced upon it by a black-lead pencil.

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[6] If the observer stand behind the paper which receives the image, the objects will appear erect.—*Ed.*

N. B. The tube in which the convex glass *C D* is fixed must be made to draw out, or push in, so as to adjust the distance of that glass from the plane mirror, in proportion to the distance of the outward objects; which the operator does, until he sees their images distinctly painted on the horizontal glass at *I K*.[7]

The forming a horizontal image as *I K*, of an upright object *A B*, depends upon the angles of incidence of the rays upon the plane mirror *E F*, being equal to their angles of reflection from it. For, if a perpendicular be supposed to be drawn to the surface of the plane mirror at *e*, where the ray *A a C e* falls upon it, that ray will be reflected upward in an equal angle with the other side of the perpendicular, in the line *e d I*. Again, if a perpendicular be drawn to the mirror from the point *f*, where the ray *A b f* falls upon it, that ray will be reflected in an equal angle from the other side of the perpendicular, in the line *f h I*. And if a perpendicular be drawn from the point *g*, where the ray *A c g* falls upon the mirror, that ray will be reflected in an equal angle from the other side of the perpendicular, in the line *g i I*. So that all the rays of the pencil *a b c*, flowing from the upper extremity of the object *A B*, and passing through the convex glass *C D*, to the plane mirror *E F*, will be reflected from the mirror and meet at *I*, where they will form the extremity *I* of the image *I K*, similar to the extremity *A* of the object *A B*. The like is to be understood of the pencil *q r s*, flowing from the lower extremity of the object *A B*, and meeting at *K* (after reflection from the plane mirror) the rays from the extremity *K* of the image, similar to the extremity *B* of the object: and so of all the pencils that flow from the intermediate points of the object to the mirror, through the convex glass.

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[7] For an account of some improvements on the camera obscura, and the description of a new portable one invented by the Rev. Mr. Thomson of Duddingston, see Appendix, Vol. II.—*Ed.*

The *Opera-glass.* If a convex glass, of a short focal distance, be placed near the plane mirror, in the end of a short tube, and a convex glass be placed in a hole in the side of the tube, so that the image may be formed between the last mentioned convex glass and the plane mirror, the image being viewed through this glass will appear magnified. In this manner the *opera-glasses* are constructed; with which a gentleman may look at any lady at a distance in the company, and the lady know nothing of it.

The common *looking-glass.* The image of any object that is placed before a plane mirror appears as big to the eye as the object itself; and is erect, distinct, and seemingly as far behind the mirror, as the object is before it: and that part of the mirror, which reflects the image of the object to the eye (the eye being supposed equally distant from the glass with the object), is just half as long and half as broad as the object itself. Let *A B* (Fig. 3), be an object placed before the reflecting surface *g h i* of the plane mirror *C D;* and let the eye be at *o*. Let *A h* be a ray of light flowing from the top *A* of the object, and falling upon the mirror at *h:* and *h m* be a perpendicular to the surface of the mirror at *h*, the ray *A h* will be reflected from the mirror to the eye at *o*, making an angle *m h o* equal to the angle *A h m:* then will the top of the image *E* appear to the eye in the direction of the reflected ray *o h* produced to *E*, where the right line *A p E*, from the top of the object, cuts the right line *o h E,* at *E*. Let *B i* be a ray of light proceeding from the foot of the object at *B* to the mirror at *i*, and *n i* a perpendicular to the mirror from the point *i*, where the ray *B i* falls upon it: this ray will be reflected in the line *i o*, making an angle *n i o*, equal to the angle *B i n*, with that perpendicular, and entering the eye at *o:* then will the foot *F* of the image appear in the direction of the reflected ray *o i*, produced to *F*, where the right line *B F* cuts the reflected ray produced to *F*. All the other rays that flow from the intermediate points of the object *A B*, and fall upon the mirror between *h* and *i*, will be reflected to the eye at *o;* and all the intermediate points of the image *E F* will appear to the eye in the direction-line of these reflected rays produced. But all the rays that flow from the object and fall upon the mirror above *h*, will be reflected back above the eye at *o*; and all the rays that flow from the object, and fall upon the mirror below *i*, will be reflected back below the eye at *o*: so that none of the rays that fall above *h*, or below *i*, can be reflected to the eye at *o*; and the distance between *h* and *i* is equal to half the length of the object *A B*.

A man will see his image in a plane looking-glass that is but half his height.

Hence it appears, that if a man see his whole image in a plane looking-glass, the part of the glass that reflects his image must be just half as long and half as broad as himself, let him stand at any distance from it whatever; and that his image may appear just as far behind the glass as he is before it. Thus, the man *A B* (Fig. 4) viewing himself in the plane mirror *C D*, which is just half as long as himself, sees his whole image as at *E F*, behind the glass, exactly equal to his own size. For, a ray *A C* proceeding from his eye, at *A*, and falling perpendicularly upon the surface of the glass at *C*, is reflected back to his eye in the same line *C A*; and the eye of his image will appear at *E*, in the same line produced to *E*, beyond the glass. And a ray *B D*, flowing from his foot, and falling obliquely on the glass at *D*, will be reflected as obliquely on the other side of the perpendicular *a b D*, in the direction *D A*; and the foot of his image will appear at *F*, in the direction of the reflected ray *A D*, produced to *F*, where it is cut by the right line *B G F*, drawn parallel to the right line *A C E*; just the same as if the glass were taken away, and a real man stood at *F*, equal in size to the man standing at *B*: for to his eye at *A*, the eye of the other man at *E* would be seen in the direction of the line *A C E*; and the foot of the man at *F* would be seen by the eye at *A*, in the direction of the line *A D F*.

If the glass be brought nearer the man *A B*, as suppose to *c b*, he will see his image as at *C D G*: for the reflected ray *C A* (being perpendicular to the glass) will shew the eye of the image as at *C*; and the incident ray *B b*, being reflected in the line *b A*, will shew the foot of his image as at *G*; the angle of reflection *a b A* being always equal to the angle of incidence *B b a*: and so of all the intermediate rays from *A* to *B*. Hence, if the man *A B* advances toward the glass *C D*, his image will approach toward it; and if he recedes from the glass, his image will also recede from it.

Different refrangibility of light.

Having already shewn, that the rays of light are refracted when they pass obliquely through different mediums, we come now to prove that some rays are more refrangible than others: and that, as they are differently refracted, they excite in our minds the ideas of different colours. This will account for the colours seen about the edges of the images of those objects which are viewed through some telescopes.

The *prism.*

Let the sun shine into a dark room through a small hole, as at *c e* (Plate IX, Fig. 5), in a window-shutter; and place a triangular prism *B C* in the beam of rays *A*, in such a manner that the beam may fall obliquely on one of the sides *a b C* of the prism; the rays will suffer different refractions by passing through the prism, so that instead of going all out of it on the side *d c C*, in one direction, they will go on from it in the different directions represented by the lines *f*, *g*, *h*, *i*, *k*, *l*, *m*, *n*; and falling upon the opposite side of the room, or on white paper placed as at *p q* to receive them, they will paint upon it a series of most beautiful lively colours (not to be equalled by art) in this order, viz. those rays which are least refracted by the prism, and will therefore go on between the lines *n* and *m*, will be of a very bright intense *red* at *n*, degenerating from thence gradually into an orange colour, as they are nearer the line *m*: the next will be of a fine *orange* colour at *m*, and from thence degenerate into a yellow colour toward *l*: at *l* they will be of a fine *yellow*, which will incline toward a green, more and more, as they are nearer and nearer *k*: at *k* they will be a pure *green*, but from thence toward *i* they will incline gradually to a blue: at *i* they will be a perfect *blue*, inclining to an indigo colour from thence toward *h*: at *h* they will be quite the colour of *indigo*, which will gradually change toward a violet, the nearer they are to *g*: and at *g* they will be of a fine *violet* colour, which will incline gradually to a red as they come nearer to *f*, where the coloured image ends.[8]

The *colours of the light.*

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[8] Since this work was composed, important discoveries have been made concerning the nature and properties of the prismatic spectrum. Let *p q* be the spectru (Plate

There is not an equal quantity of rays in each of these colours; for, if the oblong image $p\ q$ be divided into 360 equal parts, the red space R will take up 45 of these parts; the orange O, 27; the yellow Y, 48; the green G, 60; the blue B, 60; the indigo I, 40; and the violet V, 80; all which spaces are as nearly proportioned in the figure as the small space $p\ q$ would admit of.[9]

If all these colours be blended together again, they will make a pure white; as is proved thus. Take away the paper on which the colours $p\ q$ fell, and place a large convex glass D in the rays f, g, h, &c. which will refract them so as to make them unite and cross each other at W; where, if a white paper be placed to receive them, they will excite the idea of a strong lively white. But if the paper be placed farther from the glass, as at $r\ s$, the different colours will appear again upon it, in an inverted order, occasioned by the rays crossing at W.

As white is a composition of all colours, so black is a privation of them all; and, therefore, properly no colour.

Let two concentric circles be drawn on a smooth round board $A\ B\ C\ D\ E\ F\ G$ (Fig. 6), and the outermost of them divided

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IX, Fig. 5) formed by the prism $B\ C$; its illuminating power is greatest in the line k, the confines of the green and yellow; and the light decreases from G to p, and from Y to q. When Dr. Herschel was wishing to determine whether or not the power of the differently coloured rays to heat bodies varied with their power to illuminate them, he found that their heating power increased from V to R. The mercury of the thermometer rose higher when its bulb was placed in I than when it was placed in V; still higher in B; and highest of all at R. Upon placing the bulb of the thermometer below R, about q, *quite out of the spectrum*, he was surprised to find that the mercury rose highest of all; and concluded, that rays proceed from the sun which have the power of heating, but not of illuminating bodies: these rays are called *invisible solar rays*. Since the date of Dr. Herschel's discovery, Mr. Ritter and Dr. Wollaston have found, that the rays of the spectrum have different chemical properties; and that there are invisible rays at p, beyond the violet, which act chemically, while they have neither the power of heating nor illuminating bodies. Muriate of silver, exposed to the action of the red rays, became blackish; a greater effect was produced by the yellow; a still greater by the violet; and the greatest of all by the invisible rays beyond the violet. When phosphorus was exposed to the action of the invisible rays beyond the red, it emitted white fumes; but the invisible rays beyond the violet extinguished them. Dr. Morichini of Rome has found that the *violet* rays of the spectrum have the power of communicating magnetism to small needles of steel. See the *Edinburgh Philosophical Journal*, vol. i, p. 239.—*Ed.*

[9] The above measures of the coloured spaces in the prismatic spectrum do not always hold; nor are these spaces constantly proportional. There is an evident fallacy in Newton's experiment upon this subject, for when the mean refractive power of glass is given, we cannot thence infer its power of dispersion. See Appendix, Vol. II.—*Ed.*

into 360 equal parts or degrees: then, draw seven right lines, as $\odot A$, $\odot B$, &c. from the centre to the outermost circle; making the lines $\odot A$ and $\odot B$ include 80 degrees of that circle; the lines $\odot B$ and $\odot C$ 40 degrees; $\odot C$ and $\odot D$ 60; $\odot D$ and $\odot E$ 60; $\odot E$ and $\odot F$ 48; $\odot F$ and $\odot G$ 27; $\odot G$ and $\odot A$ 45. Then between these two circles paint the space *A G* red, inclining to orange near *G*; *G F* orange, inclining to yellow near *F*; *F E* yellow, inclining to green near *E*; *E D* green, inclining to blue near *D*; *D C* blue, inclining to indigo near *C*; *C B* indigo, inclining to violet near *B*; and *B A* violet, inclining to a soft red near *A*. This done, paint all that part of the board black which lies within the inner circle; and putting an axis through the centre of the board, let it be turned very swiftly round that axis, so that the rays proceeding from the above colours may be all blended and mixed together in coming to the eye; and then the whole coloured part will appear like a white ring, a little greyish; not perfectly white, because no colours prepared by art are perfect.

All the prismatic colours blended together, make a white.

Any of these colours, except red and violet, may be made by mixing together the two contiguous prismatic colours. Thus, yellow is made by mixing together a due proportion of orange and green; and green may be made by a mixture of yellow and blue.[1]

All bodies appear of that colour whose rays they reflect most; as a body appears red when it reflects most of the red-making rays, and absorbs the rest.

Transparent colours become opaque, if put together.

Any two or more colours that are quite transparent by themselves, become opaque when put together. Thus, if water, or spirits of wine, be tinged red, and put in a phial, every object seen through it will appear red, because it lets only the red rays pass through it, and stops all the rest. If water or spirits be tinged blue, and put in a phial, all objects seen through it will appear blue, because it transmits only the blue rays, and stops all the rest. But if these two phials are held close together, so that both of them may be between the eye and object, the object will no more be seen through them than through a plate of metal; for whatever rays are transmitted through the fluid in the phial next the object, are stopped by that in the phial next the eye. In this experiment, the phials ought not to be round, but square; because nothing but the light itself can be seen through a round transparent body, at any distance.

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[1] Our author is here mistaken, as *pure yellow* cannot be composed by mixing together any proportion of *green* and *orange*. It is on this account that the celebrated Tobias Mayer maintains, in opposition to Newton (*De Affinitate Colorum* in his *Opera Inedita*, p. 33, published by Litchenberg), that there are only *three* primary colours, viz. *Red*, *yellow*, and *violet*; because every other colour may be composed of a proper proportion of any two of these, while they themselves are simple, and incapable of being produced by composition.—*Ed.*

As the rays of light suffer different degrees of refraction by passing obliquely through a prism, or through a convex glass, and are thereby separated into all the seven original or primary colours; so they also suffer different degrees of refraction by passing through drops of falling rain; and then, being reflected toward the eye, from the sides of these drops which are farthest from the eye, and again refracted by passing out of these drops into the air, in which refracted directions they come to the eye, they make all the colours to appear in the form of a fine arch in the heavens, which is called the *rainbow*.

There are always two rainbows seen together, the interior of which is formed by the rays *a b* (Fig. 7), which falling upon the upper part *b* of the drop *b c d*, are refracted into the line *b c* as they enter the drop, and are reflected from the back of it at *c* in the line *c d*, and then, by passing out of the drop into air, they are again refracted at *d*; and from thence they pass on to the eye at *e*; so that to form the interior bow, the ray suffers two refractions, as at *b* and *d*, and one reflection, as at *c*.

The exterior bow is formed by rays which suffer two reflections, and two refractions, which is the occasion of its being less vivid than the interior, and also of its colours being inverted with respect to those of the interior. For, when a ray *a b* falls upon the lower part of the drop *b c d e*, it is refracted into the direction *b c* (Fig. 8) by entering the drop; and passing on to the back of the drop at *c*, it is thence reflected in the line *c d*, in which direction it is impossible for it to enter the eye at *f*: but by being again reflected from the point *d* of the drop, it goes on in the drop to *e*, where it passes out of the drop into the air, and is there refracted downward to the eye, in the direction *e f*.[2]

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[2] A rainbow is sometimes seen in the evening, when the moon is nearly full, formed by the refraction of her light through the drops of rain, and is called the Lunar iris.—*Ed.*

LECTURE VIII.

THE DESCRIPTION AND USE OF THE TERRESTRIAL GLOBE.

The *terrestrial globe.*

If a map of the world be accurately delineated, on a spherical ball, the surface thereof will represent the surface of the earth; for the highest hills are so inconsiderable with respect to the bulk of the earth, that they take off no more from its roundness than grains of sand do from the roundness of a common globe; for the diameter of the earth is 8000 miles in round numbers, and no known hill upon it is three miles in perpendicular height.

Proof of the earth's being globular,

That the earth is spherical, or round like a globe, appears, 1, From its casting a round shadow upon the moon, whatever side be turned toward her when she is eclipsed: 2, From its having been sailed round by several persons: 3, From our seeing the farther the higher we stand: 4, From our seeing the masts of a ship while the hull is hid by the convexity of the water.

And that it may be peopled on all sides, without any one's being in danger of falling away from it.

The attractive power of the earth draws all terrestrial bodies toward its centre; as is evident from the descent of bodies in lines perpendicular to the earth's surface, at the places whereon they fall, even when they are thrown off from the earth on opposite sides, and, consequently, in opposite directions: so that the earth may be compared to a great magnet rolled in filings of steel, which attracts and keeps them equally fast to its surface on all sides. Hence, as all terrestrial bodies are attracted toward the earth's centre, they can be in no danger of falling from any side of the earth, more than from any other.

Up and *down*, what.

The heaven or sky surrounds the whole earth; and, when we speak of *up* or *down*, we mean only with regard to ourselves; for no point, either in the heavens, or on the surface of the earth, is *above* or *below*, but only with respect to ourselves. And let us be upon what part of the earth we will, we stand with our feet toward its centre, and our heads toward the sky: and so we say, it is *up* toward the sky, and *down* toward the centre of the earth.

To an observer placed anywhere in the indefinite space, where there is nothing to limit his view, all remote objects appear equally distant from him; and seem to be placed in a vast concave sphere, of which his eye is the centre. Every astronomer can demonstrate, that the moon is much nearer to us than the sun is; that some of the planets are sometimes nearer to us, and sometimes farther from us, than the sun; that others of them never come so near us as the sun always is; that the remotest planet in our system is, beyond comparison, nearer to us than any of the fixed stars are; and that it is highly probable some stars are, in a manner, infinitely more distant from us than others; and yet all these celestial objects appear equally distant from us. Therefore, if we imagine a large hollow sphere of glass to have as many bright studs fixed to its inside, as there are stars visible in the heavens, and these studs to be of different magnitudes, and placed at the same angular distances from each other as the stars are, the sphere will be a true representation of the starry heavens, to an eye supposed to be in its centre, and viewing it all around. And if a small globe, with a map of the earth upon it, be placed on an axis in the centre of this starry sphere, and the sphere be made to turn round on this axis, it will represent the apparent motion of the heavens round the earth.

All objects in the heavens appear equally distant.

The face of the heavens and earth represented in a machine.

If a great circle be so drawn upon this sphere, as to divide it into two equal parts, or hemispheres, and the plane of the circle be perpendicular to the axis of the sphere, this circle will represent the *equinoctial*, which divides the heavens into two equal parts, called the *northern* and the *southern hemispheres*; and every point of that circle will be equally distant from the *poles*, or ends of the axis in the sphere. That pole which is in the middle of the northern hemisphere will be called the *north pole of the sphere*, and that which is in the middle of the southern hemisphere, the *south pole*.

The *equinoctial*.

The *poles*.

If another great circle be drawn upon the sphere, in such a manner as to cut the equinoctial at an angle of 23½ degrees in two opposite points, it will represent the *ecliptic*, or circle of the sun's apparent annual motion; one half of which is on the north side of the equinoctial, and the other half on the south.

The *ecliptic*.

If a large stud be made to move eastward in this ecliptic, in such a manner as to go quite round it, in the same time that the sphere is turned round westward 366 times upon its axis; this stud will represent the *sun*, changing his place every day a 365th part of the ecliptic; and, going round westward, the same way as the stars do; but with a motion so much slower than the motion of the stars, that they will make 366 revolutions about the axis of the sphere, in the time that the sun makes only 365. During one half of these revolutions, the sun will be on the north side of the equinoctial; during the other half, on the south; and at the end of each half, in the equinoctial.

The *sun*.

The *earth*.

If we suppose the terrestrial globe in this machine to be about one inch in diameter, and the diameter of the starry sphere to be about five or six feet, a small insect on the globe would see only a very little portion of its surface; but it would see one half of the starry sphere; the convexity of the globe hiding the other half from its view. If the sphere be turned westward round the globe, and the insect could judge of the appearances which arise from that motion, it would see some stars rising to its view in the eastern side of the sphere, while others were setting on the western; but as all the stars are fixed to the sphere, the same stars would always rise in the same points of view on the east side, and set in the same points of view on the west side. With the sun it would be otherwise, because the sun is not fixed to any point of the sphere, but moves slowly along an oblique circle in it. And if the insect should look toward the south, and call that point of the globe, where the equinoctial in the sphere seems to cut it on the left side, the *east point;* and where it cuts the globe on the right side, the *west point,* the little animal would see the sun rise north of the east, and set north of the west for 182½ revolutions; after which, for as many more, the sun would rise south of the east, and set south of the west. And in the whole 365 revolutions, the sun would rise only twice in the east point, and set twice in the west. All these appearances would be the same, if the starry sphere stood still (the sun only moving in the ecliptic) and the earthly globe were turned round the axis of the sphere eastward. For, as the insect would be carried round with the globe, he would be quite insensible of its motion; and the sun and stars would appear to move westward.

The apparent motion of the heavens.

We are but very small beings when compared with our earthly globe, and *the globe itself* is but a dimensionless point compared with the magnitude of the starry heavens. Whether the earth be at rest, and the heavens turn round it, or the heavens be at rest, and the earth turns round, the appearance to us will be exactly the same. And because the heavens are so immensely large, in comparison of the earth, we see one half of the heaven as well from the earth's surface, as we could do from its centre, if the limits of our view are not intercepted by hills.

Circles of the sphere.

We may imagine as many circles described upon the earth as we please: and we may imagine the plane of any circle described upon the earth to be continued, until it marks a circle in the concave sphere of the heavens.

The *horizon.*

The *horizon* is either *sensible* or *rational.* The *sensible* horizon is that circle, which a man standing upon a large plane, observes to terminate his view all around, where the heavens and earth seem to meet. The plane of our sensible horizon continued to the heavens, divides it into two hemispheres; one visible to us, the other hid by the convexity of the earth.

The plane of the *rational horizon* is supposed parallel to the plane of the sensible; to pass through the centre of the earth, and to be continued to the heavens. And although the plane of the sensible horizon touches the earth in the place of the observer, yet *this* plane, and that of the rational horizon, will seem to coincide in the heavens, because the whole earth is but a point compared to the sphere of the heavens.

The earth being a spherical body, the horizon, or limit of our view, must change as we change our place.

Poles.

The *poles of the earth* are those two points on its surface in which its axis terminates. The one is called the *north pole,* and the other the *south pole.*

The *poles of the heavens* are those two points in which the earth's axis produced terminates in the heavens: so that the *north pole* of the heavens is directly over the north pole of the earth; and the *south pole* of the heavens is directly over the south pole of the earth.

Equator.

The *equator* is a great circle upon the earth, every part of which is equally distant from either of the poles. It divides the earth into two equal parts, called the *northern* and *southern hemispheres.* If we suppose the plane of this circle to be extended to the heavens, it will mark the *equinoctial* therein, and will divide the heavens into two equal parts, called the *northern* and *southern hemispheres* of the heavens.

Meridian.

The *meridian* of any place is a great circle passing through that place and the poles of the earth. We may imagine as many such meridians as we please, because any place that is ever so little to the east or west of any other place, has a different meridian from that place; for no one circle can pass through any two such places and the poles of the earth.

The *meridian* of any place is divided by the poles into two semicircles: that which passes through the place is called the *geographical* or *upper meridian;* and that which passes through the opposite place is called the *lower meridian.*

Noon and midnight.

When the rotation of the earth brings the plane of the geographical meridian to the sun, it is *noon,* or *mid-day,* to that place; and when our lower meridian comes to the sun, it is *midnight.*

All places lying under the same geographical meridian have their noon at the same time, and, consequently, all the other hours. All those places are said to have the same *longitude,* because no one of them lies either eastward or westward from any of the rest.

Hour circles.

If we imagine 24 semicircles, one of which is the geographical meridian of a given place, to meet at the poles, and to divide the equator into 24 equal parts, each of these meridians will come round to the sun in 24 hours, by the earth's equable motion round its axis in that time. And, as the equator contains 360 degrees, there will be 15 degrees contained between any two of these meridians which are nearest to one another; for 24 times 15 is 360. And, as the earth's motion is eastward, the sun's apparent motion will be westward, at the rate of 15 degrees each hour. Therefore, they whose geographical meridian is 15 degrees eastward from us, have noon, and every other hour, an hour sooner than we have. They whose meridian is fifteen degrees westward from us, have noon, and every other hour, an hour later than we have: and so on in proportion, reckoning one hour for every fifteen degrees.

Longitude.

As the earth turns round its axis once in 24 hours, and shews itself all around to the sun in that time, so it goes round the sun once a-year, in a great circle called the *ecliptic*, which crosses the equinoctial in two opposite points, making an angle of 23½ degrees with the equinoctial on each side. So that one half of the ecliptic is in the northern hemisphere, and the other in the southern. It contains 360 equal parts, called degrees (as all other circles do, whether great or small); and, as the earth goes once round it every year, the sun will appear to do the same, changing his place almost a degree, at a mean rate, every 24 hours. So that, whatever place, or degree of the ecliptic, the earth is in at any time, the sun will then appear in the opposite. And as one half of the ecliptic is on the north side of the equinoctial, and the other half on the south, the sun, as seen from the earth, will be half a year on the south side of the equinoctial, and half a year on the north; and twice a-year in the equinoctial itself.

Ecliptic.

Signs and degrees.

The ecliptic is divided by astronomers into 12 equal parts, called *signs*, each sign into 30 *degrees*, and each degree into 60 *minutes*: but in using the globes, we seldom want the sun's place nearer than half a degree of the truth.

The names and characters of the 12 signs are as follow: beginning at that point of the ecliptic where it crosses the equinoctial to the northward, and reckoning eastward round to the same point again. And the days of the months on which the sun now enters the signs, are set down below them.

Aries, ♈	*Taurus*, ♉	*Gemini*, ♊	*Cancer*, ♋
March 20	April 19	May 20	June 21
Leo, ♌	*Virgo*, ♍	*Libra*, ♎	*Scorpio*, ♏
July 22	August 22	September 22	October 22
Sagittarius, ♐	*Capricornus*, ♑	*Aquarius*, ♒	*Pisces*, ♓
November 21	December 21	January 19	February 18

By remembering on what day the sun enters any particular sign, we may easily find his place any day afterward, while he is in that sign, by reckoning a degree for each day, which will occasion no error of consequence in using the globes.

When the sun is at the beginning of Aries, he is in the equinoctial; and from that time he declines northward every day, until he comes to the beginning of Cancer, which is 23½° from the equinoctial; from thence he recedes southward every day, for half a year; in the middle of which half, he crosses the equinoctial at the beginning of Libra, and at the end of that half year he is at his greatest south declination, in the beginning of Capricorn, which is also 23½° from the equinoctial. Then he returns northward from Capricorn every day, for half a year; in the middle of which half, he crosses the equinoctial at the beginning of Aries, and, at the end of it, he arrives at Cancer.

The sun's motion in the ecliptic is not perfectly equable, for he continues eight days longer in the northern half of the ecliptic than in the southern: so that the summer half year, in the northern hemisphere, is eight days longer than the winter half year; and the contrary in the southern hemisphere.

Tropics.

The *tropics* are lesser circles in the heavens, parallel to the equinoctial; one on each side of it, touching the ecliptic in the points of its greatest declination; so that each tropic is 23½° from the equinoctial, one on the north side of it, and the other on the south. The northern tropic touches the ecliptic at the beginning of Cancer, the southern at the beginning of Capricorn; for which reason the former is called the tropic of Cancer, and the latter the tropic of Capricorn.

Polar circles.

The *polar circles* in the heavens are each 23½° from the poles, all around. That which goes round the north pole is called the *arctic circle*, from ἄρκτος, which signifies a *bear;* there being a collection or group of stars near the north pole, which goes by that name. The south polar circle is called the *antarctic circle*, from its being opposite to the arctic.

The ecliptic, tropics, and polar circles, are drawn upon the terrestrial globe, as well as upon the celestial. But the ecliptic, being a great fixed circle in the heavens, cannot properly be said to belong to the terrestrial globe: and is laid down upon it only for the conveniency of solving some problems. So that, if this circle on the terrestrial globe were properly divided into the months and days of the year, it would not only suit the globe better, but would also make the problems thereon much easier.

In order to form a true idea of the earth's motion round its axis every 24 hours, which is the cause of day and night; and of its motion in the ecliptic round the sun every year, which is the cause of the different lengths of days and nights, and of the vicissitude of seasons,—take the following method, which will be both easy and pleasant.

An idea of the seasons.

Let a small terrestrial globe, of about three inches diameter, be suspended by a long thread of twisted silk, fixed to its north pole: then having placed a lighted candle on a table, to represent the sun, in the centre of a hoop of a large cask, which may represent the ecliptic, the hoop making an angle of 23½° with the plane of the table; hang the globe within the hook near to it; and if the table be level, the equator of the globe will be parallel to the table, and the plane of the hoop will cut the equator at an angle of 23½°: so that one half of the equator will be above the hoop, and the other half below it; and the candle will enlighten one half of the globe, as the sun enlightens one half of the earth, while the other half is in the dark.

Things being thus prepared, twist the thread toward the left hand, that it may turn the globe the same way by untwisting; that is, from west, by south, to east. As the globe turns round its axis or thread, the different places of its surface will go regularly through the light and dark; and have, as it were, an alternate return of day and night in each rotation. As the globe continues to turn round, and to shew itself all around to the candle, carry it slowly round the hoop by the thread, from west, by south, to east; which is the way that the earth moves round the sun, once a year, in the ecliptic; and you will see, that while the globe continues in the lower part of the hoop, the candle (being then north of the equator) will constantly shine round the north pole; and all the northern places which go through any part of the dark, will go through a less portion of it than they do of the light, and the more so, the farther they are from the equator; consequently their days are then longer than their nights. When the globe comes to a point in the hoop, mid-way, between the highest and lowest points, the candle will be directly over the equator, and will enlighten the globe just from pole to pole; and then every place on the globe will go through equal portions of light and darkness, as it runs round its axis; and, consequently, the day and night will be of equal length at all places upon it. As the globe advances thenceforward, toward the highest part of the hoop, the candle will be on the south side of the equator, shining farther and farther round the south pole, as the globe rises higher and higher in the hoop: leaving the north pole as much in darkness, as the south pole is then in the light, and making long days and short nights on the south side of the equator, and the contrary on the north side, while the globe continues in the northern or higher side of the hoop: and when it comes to the highest point, the days will be at the longest, and the nights at the shortest, in the southern hemisphere; and the reverse in the northern. As the globe advances and descends in the hoop, the light will gradually recede from the south pole, and approach toward the north pole, which will cause the northern days to lengthen, and the southern days to shorten, in the same proportion. When the globe comes to the middle point, between the highest and lowest points of the hoop, the candle will be over the equator, enlightening the globe just from pole to pole, when every place of the earth (except the poles) will go through equal portions of light and darkness; and, consequently, the day and night will be then equal all over the globe.

And thus, at a very small expence, one may have a delightful and demonstrative view of the cause of days and nights, with their gradual increase and decrease in length, through the whole year together, with the vicissitudes of spring, summer, autumn, and winter, in each annual course of the earth round the sun.[1]

If the hoop be divided into 12 equal parts, and the signs be marked in order upon it, beginning with Cancer at the highest point of the hoop, and reckoning eastward (or contrary to the apparent motion of the sun), you will see how the sun appears to change his place every day in the ecliptic, as the globe advances eastward along the hoop, and turns round its own axis: and that when the earth is in a low sign, as at Capricorn, the sun must appear in a high sign, as at Cancer, opposite to the earth's real place; and that while the earth is in the southern half of the ecliptic, the sun appears in the northern half, and *vice versa*: that the farther any place is from the equator, between it and the polar circle, the greater is the difference between the longest and shortest day at that place; and that the poles have but one day and one night in the whole year.

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[1] This method of exhibiting the change of seasons is represented in Plate IV, Fig. 3, of Ferguson's *Astronomy*.—*Ed.*

These things premised, we shall proceed to the description and use of the terrestrial globe, and explain the geographical terms as they occur in the problems.

The terrestrial globe described.

This globe has the boundaries of land and water laid down upon it; the countries and kingdoms divided by dots, and coloured, to distinguish them; the islands properly situated, the rivers and principal towns inserted, as they have been ascertained upon the earth by measurement and observation.

The equator, ecliptic, tropics, polar circles, and meridians, are laid down upon the globe in the manner already described. The ecliptic is divided into twelve signs, and each sign into 30°, which are generally subdivided into halves, and into quarters, if the globe is large. Each tropic is 23½° from the equator, and each polar circle 23½° from its respective pole. Circles are drawn parallel to the equator at every 10° distance from it on each side to the poles: these circles are called *parallels of latitude.* On large globes there are circles drawn perpendicularly through every tenth degree of the equator, intersecting each other at the poles: but on globes of or under a foot diameter, they are only drawn through every fifteenth degree of the equator; these circles are generally called *meridians*, sometimes *circles of longitude*, and at other times *hour circles.*

The globe is hung in a brass ring, called the *brazen meridian;* and turns upon a wire in each pole sunk half its thickness into one side of the meridian ring: by which means, *that* side of the ring divides the globe into two equal parts, called the *eastern* and *western hemispheres;* as the equator divides it into two equal parts, called the *northern* and *southern hemispheres.* This ring is divided into 360 equal parts or degrees, on the side wherein the axis of the globe turns. One half of these degrees are numbered, and reckoned from the equator to the poles, where they end at 90: their use is to shew the latitudes of places. The degrees on the other half of the meridian ring are numbered from the poles to the equator, where they end at 90: their use is to shew how to elevate either the north or south pole above the horizon, according to the latitude of any given place, as it is north or south of the equator.

The brazen meridian is let into two notches made in a broad flat ring, called the *wooden horizon*, the upper surface of which divides the globe into two equal parts, called the *upper* and *lower hemispheres.* One notch is in the north point of the horizon, and the other in the south. On this horizon are several concentric circles which contain the months and days of the year, the signs and degrees answering to the sun's place for each month and day, and the 32 points of the compass. The graduated side of the brass meridian lies toward the east side of the horizon, and should be generally kept toward the person who works problems by the globes.

There is a small *horary* circle, so fixed to the north part of the brazen meridian, that the wire in the north pole of the globe is in the centre of that circle; and on the wire is an *index*, which goes over all the 24 hours of the circle, as the globe is turned round its axis. Sometimes there are two horary circles, one between each pole of the globe and the brazen meridian; which is the contrivance of the late ingenious Mr. Joseph Harris, master of the Assay-office in the Tower of London; and makes it very convenient for putting the poles of the globe through the horizon, and for elevating the pole to small latitudes, and declinations of the sun; which cannot be done where there is only one horary circle fixed to the outer edge of the brazen meridian.

There is a thin slip of brass called the *Quadrant of Altitude*, which is divided into 90 equal parts or degrees, answering exactly to so many degrees of the equator. It is occasionally fixed to the uppermost point of the brazen meridian by a nut and screw. The divisions end at the nut, and the quadrant is turned round upon it.

As the globe has been seen by most people, and upon the figure of which, in a plate, neither the circles nor countries can be properly expressed, we judge it would signify very little to refer to a figure of it; and shall therefore only give some directions how to choose a globe, and then describe its use.

Directions for choosing globes.

1. See that the papers be well and neatly pasted on the globes, which you may know, if the lines and circles thereon meet exactly, and continue all the way even and whole; the circles not breaking into several arches, nor the papers either coming short, or lapping over one another.

2. See that the colours be transparent, and not laid too thick upon the globe to hide the names of places.

3. See that the globe hangs evenly between the brazen meridian and the wooden horizon; not inclining either to one side or to the other.

4. See that the globe be as close to the horizon and meridian as it conveniently may; otherwise you will be too much puzzled to find against what part of the globe any degree of the meridian or horizon is.

5. See that the equinoctial line be even with the horizon all around, as the north or south pole is elevated 90° above the horizon.

6. See that the equinoctial line cuts the horizon in the east and west points, in all elevations of the pole from 0 to 90.°

7. See that the degree of the brazen meridian marked with 0, be exactly over the equinoctial line of the globe.

8. See that there be exactly half of the brazen meridian above the horizon; which you may know, if you bring any of the decimal divisions on the meridian to the north point of the horizon, and find their complement to 90 in the south point.

9. See that when the quadrant of altitude is placed as far from the equator, or the brazen meridian, as the pole is elevated above the horizon, the beginning of the degrees of the quadrant reaches just to the plane surface of the horizon.

10. See that while the index of the hour-circle (by the motion of the globe) passes from one hour to another, 15 degrees of the equator pass under the graduated edge of the brazen meridian.

11. See that the wooden horizon be made substantial and strong: it being generally observed, that in most globes the horizon is the first part that fails, on account of its having been made too slight.

Directions for using them.

In using the globes, keep the east side of the horizon toward you (unless your problem requires the turning of it), which side you may know by the word *east* upon the horizon; for then you have the graduated side of the meridian toward you, the quadrant of altitude before you, and the globe divided exactly into two equal parts, by the graduated side of the meridian.

In working some problems, it will be necessary to turn the whole globe and horizon about, that you may look on the west side thereof: which turning will be apt to jog the ball so as to shift away that degree of the globe which was before set to the horizon or meridian: to avoid which inconvenience you may thrust in the feather-end of a quill between the ball of the globe and the brazen meridian; which, without hurting the ball, will keep it from turning in the meridian, while you turn the west side of the horizon toward you.

PROB. I.—*To find the Longitude*[1] *and Latitude*[2] *of any given place upon the globe.*

Turn the globe on its axis, until the given place comes exactly under that graduated side of the brazen meridian, on which the degrees are numbered from the equator; and observe what degree of the meridian the place then lies under; which is its latitude, north or south, as the place is north or south of the equator.

The globe remaining in this position, the degree of the equator, which is under the brazen meridian, is the longitude of the place (from the meridian of London on the English globes), which is east or west, as the place lies on the east or west side of the first meridian of the globe. All the Atlantic ocean, and America, is on the west side of the meridian of London; and the greatest part of Europe, and of Africa, together with all Asia, is on the east side of the meridian of London, which is reckoned the *first meridian* of the globe by the British geographers and astronomers.

PROB. II.—*The Longitude and Latitude of a Place being given, to find that Place on the globe.*

Look for the given longitude in the equator (counting it eastward or westward from the first meridian, as it is mentioned to be east or west), and bring the point of longitude in the equator to the brazen meridian, on that side which is above the south point of the horizon ; then count from the equator, on the brazen meridian, to the degree of the given latitude, toward the north or south pole, according as the latitude is north and south ; and under that degree of latitude on the meridian, you will have the place required.

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[1] The *latitude* of a place is its distance from the equator, and is *north* or *south*, as the place is north or south of the equator. Those who live at the equator have no latitude, because it is there that the latitude begins.

[2] The *longitude* of a place is the number of degrees (reckoned upon the equator) that the meridian of the said place is distant from the meridian of any other place from which we reckon, either *eastward* or *westward*, for 180° or half round the globe. The British reckon the longitude from the meridian of London, or rather from the observatory of Greenwich, and the French from the meridian of Paris. The meridian of that place, from which the longitude is reckoned, is called the *first meridian*. The places upon this meridian have no longitude, because it is there that the longitude begins.

PROB. III.—*To find the difference of longitude, or difference of latitude, between any two given places.*

Bring each of these places to the brazen meridian, and see what its latitude is : the lesser latitude subtracted from the greater, if both places are on the same side of the equator, or both latitudes added together, if they are on different sides of it, is the difference of latitude required. And the number of degrees contained between these places, reckoned on the equator, when they are brought separately under the brazen meridian, is their difference of longitude, if it be ess than 180 : but if more, let it be subtracted from 360, and the remainder is the difference of longitude required. Or,

Having brought one of the places to the brazen meridian, and set the hour index to XII, turn the globe until the other place comes to the brazen meridian, and the number of hours and parts of an hour, passed over by the index, will give the longitude in time ; which may be easily reduced to degrees, by allowing 15° for every hour, and one degree for every four minutes.

N. B.—When we speak of bringing any place to the brazen meridian, it is the graduated side of the meridian that is meant.

PROB. IV.—*Any place being given, to find all those places that have the same longitude or latitude with it.*

Bring the given place to the brazen meridian ; then all those places which lie under that side of the meridian, from pole to pole, have the same longitude with the given place. Turn the globe round its axis, and all those places which pass under the same degree of the meridian that the given place does, have the same latitude with that place.

Since all latitudes are reckoned from the equator, and all longitudes are reckoned from the first meridian, it is evident that the point of the equator which is cut by the first meridian, has neither latitude nor longitude.—The greatest latitude is 90°, because no place is more than 90° from the equator. And the greatest longitude is 180°, because no place is more than 180° from the first meridian.

PROB. V.—*To find the Antœci,*[3] *Periœci,*[4] *and Antipodes,*[5] *of any given place.*

Bring the given place to the brazen meridian, and having found its latitude, keep the globe in that situation, and count the same number of degrees of latitude from the equator toward the contrary pole, and where the reckoning ends, you have the *antœci* of the given place upon the globe. Those who live at the equator have no *antœci.*

The globe remaining in the same position, set the hour-index to the upper XII, on the horary circle, and turn the globe until the index comes to the lower XII; then, the place which lies under the meridian, in the same latitude with the given place, is the *periœci* required. Those who live at the poles have no *periœci.*

As the globe now stands (with the index at the lower XII) the *antipodes* of the given place will be under the same point of the brazen meridian where its *antœci* stood before. Every place upon the globe has its *antipodes.*

PROB. VI.—*To find the distance between any two places on the globe.*

Lay the graduated edge of the quadrant of altitude over both the places, and count the number of degrees intercepted between them on the quadrant; then multiply these degrees by 60, and the product will give the distance in geographical miles: but to find the distance in English miles, multiply the degrees by $69\frac{1}{2}$, and the product will be the number of miles required. Or, take the distance between any two places with a pair of compasses, and apply that extent to the equator; the number of degrees, intercepted between the points of the compasses, is the distance in degrees of a great circle;[6] which may be reduced either to geographical miles, or to English miles, as above.

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[3] The *Antœci* are those who live on the same meridians, and in equal latitudes, on different sides of the equator. Being on the same meridian, they have the same hours; that is, when it is noon to the one, it is also noon to the other; and when it is midnight to the one, it is also midnight to the other, &c. Being on different sides of the equator, they have different or opposite seasons at the same time; the length of any day to the one is equal to the length of the night of that day to the other; and they have equal elevations of the different poles.

[4] The *Periœci* are those people who live on the same parallel of latitude, but on opposite meridians: so that though their latitude be the same, their longitude differs 180 degrees. By being in the same latitude, they have equal elevations of the same pole (for the elevation of the pole is always equal to the latitude of the place), the same length of days or nights, and the same seasons. But being on opposite meridians, when it is noon to the one, it is midnight to the other.

[5] The *Antipodes* are those who live diametrically opposite to one another upon the globe, standing with feet toward feet, on opposite meridians and parallels. Being on opposite sides of the equator, they have opposite seasons, winter to one, when it is summer to the other; being equally distant from the equator, they have their contrary poles equally elevated above the horizon; being on opposite meridians, when it is noon to the one, it must be midnight to the other; and as the sun recedes from the one when he approaches to the other, the length of the day to one must be equal to the length of the night at the same time to the other.

Prob. VII.—*A place on the globe being given, and its distance from any other place, to find all the other places upon the globe which are at the same distance from the given place.*

Bring the given place to the brazen meridian, and screw the quadrant of altitude to the meridian, directly over that place; then keeping the globe in that position, turn the quadrant quite round upon it, and the degree of the quadrant that touches the second place, will pass over all the other places which are equally distant with it from the given place.

This is the same as if one foot of a pair of compasses was set in the given place and the other foot extended to the second place, whose distance is known; for if the compasses be then turned round the first place as a centre, the moving foot will go over all those places which are at the same distance with the second from it.

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[6] Any circle that divides the globe into two equal parts, is called a *Great Circle*, as the equator or meridian. Any circle that divides the globe into two unequal parts (which every parallel of latitude does) is called a *Lesser Circle*. Now, as every circle, whether great or small, contains 360° and a degree upon the equator or meridian contains 60 geographical miles, it is evident, that a degree of longitude upon the equator is longer than a degree of longitude upon any parallel of latitude, and must therefore contain a greater number of miles. So that, although all the degrees of latitude are equally long upon an artificial globe (though not precisely so upon the earth itself), yet the degrees of longitude decrease in length as the latitude increases, but not in the same proportion. The table in page 208 shews the length of a degree of longitude, in geographical miles, and hundredth parts of a mile, for every degree of latitude, from the equator to the poles: a degree on the equator being 60 geographical miles.

N. B. As the table given by our author supposed the earth to be a perfect sphere, we have computed it anew for an oblate spheroidal figure, or an ellipsoid, the compression at its poles, or the difference between the equatoreal and polar axes being $\frac{1}{300}$.—*Ed.*

A TABLE shewing the number of miles in a degree of longitude, in any given degree of latitude, the earth being supposed an oblate spheriod, and the flattening at its poles $\frac{1}{300}$.

Degrees of latitude.	Length of a degree of longitude.	Degrees of latitude.	Length of a degree of longitude.	Degrees of latitude.	Length of a degree of longitude.	Degrees of latitude.	Length of a degree of longitude.
	Miles. Dec.		Miles. Dec.		Miles. Dec.		Miles. Dec.
1	59.99	24	54.84	47	40.99	70	20.58
2	59.96	25	54.41	48	40.22	71	19.59
3	59.92	26	53.96	49	39.44	72	18.60
4	59.86	27	53.50	50	38.64	73	17.60
5	59.77	28	53.02	51	37.84	74	16.59
6	59.67	29	52.52	52	37.02	75	15.58
7	59.56	30	52.01	53	36.19	76	14.56
8	59.42	31	51.48	54	35.34	77	13.54
9	59.27	32	50.93	55	34.49	78	12.51
10	59.10	33	50.37	56	33.63	79	11.49
11	58.91	34	49.79	57	32.76	80	10.45
12	58.70	35	49.20	58	31.87	81	9.42
13	58.47	36	48.60	59	30.98	82	8.38
14	58.23	37	47.98	60	30.08	83	7.34
15	57.97	38	47.34	61	29.16	84	6.29
16	57.69	39	46.69	62	28.24	85	5.25
17	57.40	40	46.03	63	27.31	86	4.20
18	57.08	41	45.35	64	26.37	87	3.15
19	56.75	42	44.66	65	25.43	88	2.10
20	56.40	43	43.95	66	24.47	89	1.05
21	56.04	44	43.23	67	23.51	90	0.00
22	55.66	45	42.50	68	22.54		
23	55.26	46	41.75	69	21.56		

PROB. VIII.—*The hour of the day at any place being given, to find all those places where it is noon at that time.*

Bring the given place to the brazen meridian, and set the index to the given hour; this done, turn the globe until the index points to the upper XII, and then, all the places that lie under the brazen meridian have noon at that time.

N. B. The upper XII always stands for noon; and when the bringing of any place to the brazen meridian is mentioned, the side of that meridian on which the degrees are reckoned from the equator is meant, unless the contrary side be mentioned.

Prob. IX.—*The hour of the day at any place being given, to find what time it then is at any other place.*

Bring the given place to the brazen meridian, and set the index to the given hour; then turn the globe, until any place where the time is required comes to the brazen meridian, and the index will point out the time at that place.

Prob. X.—*To find the sun's place in the ecliptic, and his declination,*[7] *for any given day of the year.*

Look on the horizon for the given day, and right against it you have the degree of the sign in which the sun is (or his place) on that day at noon. Find the same degree of that sign in the ecliptic line upon the globe, and having brought it to the brazen meridian, observe what degree of the meridian stands over it; for that is the sun's declination, reckoned from the equator.

Prob. XI.—*The day of the month being given, to find all those places of the earth over which the sun will pass vertically on that day.*

Find the sun's place in the ecliptic for the given day, and having brought it to the brazen meridian, observe what point of the meridian is over it; then, turning the globe round its axis, all those places which pass under that point of the meridian are the places required: for as their latitude is equal, in degrees and parts of a degree, to the sun's declination, the sun must be vertical (or directly over head) to each of them at its respective noon.

Prob. XII.—*A place being given in the torrid zone,*[8] *to find those two days of the year, on which the sun shall be vertical to that place.*

Bring the given place to the brazen meridian, and mark the degree of latitude that is exactly over it on the meridian; then turn e globe round its axis, and observe the two degrees of the ecliptic which pass exactly under that degree of latitude: lastly, find on the wooden horizon the two days of the year on which the sun is in those degrees of the ecliptic, and they are the days required: for on them, and none else, the sun's declination is equal to the latitude of the given place: and, consequently, he will then be vertical to it at noon.

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[7] The sun's declination is his distance from the equinoctial in degrees, &c. and is north or south, according as the sun is between the equinoctial and the north or south pole.

[8] The globe is divided into five zones; one torrid, two temperate, and two frigid. The *torrid* zone lies between the two tropics, and is 47° in breadth, or 23½ on each

Prob. XIII.—*To find all those places of the north frigid zone, where the sun begins to shine constantly without setting, on any given day, from the 20th of March to the 22d of September.*

On these two days, the sun is in the equinoctial, and enlightens the globe exactly from pole to pole: therefore, as the earth turns round its axis, which terminates in the poles, every place upon it will go equally through the light and the dark, and so make the day and night equal to all places of the earth. But as the sun declines from the equator, toward either pole, he will shine just as many degrees round that pole, as are equal to his declination from the equator; so that no place within the distance of the pole will then go through any part of the dark, and, consequently, the sun will not set to it. Now, as the sun's declination is northward, from the 20th of March to the 22d of September, he must constantly shine round the north pole all that time; and on the day that he is in the northern tropic, he shines upon the whole north frigid zone; so that no place within the north polar circle goes through any part of the dark on that day. Therefore, having brought the sun's place for the given day to the brazen meridian, and found his declination (by Prob. X), count as many degrees on the meridian, from the north pole, as are equal to the sun's declination from the equator, and mark that degree from the pole where the reckoning ends: then, turning the globe round its axis, observe what places in the north frigid zone pass directly under the mark; for they are the places required.

The like may be done for the south frigid zone, from the

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side of the equator: the *temperate zones* lie between the tropics and polar circles, or from 23½° of latitude, to 66½, on each side of the equator; and are each 43° in breadth: the *frigid zones* are the spaces included within the polar circles, which being each 23½° from their respective poles, the diameter of each of these zones is 47°. As the sun never goes without the tropics, he must every moment be vertical to some place or other in the torrid zone.

22d of September to the 20th of March, during which time the sun shines constantly on the south pole.

Prob. XIV.—*To find the place over which the sun is vertical, at any hour of a given day.*

Having found the sun's declination for the given day (by Prob. X), mark it with a chalk on the brazen meridian: then bring the place where you are (suppose London) to the brazen meridian, and set the index to the given hour; which done, turn the globe on its axis, until the index points to XII at noon; and the place on the globe, which is then directly under the point of the sun's declination marked upon the meridian, has the sun that moment in the zenith, or directly over head.[9]

Prob. XV.—*The day and hour at any place being given, to find all those places where the sun is then rising, or setting, or on the meridian; consequently all those places which are enlightened at that time, and those which are in the dark.*

This problem cannot be solved by any globe fitted up in the common way with the hour-circle fixed upon the brass meridian; unless the sun be on or near some of the tropics on the given day. But by a globe fitted up according to Mr. Joseph Harris's invention[1] (already mentioned), where the hour-circle lies on the surface of the globe, below the meridian, it may be solved for any day in the year, according to his method, which is as follows.

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[9] This problem may be easily solved without the assistance of a globe. The latitude of the place where the sun is vertical at any given time, is always equal to the sun's declination, and of the same name; that is, if the sun's declination is north, the latitude of the place will be north, and if south, the latitude will be south. The longitude of the place is always equal to the number of hours (converted into degrees, at the rate of 15° for every hour) that the given time is from noon, and is *East* if the given time is before noon, and *West* if it is after noon. The latitude and longitude, therefore, being known, the corresponding place may be readily found upon a common map. The same method will answer for the moon and stars, with this difference only, that the longitude of the place is equal to the number of hours between the given time and the southing of these celestial bodies.—*Ed.*

[1] In all new globes the hour-circle is placed upon the surface of the globe; and sometimes the hours are marked upon the equator, without any hour-circle at all. A brass wire surrounds the globe, passing through two holes in the brazen meridian, and lying in the plane of the equator; and a small index moves upon it to point out the hour. This is an excellent contrivance, as it enlarges the hour spaces, and enables us to work problems with the greatest accuracy.—*Ed.*

Having found the place to which the sun is vertical at the given hour, if the place be in the northern hemisphere, elevate the north pole as many degrees above the horizon, as are equal to the latitude of that place; if the place be in the southern hemisphere, elevate the south pole accordingly; and bring the said place to the brazen meridian. Then, all those places which are in the western semicircle of the horizon, have the sun rising to them at that time; and those in the eastern semicircle have it setting: to those under the upper semicircle of the brass meridian, it is noon; and to those under the lower semicircle, it is midnight. All those places which are above the horizon, are enlightened by the sun, and have the sun just as many degrees high to them, as they themselves are above the horizon: and this height may be known, by fixing the quadrant of altitude on the brazen meridian over the place to which the sun is vertical; and then, laying it over any other place, observe what number of degrees on the quadrant are intercepted between the said place and the horizon. In all those places that are 18° below the western semicircle of the horizon, the morning twilight is just beginning; in all those places that are 18° below the eastern semicircle of the horizon, the evening twilight is ending; and all those places that are lower than 18° have dark night.

If any place be brought to the upper semicircle of the brazen meridian, and the hour-index be set to the upper XII or noon, and then the globe be turned round eastward on its axis; when the place comes to the western semicircle of the horizon, the index will shew the time of sun-rising at that place; and when the same place comes to the eastern semicircle of the horizon, the index will shew the time of sun-set.

To those places which do not go under the horizon, the sun sets not on that day; and to those which do not come above it, the sun does not rise.

PROB. XVI.—*The day and hour of a lunar eclipse being given, to find all those places of the earth to which it will be visible.*

The moon is never eclipsed but when she is full, and so directly opposite to the sun, that the earth's shadow falls upon her. Therefore, whatever place of the earth the sun is vertical to at that time, the moon must be vertical to the antipodes of that place; so that the sun will be then visible to one half of the earth, and the moon to the other.

Find the place to which the sun is vertical at the given hour (by Prob. XIV); elevate the pole to the latitude of that place, and bring the place to the upper part of the brazen meridian, as in the former problem: then, as the sun will be visible to all those parts of the globe which are above the horizon, the moon will be visible to all those parts of the globe which are below it, at the time of her greatest obscuration.

But with regard to an eclipse of the sun, there is no such thing as shewing to what places it will be visible, with any degree of certainty, by a common globe, because the moon's shadow covers but a small portion of the earth's surface; and her latitude, or declination from the ecliptic, throws her shadow so variously upon the earth, that to determine the places on which it falls, recourse must be had to long calculations.

PROB. XVII.—*To rectify the globe for the latitude, the zenith,*[a] *and the sun's place.*

Find the latitude of the place (by Prob. I), and if the place be in the northern hemisphere, raise the north pole above the north point of the horizon, as many degrees (counted from the pole upon the brazen meridian) as are equal to the latitude of the place. If the place be in the southern hemisphere, raise the south pole above the south point of the horizon, as many degrees as are equal to the latitude. Then turn the globe till the place comes under its latitude on the brazen meridian, and fasten the quadrant of altitude so that the chamfered edge of its nut (which is even with the graduated edge) may be joined to the zenith, or point of latitude. This done, bring the sun's place in the ecliptic for the given day (found by Prob. X) to the graduated side of the brazen meridian, and set the hour-index to XII at noon, which is the uppermost XII on the hour-circle; and the globe will be rectified.

Remark.

The latitude of any place is equal to the elevation of the nearest pole of the heavens above the horizon of that place; and the poles of the heavens are directly over the poles of the earth, each 90° from the equinoctial line. Let us be upon what place of the earth we will, if the limits of our view be not intercepted by hills, we shall see one-half of the heavens, or 90° every way round, from that point which is over our heads. Therefore, if we were upon the equator, the poles of the heavens would lie in our horizon, or limit of our view; if we go from the equator, toward either pole of the earth, we shall see the corresponding pole of the heavens rising gradually above our horizon, just as many degrees as we have gone from the equator: and if we were at either of the earth's poles, the corresponding pole of the heavens would be directly over our head. Consequently, the elevation or height of the pole in degrees above the horizon, is equal to the number of degrees that the place is from the equator.

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[a] The *Zenith,* in this sense, is the highest point of the brazen meridian above the horizon; but in the proper sense it is that point of the heavens which is directly vertical to any given place, at any given instant of time.

Prob. XVIII.—*The latitude of any place, not exceeding* 66½°,[3] *and the day of the month being given, to find the time of sun-rising and setting, and consequently the length of the day and night.*

Having rectified the globe for the latitude, and for the sun's place on the given day (as directed in the preceding problem), bring the sun's place in the ecliptic to the eastern side of the horizon, and the hour-index will shew the time of sun-rising; then turn the globe on its axis, until the sun's place comes to the western side of the horizon, and the index will shew the time of sun-setting.

The hour of sun-setting doubled, gives the length of the day; and the hour of sun-rising doubled, gives the length of the night.

Prob. XIX.—*The latitude of any place, and the day of the month, being given, to find when the morning twilight begins, and the evening twilight ends, at that place.*

This problem is often limited; for, when the sun does not go 18° below the horizon, the twilight continues the whole night, and for several nights together in summer, between 49 and 66½° of latitude; and the nearer to 66½, the greater is the number of these nights. But when it does begin and end, the following method will shew the time for any given day.

Rectify the globe, and bring the sun's place in the ecliptic to the eastern side of the horizon; then mark that point of the ecliptic with a chalk which is in the western side of the horizon, it being the point opposite to the sun's place: this done, lay the quadrant of altitude over the said point, and turn the globe eastward, keeping the quadrant at the chalk-mark, until it is just 18° high on the quadrant; and the index will point out the time when the morning twilight begins: for the sun's place will then be 18° below the eastern side of the horizon. To find the time when the evening twilight ends, bring the sun's place to the western side of the horizon; and the point opposite to it, which was marked with the chalk, will be rising in the east: then, bring the quadrant over that point, and keeping it thereon, turn the globe westward, until the said point be 18° above the horizon on the quadrant, and the index will shew the time when the evening twilight ends; the sun's place being then 18° below the western side of the horizon.

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[3] All places whose latitude is more than 66½°, are in the frigid zones; and to those places the sun does not set in summer, for a certain number of diurnal revolutions, which occasions this limitation of latitud

PROB. XX.—*To find on what day of the year the sun begins to shine constantly, without setting, on any given place in the north frigid zone, and how long he continues to do so.*

Rectify the globe to the latitude of the place, and turn it about until some point of the ecliptic, between Aries and Cancer, coincides with the north point of the horizon, where the brazen meridian cuts it; then find, on the wooden horizon, what day of the year the sun is in that point of the ecliptic; for that is the day on which the sun begins to shine constantly on the given place, without setting. This done, turn the globe until some point of the ecliptic, between Cancer and Libra, coincides with the north point of the horizon, where the brazen meridian cuts it; and find, on the wooden horizon, on what day the sun is in that point of the ecliptic; which is the day that the sun leaves off constantly shining on the said place, and rises and sets to it as to other places on the globe. The number of natural days, or complete revolutions of the sun about the earth, between the two days above found, is the time that the sun keeps constantly above the horizon without setting: for all the portion of the ecliptic, that lies between the two points which intersect the horizon in the very north, never sets below it: and there is just as much of the opposite part of the ecliptic that never rises: therefore, the sun will keep as long constantly below the horizon in winter as above it in summer.

Whoever considers the globe, will find, that all places of the earth do equally enjoy the benefit of the sun, in respect of time, and are equally deprived of it. For, the days and nights are always equally long at the equator: and in all places that have latitude, the days at one time of the year are exactly equal to the nights at the opposite season.

Prob. XXI.—*To find in what latitude the sun shines constantly, without setting, for any length of time less than* 182½ *of our days and nights.*[4]

Find a point in the ecliptic half as many degrees from the beginning of Cancer (either toward Aries or Libra) as there are natural days[5] in the time given; and bring that point to the north side of the brazen meridian, on which the degrees are numbered from the pole toward the equator; then keep the globe from turning on its axis, and slide the meridian up or down, until the aforesaid point of the ecliptic comes to the north point of the horizon, and then, the elevation of the pole will be equal to the latitude required.

Prob. XXII.—*The latitude of a place, not exceeding* 66½°, *and the day of the month, being given, to find the sun's amplitude, or point of the compass on which he rises or sets on that day.*

Rectify the globe, and bring the sun's place to the eastern side of the horizon; then observe what point of the compass on the horizon stands right against the sun's place, for that is his amplitude at rising. This done, turn the globe westward, until the sun's place comes to the western side of the horizon, and it will cut the point of his amplitude at setting. Or, you may count the rising amplitude in degrees, from the east point of the horizon, to that point where the sun's place cuts it; and the setting amplitude, from the west point of the horizon, to the sun's place at setting.

Prob. XXIII.—*The latitude, the sun's place, and his altitude,*[6] *being given, to find the hour of the day, and the sun's azimuth, or number of degrees that he is distant from the meridian.*

Rectify the globe, and bring the sun's place to the given height upon the quadrant of altitude; on the eastern side of the horizon, if the time be in the forenoon; or the western side, if it be in the afternoon; then, the index will shew the hour; and the number of degrees in the horizon intercepted between the quadrant of altitude and the south point, will be the sun's true azimuth at that time.

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[4] The reason of this limitation is, that 182½ of our days and nights make half a year, which is the longest time that the sun shines without setting, even at the poles of the earth.

[5] A natural day contains the whole 24 hours: an artificial day, the time that the sun is above the horizon.

[6] The sun's altitude, at any time, is his height in degrees above the horizon at that time.

N. B. Always when the quadrant of altitude is mentioned in working any problem, the graduated edge of it is meant.

If this be done at sea, and compared with the sun's azimuth, as shewn by the compass, if they agree, the compass has no variation in that place: but if they differ, the compass does vary: and the variation is equal to this difference.

PROB. XXIV.—*The latitude, hour of the day, and the sun's place, being given, to find the sun's altitude and azimuth.*

Rectify the globe, and turn it until the index points to the given hour; then lay the quadrant of altitude over the sun's place in the ecliptic, and the degree of the quadrant cut by the sun's place is his altitude at that time above the horizon; and the degree of the horizon cut by the quadrant is the sun's azimuth, reckoned from the south.

PROB. XXV.—*The latitude, the sun's altitude, and his azimuth, being given, to find his place in the ecliptic, the day of the month, and hour of the day, though they had all been lost.*

Rectify the globe for the latitude and zenith,[7] and set the quadrant of altitude to the given azimuth in the horizon; keeping it there, turn the globe on its axis until the ecliptic cuts the quadrant in the given altitude: that point of the ecliptic which cuts the quadrant there, will be the sun's place; and the day of the month answering thereto, will be found over the like place of the sun on the wooden horizon. Keep the quadrant of altitude in that position, and having brought the sun's place to the brazen meridian, and the hour-index to XII at noon, turn back the globe until the sun's place cuts the quadrant of altitude again, and the index will shew the hour.

Any two points of the ecliptic which are equi-distant from the beginning of Cancer or of Capricorn, will have the same altitude and azimuth at the same hour, though the months be different; and therefore it requires some care in this problem, not to mistake both the month, and the day of the month; to avoid which, observe, that from the 20th of March to the 21st of June, that part of the ecliptic which is between the beginning of Aries and the beginning of Cancer is to be used: from the 21st of June to the 22d of September, between the beginning of Cancer and the beginning of Libra: from the 22d of September to the 21st of December, between the beginning of Libra and the beginning of Capricorn: and from the 21st of December to the 20th of March, between the beginning of Capricorn and the beginning of Aries. And as one can never be at a loss to know in what quarter of the year he takes the sun's altitude and azimuth, the above caution, with regard to the quarters of the ecliptic, will keep him right as to the month and day thereof.

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[7] By rectifying the globe for the zenith, is meant screwing the quadrant of altitude to the given latitude on the brass meridian.

Prob. XXVI.—*To find the length of the longest day at any given place.*

If the place be on the north side of the equator, find its latitude (by Prob. I), and elevate the north plole to that latitude; then, bring the beginning of Cancer ♋ to the brazen meridian, and set the hour-index to XII at noon. But if the given place be on the south side of the equator, elevate the south pole to its latitude, and bring the beginning of Capricorn ♑ to the brass meridian, and the hour-index to XII. This done, turn the globe westward, until the beginning of Cancer or Capricorn (as the latitude is north or south) comes to the horizon; and the index will then point out the time of sun-setting, for it will have gone over all the afternoon hours, between mid-day and sun-set; which length of time being doubled, will give the whole length of the day, from sun-rising to sun-setting. For, in all latitudes, the sun rises as long before mid-day as he sets after it.

Prob. XXVII.—*To find in what latitude the longest day is of any given length less than 24 hours.*

If the latitude be north, bring the beginning of Cancer to the brazen meridian, and elevate the north pole to about 66½°; but if the latitude be south, bring the beginning of Capricorn to the meridian, and elevate the south pole to about 66½°; because the longest day in north latitude, is when the sun is in the first point of Cancer; and in south latitude, when he is in the first point of Capricorn. Then set the hour-index to XII at noon, and turn the globe westward, until the index points at half the number of hours given: which done, keep the globe from turning on its axis, and slide the meridian down in the notches, until the aforesaid point of the ecliptic (viz. Cancer or Capricorn) comes to the horizon; then, the elevation of the pole will be equal to the latitude required.

PROB. XXVIII.—*The latitude of any place, not exceeding 66½°, being given, to find in what climate*[8] *the place is.*

Find the length of the longest day at the given place by problem XXVI, and whatever be the number of hours whereby it exceeds twelve, double that number, and the sum will answer to the climate in which the place is.

PROB. XXIX.—*The latitude and the day of the month being given, to find the hour of the day when the sun shines.*

Set the wooden horizon truly level, and the brazen meridian due north and south by a mariner's compass; then, having rectified the globe, stick a small sewing needle into the sun's place in the ecliptic, perpendicular to that part of the surface of the globe: this done, turn the globe on its axis, until the needle comes to the brazen meridian, and set the hour-index to XII at noon; then, turn the globe on its axis, until the needle points exactly toward the sun (which it will do when it casts no shadow on the globe), and the index will shew the hour of the day.

PROB. XXX.—*A pleasant way of shewing all those places of the earth which are enlightened by the sun, and also the time of the day when the sun shines.*

Take the terrestrial ball out of the wooden horizon, and also out of the brazen meridian; then set it upon a pedestal in sun-shine, in such a manner, that its north pole may point directly toward the north pole of the heavens, and the meridian of the place where you are be directly toward the south. Then, the sun will shine upon all the like places of the globe, that he does on the real earth, rising to some when he is setting to others; as you may perceive by that part where the enlightened half of the globe is divided from the half in the shade, by the boundary of the light and darkness: all those places on which the sun shines, at any time, having day; and all those, on which he does not shine, having night.

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[8] A *Climate*, from the equator to either of the polar circles, is a tract of the earth's surface, included between two such parallels of latitude, that the length of the longest day in the one exceeds that in the other by half an hour; but, from the polar circles to the poles, where the sun keeps long above the horizon without setting, each climate differs a whole month from the one next to it. There are twenty-four climates between the equator and each of the polar circles, and six from each polar circle to its respective pole.

If a narrow slip of paper be put round the equator, and divided into twenty-four equal parts, beginning at the meridian of your place, and the hours be set to those divisions in such a manner, that one of the VI's may be upon your meridian; the sun being upon that meridian at noon, will then shine exactly to the two XII's; and at one to the two I's, &c. So that the place where the enlightened half of the globe is parted from the shaded half, in this circle of hours, will shew the time of the day.

The principles of Dialling shall be explained farther on, by the terrestrial globe. At present we shall only add the following observations upon it; and then proceed to the use of the celestial globe.

1. The latitude of any place is equal to the elevation of the pole above the horizon of that place, and the elevation of the equator is equal to the complement of the latitude, that is, to what the latitude wants of 90°.

2. Those places which lie on the equator, have no latitude, it being there that the latitude begins; and those places which lie on the first meridian have no longitude, it being there that the longitude begins. Consequently, *that* particular place of the earth, where the first meridian intersects the equator, has neither longitude nor latitude.

3. At all places of the earth, except the poles, all the points of the compass may be distinguished in the horizon: but from the north pole every place is south; and from the south pole every place is north. Therefore, as the sun is constantly above the horizon of each pole for half a year in its turn, he cannot be said to depart from the meridian of either pole for half a year together. Consequently, at the north pole

it may be said to be noon every moment for half a year; and let the winds blow from what part they will, they must always blow from the south; and at the south pole, from the north.

4. Because one half of the ecliptic is above the horizon of the pole, and the sun, moon, and planets, move in (or nearly in) the ecliptic; they will all rise and set to the poles. But, because the stars never change their declinations from the equator (at least not sensibly in one age), those which are once above the horizon of either pole, never set below it; and those which are once below it, never rise.

5. All places of the earth do equally enjoy the benefit of the sun, in respect of time, and are equally deprived of it.

6. All places upon the equator have their days and nights equally long, that is, twelve hours each, at all times of the year. For, although the sun declines, alternately, from the equator toward the north and toward the south, yet, as the horizon of the equator cuts all the parallels of latitude and declination in halves, the sun must always continue above the horizon for one half a diurnal revolution about the earth, and for the other half below it.

7. When the sun's declination is greater than the latitude of any place, upon either side of the equator, the sun will come twice to the same azimuth, or point of the compass, in the forenoon, at that place, and twice to a like azimuth in the afternoon; that is, he will go twice back every day, while his declination continues to be greater than the latitude. Thus, suppose the globe rectified to the latitude of Barbadoes, which is 13° north; and the sun to be anywhere in the ecliptic, between the middle of Taurus and the middle of Leo; if the quadrant of altitude be set to about 18°[9] north of the east in the horizon, the sun's place be marked with a chalk upon the ecliptic, and the globe be then turned westward on its axis, the said mark will rise in the horizon a little to the north of the quadrant, and thence ascending, it will cross the quadrant toward the south; but before it arrives at the meridian, it will cross the quadrant again, and pass over the meridian northward of Barbadoes. And if the quadrant be set about 18° north of the west, the

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[9] From the middle of Gemini to the middle of Cancer, the quadrant may be set at 20°.

sun's place will cross it twice, as it descends from the meridian toward the horizon, in the afternoon.

8. In all places of the earth between the equator and poles, the days and nights are equally long, viz. 12 hours each, when the sun is in the equinoctial; for, in all elevations of the pole short of 90° (which is the greatest), one half of the equator or equinoctial will be above the horizon, and the other half below it.

9. The days and nights are never of an equal length at any place between the equator and polar circles, but when the sun enters the signs ♈ Aries and ♎ Libra: for in every other part of the ecliptic the circle of the sun's daily motion is divided into two unequal parts by the horizon.

10. The nearer any place is to the equator, the less is the difference between the length of the days and nights in that place; and the more remote, the contrary. The circles which the sun describes in the heavens every twenty-four hours, being cut more nearly equal in the former case, and more unequally in the latter.

11. In all places lying upon any given parallel of latitude, however long or short the day or night be at any one of these places, at any time of the year, it is then of the same length at all the rest; for in turning the globe round its axis (when rectified according to the sun's declination) all these places will keep equally long above or below the horizon.

12. The sun is vertical twice a-year to every place between the tropics; to those under the tropics, once a-year, but never anywhere else: for there can be no place between the tropics but that there will be two points in the ecliptic, whose declination from the equator is equal to the latitude of that place; and but one point of the ecliptic which has a declination equal to the latitude of places on the tropic which that point of the ecliptic touches; and as the sun never goes without the tropics, he can never be vertical to any place that lies without them.

13. To all places in the torrid zone,[1] the duration of the twilight is least, because the sun's daily motion is the most perpendicular to the horizon. In the frigid zones,[2] greatest; be-

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[1] Between the tropics.

[2] Between the polar circles and poles.

cause the sun's daily motion is nearly parallel to the horizon; and therefore he is the longer of getting 18° below it, till which time the twilight always continues. And in the temperate zones[3] it is at a medium between the two, because the obliquity of the sun's daily motion is so.

14. In all places lying exactly under the polar circles, the sun, when he is in the nearest tropic, continues twenty-four hours above the horizon without setting; because no part of that tropic is below their horizon. And when the sun is in the farthest tropic, he is for the same length of time without rising; because no part of that tropic is above their horizon. But, at all other times of the year, he rises and sets there, as in other places; because all the circles that can be drawn parallel to the equator, between the tropics, are more or less cut by the horizon, as they are farther from, or nearer to, that tropic, which is all above the horizon: and when the sun is not in either of the tropics, his diurnal course must be in one or other of these circles.

15. To all places in the northern hemisphere, from the equator to the polar circle, the longest day and shortest night is when the sun is in the northern tropic; and the shortest day and longest night is when the sun is in the southern tropic; because no circle of the sun's daily motion is so much above the horizon, and so little below it, as the northern tropic; and none so little above it, and so much below it, as the southern. In the southern hemisphere the contrary.

16. In all places between the polar circles and poles, the sun appears for a number of days (or rather diurnal revolutions) without setting; and at the opposite time of the year without rising; because some part of the ecliptic never sets in the former case, and as much of the opposite part never rises in the latter. And the nearer unto, or the more remote from, the pole, these places are, the longer or shorter is the sun's continuing presence or absence.

17. If a ship sets out from any port, and sails round the earth eastward to the same port again, let her take what time she will to do it in, the people in that ship, in reckoning their time, will gain one complete day at their return, or count one day more than those who reside at the same port; because, by

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[3] Between the tropics and polar circles.

going contrary to the sun's diurnal motion, and being forwarder every evening than they were in the morning, their horizon will get so much the sooner above the setting sun, than if they had kept for a whole day at any particular place. And thus, by cutting off a part, proportional to their own motion, from the length of every day, they will gain a complete day of that sort at their return; without gaining one moment of absolute time more than is elapsed, during their course, to the people at the port. If they sail westward, they will reckon one day less than the people do who reside at the said port; because, by gradually following the apparent diurnal motion of the sun, they will keep him each particular day so much longer above their horizon, as answers to that day's course; and by that means they cut off a whole day in reckoning, at their return, without losing one moment of absolute time.

Hence, if two ships should set out at the same time from any port, and sail round the globe, one eastward, and the other westward, so as to meet at the same port on any day whatever, they will differ two days in reckoning their time, at their return. If they sail twice round the earth, they will differ four days; if thrice, then six, &c.

LECTURE IX.

THE USE OF THE CELESTIAL GLOBE, AND ARMILLARY SPHERE.

The *celestial globe.*

HAVING done for the present with the terrestrial globe, we shall proceed to the use of the celestial; first, premising, that, as the equator, ecliptic, tropics, polar circles, horizon, and brazen meridian, are exactly alike on both globes, all the former problems concerning the sun are solved the same way by both globes. The method also of rectifying the celestial globe is the same as rectifying the terrestrial, viz. elevate the pole according to the latitude of your place, then screw the quadrant of altitude to the zenith, on the brass meridian; bring the sun's place in the ecliptic to the graduated edge of the brass meridian, on the side which is above the south point of the wooden horizon, and set the hour-index to the uppermost XII, which stands for noon.

To rectify it.

N.B.—The sun's place for any day of the year stands directly over that day on the horizon of the celestial globe, as it does on that of the terrestrial.

Latitude and *longitude* of the *stars.*

The latitude and longitude of the stars, and of all other celestial phenomena, are reckoned in a very different manner from the latitude and longitude of places on the earth: for all terrestrial latitudes are reckoned from the equator; and longitudes from the meridian of some remarkable place, as of London by the British, and of Paris by the French, though most of the French maps begin their longitude at the meridian of the island Ferro. But the astronomers of all nations agree in reckoning the latitudes of the moon, stars, planets, and comets, from the ecliptic; and their longitudes from the equinoctial colure,[1] in that semicircle of it which cuts the ecliptic at the beginning of Aries, ♈; and thence eastward, quite round, to the same semicircle again. Consequently those stars which lie between the equinoctial and the northern half of the ecliptic, have north declination and south latitude; those which lie between the equinoctial and the southern half of the ecliptic, have south declination and north latitude; and all those which lie between the tropics and poles have their declinations and latitudes of the same denomination.

There are six great circles on the celestial globe, which cut the ecliptic perpendicularly, and meet in two opposite points in the polar circles; which points are each ninety degrees from the ecliptic, and are called its poles. These polar points divide those circles into twelve semicircles; which cut the ecliptic at the beginnings of the twelve signs. They resemble so many meridians on the terrestrial globe; and as all places which lie under any particular meridian semicircle on that globe, have the same longitude, so all those points of the heavens, through which any one of the above semicircles are drawn, have the same longitude. And as the greatest latitudes on the earth are at the north and south poles of the earth, so the greatest latitudes in the heavens, are at the north and south poles of the ecliptic.

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[1] *The great circle* that passes through the *equinoctial points* at the beginning of ♈ and ♎, and through the *poles of the world* (which are two opposite points, each 90° from the equinoctial), is called the *equinoctial colure:* and the *great circle* that passes through the *beginning* of ♋ and ♑, and also through the *poles of the ecliptic*, and poles of the world, is called the *solstitial colure.*

Constellations.

In order to distinguish the stars, with regard to their situations and positions in the heavens, the ancients divided the whole visible firmament of stars into particular systems, which they called *constellations;* and digested them into the forms of such animals as are delineated upon the celestial globe. And those stars which lie between the figures of those imaginary animals, and could not be brought within the compass of any of them, were called *unformed stars.*

Because the moon and all the planets were observed to move in circles or orbits which cross the ecliptic (or line of the sun's path) at small angles, and to be on the north side of the ecliptic for one half of their course round the heaven of stars, and on the south side of it for the other half, but never to go quite 8° from it on either side, the ancients distinguished that space by two lesser circles, parallel to the ecliptic (one on each side) at 8° distance from it. And the space included between the circles, they called the *zodiac,*[2] because most of the twelve constellations placed therein resemble some living creature. These constellations are, 1, Aries, ♈, the ram; 2, Taurus, ♉, the bull; 3, Gemini, ♊, the twins; 4, Cancer, ♋, the crab; 5, Leo, ♌, the lion; 6, Virgo, ♍, the virgin; 7, Libra, ♎, the balance; 8, Scorpio, ♏, the scorpion; 9, Sagittarius, ♐, the archer; 10, Capricornus, ♑, the goat; 11, Aquarius, ♒, the water-bearer; and 12, Pisces, ♓, the fishes.

Zodiac.

Remark.

It is to be observed, that in the infancy of astronomy, these twelve constellations stood at or near the places of the ecliptic, where the above characteristics are marked upon the globe: but now each constellation has got a whole sign forwarder, on account of the recession of the equinoctial points from their former places. So that the constellation of Aries is now in the former place of Taurus; that of Taurus in the former place of Gemini; and so on.

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[2] Since this work was written, five new planets have been added to our system, by the indefatigable exertions of modern astronomers—the *Georgium Sidus*, discovered by Dr. Herschel; *Ceres*, by Piazzi; *Pallas*, by Dr. Olbers; *Juno*, by Mr. Harding, and *Vesta*, by Dr. Olbers. The Georgium Sidus is situated beyond the orbit of Saturn, and the other three between the orbits of Mars and Jupiter, and almost at *equal* distances from the sun. The inclination of Ceres' orbit to the ecliptic is about 10°; the inclination of Juno's orbit 13°; that of Pallas 25°; and that of Vesta 7°; so that the limits of the zodiac must now be extended, from 8° on each side of the ecliptic, to 25° on each side of it, in order to comprehend these newly discovered planets. The satellites of the Georgium Sidus, indeed, move almost at right angles to the ecliptic; and therefore, properly speaking, there can be no zodiac at all. See Ferguson's *Astronomy*, vol. ii.—*Ed.*

The stars appear of different magnitudes to the eye, probably because they are at different distances from us. Those which appear brightest and largest are called *stars of the first magnitude;* the next to them in size and lustre, are called *stars of the second magnitude;* and so on to the *sixth,* which are the smallest that can be discerned by the bare eye.

Some of the most remarkable stars have names given them, as Castor and Pollux, in the heads of the Twins; Sirius, in the mouth of the Great Dog; Procyon, in the side of the Little Dog; Rigel, in the left foot of Orion; Arcturus, near the thigh of Bootes, &c.[3]

These things being premised, which I think are all that the young tyro need be acquainted with, before he begins to work any problem by this globe, we shall now proceed to the most useful of those problems, omitting several which are of little or no consequence.

PROB. I.—*To find the right ascension,[4] and declination,[5] of the sun, or any fixed star.*

Bring the sun's place in the ecliptic to the brazen meridian, then that degree in the equinoctial which is cut by the meridian is the sun's *right ascension;* and that degree of the meridian which is over the sun's place is his *declination.* Bring any fixed star to the meridian, and its right ascension will be cut by the meridian in the equinoctial; and the degree of the meridian that stands over it is its declination.

So that right ascension and declination on the celestial globe, are found in the same manner as longitude and latitude on the terrestrial.

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[3] The stars of each constellation are distinguished from one another, by prefixing the letters of the Greek alphabet to the name of the constellation. The brightest star, for instance, of the constellation *Leo,* is called *α Leonis;* the next brightest, *β Leonis,* and so on. It must be remembered, however, that the Greek letters do not indicate the magnitude of the stars which they represent, but merely the relative magnitude of those in the same constellation. Thus, *α Virginis* is a star of the *first* magnitude; *α Libræ,* a star of the *second* magnitude; and *α Aquarii,* a star of the *third* magnitude.—*Ed.*

[4] The degree of the equinoctial, reckoned from the beginning of Aries, that comes to the meridian with the sun or star, is its *Right Ascension.* The *Longitude* of any celestial body is counted on the ecliptic, and its *Right Ascension* upon the equinoctial.

[5] The distance of the sun or stars in degrees from the equinoctial, toward either of the poles, north or south, is its *Declination,* which is north or south accordingly.

Prob. II.—*To find the latitude and longitude of any star.*

If the given star be on the north side of the ecliptic, place the 90° of the quadrant of altitude on the north pole of the ecliptic, where the twelve semicircles meet, which divide the ecliptic into the twelve signs: but if the star be on the south side of the ecliptic, place the 90° of the quadrant on the south pole of the ecliptic; keeping the 90° of the quadrant on the proper pole, turn the quadrant about, until its graduated edge cuts the star; then, the number of degrees in the quadrant, between the ecliptic and the star, is its latitude; and the degree of the ecliptic cut by the quadrant, is the star's longitude, reckoned according to the sign in which the quadrant then is.

Prob. III.—*To represent the face of the starry firmament, as seen from any given place of the earth, at any hour of the night.*

Rectify the celestial globe for the given latitude, the zenith, and sun's place, in every respect, as taught by the 17th problem, for the terrestrial; and turn it about, until the index points to the given hour: then, the upper hemisphere of the globe will represent the visible half of the heavens for that time: all the stars upon the globe being then in such situations, as exactly correspond to those in the heavens. And if the globe be placed duly north and south, by means of a small sea-compass, every star on the globe will point toward the like star in the heavens: by which means, the constellations and remarkable stars may be easily known. All those stars which are in the eastern side of the horizon, are then rising in the eastern side of the heavens: all those in the western, are setting in the western side: and all those under the upper part of the brazen meridian, between the south point of the horizon and the north pole, are at their greatest altitude, if the latitude of the place be north: but if the latitude be south, those stars which lie under the upper part of the meridian, between the north point of the horizon and the south pole, are at their greatest altitude.

Prob. IV.—*The latitude of the place, and day of the month, being given, to find the time when any known star will rise, or be on the meridian, or set.*

Having rectified the globe, turn it about until the given star comes to the eastern side of the horizon, and the index will shew the time of the star's rising; then turn the globe westward, and when the star comes to the brazen meridian, the index will shew the time of the star's coming to the meridian of your place; lastly, turn on, until the star comes to the western side of the horizon, and the index will shew the time of the star's setting.

N. B.—In northern latitudes, those stars which are less distant from the north pole, than the quantity of its elevation above the north point of the horizon, never set; and those which are less distant from the south pole, than the number of degrees by which it is depressed below the horizon, never rise; and *vice versa* in southern latitudes.[6]

PROB. V.—*To find at what time of the year a given star will be upon the meridian, at a given hour of the night.*

Bring the given star to the upper semicircle of the brass meridian, and set the index to the given hour; then turn the globe until the index points to XII at noon, and the upper semicircle of the meridian will then cut the sun's place, answering to the day of the year sought, which day may be easily found against the like place of the sun among the signs on the wooden horizon.

PROB. VI.—*The latitude, day of the month, and azimuth*[7] *of any known star, being given, to find the hour of the night.*

Having rectified the globe for the latitude, zenith, and sun's place, lay the quadrant of altitude to the given degree of azimuth in the horizon: then turn the globe on its axis, until the star comes to the graduated edge of the quadrant; and when it does, the index will point out the hour of the night.

PROB. VII.—*The latitude of the place, the day of the month, and altitude*[8] *of any known star, being given, to find the hour of the night.*

Rectify the globe as in the former problem, guess at the hour of the night, and turn the globe until the index points at the supposed hour; then lay the graduated edge of the quadrant of altitude over the known star, and if the degree of the star's height in the quadrant upon the globe, answers exactly to the degree of the star's observed altitude in the heavens, you have guessed exactly: but if the star on the globe is higher or lower than it was observed to be in the heavens, turn the globe backward or forward, keeping the edge of the quadrant upon the star, until its centre comes to the observed altitude in the quadrant: and then the index will shew the true time of the night.

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[6] A circle, described upon the north pole as a centre, and comprehending all those stars which never set in any latitude, is called the *Circle of perpetual apparition;* and another circle, described upon the south pole as a centre, and comprehending those stars which never rise, is called the *Circle of perpetual occultation.—Ed.*

[7] The number of degrees that the sun, moon, or any star, is from the meridian, either to the east or west, is called its *Azimuth.*

[8] The number of degrees that the star is above the horizon, as observed by means of a common quadrant, is called its *Altitude.*

Prob. VIII.—*An easy method of finding the hour of the night by any two known stars, without knowing either their altitude or azimuth; and then, of finding both their latitude and azimuth, and thereby the true meridian.*

Tie one end of a thread to a common musket bullet: and having rectified the globe as above, hold the other end of the thread in your hand, and carry it slowly round between your eye and the starry heavens, until you find it cuts any two known stars at once. Then, guessing at the hour of the night, turn the globe until the index points to the time in the hour-circle; which done, lay the graduated edge of the quadrant over any one of these two stars on the globe, which the thread cuts in the heavens. If the said edge of the quadrant cuts the other star also, you have guessed the time exactly; but if it does not, turn the globe slowly backward or forward, until the quadrant (kept upon either star) cuts them both through their centres: and then the index will point out the exact time of the night; the degree of the horizon, cut by the quadrant, will be the true azimuth of both these stars from the south; and the stars themselves will cut their true altitudes in the quadrant. At which moment, if a common azimuth compass be so set upon a floor or level pavement, that these stars in the heavens may have the same bearing upon it (allowing for the variation of the needle), as the quadrant of altitude has in the wooden horizon of the globe, a thread extended over the north and south points of that compass will be directly in the plane of the meridian; and if a line be drawn upon the floor or pavement, along the course of the thread, and an upright wire be placed in the southernmost end of the line, the shadow of the wire will fall upon that line, when the sun is on the meridian, and shines upon the pavement.

Prob. IX.—*To find the place of the moon, or of any planet, and thereby to shew its time of rising, southing and setting.*

Seek in the Nautical Almanack, the geocentric place[9] of the moon or planet in the ecliptic for the given day of the month, and, according as its longitude and latitude is found, mark the same with a chalk upon the globe. Then, having rectified the globe, turn it round its axis westward; and as the said mark comes to the eastern side of the horizon, to the brazen meridian, and to the western side of the horizon, the index will shew at what time the planet rises, comes to the meridian, and sets, in the same manner as it would do for a fixed star.

Prob. X.—*To explain the phenomena of the harvest moon.*

In order to do this, we must premise the following things: 1, That as the sun goes only once a year round the ecliptic, he can be but once a year in any particular point of it: and that his motion is almost a degree every 24 hours, at a mean rate. 2, That as the moon goes round the ecliptic once in 27 days and 8 hours, she advances $13\frac{1}{6}°$ in it every day, at a mean rate. 3, That as the sun goes through part of the ecliptic in the time the moon goes round it, the moon cannot at any time be either in conjunction with the sun, or opposite to him, in that part of the ecliptic where she was so the last time before; but must travel as much forwarder, as the sun has advanced in the said time: which being $29\frac{1}{2}$ days, makes almost a whole sign. Therefore, 4, The moon can be but once a year opposite to the sun, in any particular part of the ecliptic. 5, That the moon is never full but when she is opposite to the sun, because at no other time can we see all that half of her which the sun enlightens. 6, That when any point of the ecliptic rises, the opposite point sets. Therefore, when the moon is opposite to the sun, she must rise at[1] sun-set. 7, That the different signs of the ecliptic rise at very different angles or degrees of obliquity with the horizon, especially in considerable latitudes; and that the smaller this angle is, the greater is the portion of the ecliptic that rises in any small part of time; and *vice versa.* 8, That, in northern latitudes, no part of the ecliptic rises at so small an angle with the horizon, as Pisces and Aries do; therefore, a greater portion of the ecliptic rises in one hour, about these signs, than about any of the rest. 9, That the moon can never be full in Pisces and Aries but in our autumnal months; for at no other time of the year is the sun in the opposite signs Virgo and Libra.

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[9] The place of the moon or planet, as seen from the earth, is called its *Geocentric* place, and its place, as seen from the sun, is called its *Heliocentric* place.

[1] This is not always strictly true, because the moon does not keep in the ecliptic, but crosses it twice every month. However, the difference need not be regarded in a general explanation of the cause of the harvest moon.

These things premised, take $13\frac{1}{6}°$ of the ecliptic in your compasses, and, beginning at Pisces, carry that extent all round the ecliptic, marking the places with a chalk where the points of the compasses successively fall. So you will have the moon's daily motion marked out for one complete revolution in the ecliptic; according to § 2 of the last paragraph.

Rectify the globe for any considerable northern latitude (as suppose that of London), and then, turning the globe round its axis, observe how much of the hour-circle the index has gone over, at the rising of each particular mark on the ecliptic; and you will find that seven of the marks (which take in as much of the ecliptic as the moon goes through in a week), will all rise successively about Pisces and Aries, in the time that the index goes over two hours: therefore, while the moon is in Pisces and Aries, she will not differ in general above two hours in her rising for a whole week. But if you take notice of the marks on the opposite signs, Virgo and Libra, you will find that seven of them take nine hours to rise: which shews, that when the moon is in these two signs, she differs nine hours in her rising within the compass of a week. And so much later as every mark is of rising than the one that rose next before it, so much later will the moon be of rising on any day than she was on the day before, in the corresponding part of the heavens. The marks about Cancer and Capricorn rise a mean difference of time between those about Aries and Libra.

Now, although the moon is in Pisces and Aries every month, and therefore must rise in those signs within the space of two hours later for a whole week, or only about 17 minutes later every day than she did on the former; yet she is never full in these signs, but in our autumnal months, August and September, when the sun is in Virgo and Libra. Therefore, no full moon in the year will continue to rise so near the time of sunset for a week or so, as these two full moons do, which fall in the time of harvest.

In the winter months, the moon is in Pisces and Aries about her first quarter; and as these signs rise about noon in winter, the moon's rising in them passes unobserved. In the spring months, the moon changes in these signs, and consequently rises at the same time as the sun; so that it is impossible to see her at that time. In the summer months she is in these signs about her third quarter, and rises not until midnight, when her rising is but very little taken notice of; especially as she is on the decrease. But in the harvest month she is at the full when in these signs, and being opposite to the sun, she rises when the sun sets (or soon after), and shines all the night.

In southern latitudes, Virgo and Libra rise at as small angles with the horizon as Pisces and Aries do in the northern; and as our spring is at the time of their harvest, it is plain their harvest full moons must be in Virgo and Libra; and will therefore rise with as little difference of time as ours do in Pisces and Aries.

For a fuller account of this matter, I must refer the reader to my *Astronomy*, in which it is described at large.

PROB. XI.—*To explain the equation of time, or difference of time between well-regulated clocks and true sun-dials.*

The earth's motion on its axis being perfectly equable, and thereby causing an apparent equable motion in the starry heavens round the same axis, produced to the poles of the heavens; it is plain that equal portions of the celestial equator pass over the meridian in equal parts of time, because the axis of the world is perpendicular to the plane of the equator. And therefore, if the sun kept his annual course in the celestial equator, he would always revolve from the meridian to the meridian again in 24 hours exactly, as shewn by a well-regulated clock.

But as the sun moves in the ecliptic, which is oblique both to the plane of the equator and the axis of the world, he cannot always revolve from the meridian to the meridian again in 24 equal hours; but sometimes a little sooner, and at other times a little later, because equal portions of the ecliptic pass over the meridian in unequal parts of time on account of its obliquity. And this difference is the same in all latitudes.

To shew this by a globe, make chalk-marks all round the equator and ecliptic, at equal distances from one another (suppose 10°), beginning at Aries or at Libra, where these two circles intersect each other. Then turn the globe round its axis, and you will see that all the marks in first quadrant of the ecliptic, or from the beginning of Aries to the beginning of Cancer, come sooner to the brazen meridian than their corresponding marks do on the equator; those on the second quadrant, or from the beginning of Cancer to the beginning of Libra, come later: those in the third quadrant, from Libra to Capricorn, sooner; and those in the fourth, from Capricorn to Aries, later. But those at the beginning of each quadrant come to the meridian at the same time with their corresponding marks on the equator.

Therefore, while the sun is in the first and third quadrants of the ecliptic, he comes sooner to the meridian every day than he would do if he kept in the equator; and consequently he is faster than a well-regulated clock, which always keeps equable or equatoreal time: and, while he is in the second or fourth quadrants, he comes later to the meridian every day than he would do if he kept in the equator; and is therefore slower than the clock. But at the beginning of each quadrant, the sun and clock are equal.

And thus, if the sun moved equally in the ecliptic, he would be equal with the clock on four days of the year, which would have equal intervals of time between them. But as he moves faster at some times than at others (being eight days longer in the northern half of the ecliptic than in the southern), this will cause a second inequality; which, combined with the former, arising from the obliquity of the ecliptic to the equator, makes up that difference, which is shewn by the common equation tables to be between good clocks and true sun-dials.

THE DESCRIPTION AND USE OF THE ARMILLARY SPHERE.

Plate IX. Fig. 9.

Whoever has seen a common *armillary sphere*, and understands how to use it, must be sensible that the machine here referred to, is of a very different, and much more advantageous, construction. And those who have seen the curious glass sphere invented by Dr. Long, or the figure of it in his Astronomy, must know that the furniture of the terrestrial globe in this machine, the form of the pedestal, and the manner of turning either the earthly globe, or the circles which surround it, are all copied from the Doctor's glass sphere; and that the only difference is, a parcel of rings instead of a glass celestial globe; and all the additions are a moon within the sphere, and a semicircle upon the pedestal.

The *Armillary sphere.*

The exterior parts of this machine are a compages of brass rings, which represent the principal circles of the heavens, viz. 1, The equinoctial *A A*, which is divided into 360° (beginning at its intersection with the ecliptic in Aries), for shewing the sun's right ascension in degrees; and also into 24 hours, for shewing his right ascension in time. 2, The ecliptic *B B* which is divided into 12 signs, and each sign into 30°, and also into the months and days of the year; in such a manner, that the degree or point of the ecliptic in which the sun is, on any given day, stands over that day in the circle of months. The tropic of Cancer *C C*, touching the ecliptic at the beginning of Cancer in *e*, and the tropic of Capricorn *D D*, touching the ecliptic at the beginning of Capricorn in *f*; each 23½° from the equinoctial circle. 4, The arctic circle *E*, and the antarctic circle *F*, each 23½° from its respective pole at *N* and *S*. 5, The equinoctial colure *G G*, passing through the north and south pole of the heavens at *N* and *S*, and through the equinoctial points Aries and Libra in the ecliptic. 6, The solstitial colure *H H*, passing through the poles of the heavens, and through the solstitial points Cancer and Capricorn in the ecliptic. Each quarter of the former of these colures is divided into 90°, from the equinoctial to the poles of the world, for shewing the declination of the sun, moon, and stars; and each quarter of the latter, from the ecliptic at *e* and *f*, to its poles *b* and *d*, for shewing the latitude of the stars.

In the north pole of the ecliptic is a nut *b*, to which is fixed one end of a quadrantal wire, and to the other end a small sun *Y*, which is carried round the ecliptic *B B*, by turning the nut: and in the south pole of the ecliptic is a pin at *d*, on which is another quadrantal wire, with a small moon *Z* upon it, which may be moved round by the hand; but there is a particular contrivance for causing the moon to move in an orbit which crosses the ecliptic at an angle of $5\frac{1}{3}$° in two opposite points, called the *moon's nodes*: and also for shifting these points backward in the ecliptic, as the *moon's nodes* shift in the heavens.

Within these circular rings is a small terrestrial globe *I*, fixed on an axis *K K*, which extends from the north and south poles of the globe at *n* and *s*, to those of the celestial sphere at *N* and *S*. On this axis is fixed the flat celestial meridian *L L*, which may be set directly over the meridian of any place on the globe, and then turned round with the globe, so as to keep over the same meridian upon it. This flat meridian is graduated the same way as the brass meridian of a common globe, and its use is much the same. To this globe is fitted the moveable horizon *M M*, so as to turn upon two strong wires proceeding from its east and west points to the globe, and entering the globe at opposite points of its equator, which is a moveable brass ring let into the globe in a groove all around its equator. The globe may be turned by hand within this ring, so as to place any given meridian upon it, directly under the celestial meridian *L L*. The horizon is divided into 360° all around its outermost edge, within which are the points of the compass, for shewing the amplitude of the sun and moon both in degrees and points. The celestial meridian *L L* passes through two notches in the north and south points of the horizon, as in a common globe: but here, if the globe be turned round, the horizon and meridian turn with it. At the south pole of the sphere is a circle of 24 hours, fixed to the rings, and on the axis is an index which goes round that circle, if the globe be turned round its axis.

The whole fabric is supported on a pedestal *N*, and may be elevated or depressed upon the joint *O*, to any number of degrees from 0 to 90, by means of the arc *P*, which is fixed in the strong brass arm *Q*, and slides in the upright piece *R*, in which is a screw at *r*, to fix it at any proper elevation.

In the box *T* are two wheels (as in Dr. Long's sphere) and two pinions, whose axes come out at *V* and *U*; either of which may be turned by the small winch *W*. When the winch is put upon the axis *V*, and turned backward, the terrestrial globe, with its horizon and celestial meridian, keep at rest; and the whole sphere of circles turn round from east, by south, to west, carrying the sun *Y* and moon *Z*, round the same way, and causing them to rise above, and set below, the horizon. But when the winch is put upon the axis *U*, and turned forward, the sphere with the sun and moon keep at rest; and the earth, with its horizon and meridian, turn round from west, by south, to east: and bring the same points of the horizon to the sun and moon, to which these bodies came when the earth kept at rest, and they were carried round it; shewing that they rise and set in the same points of the horizon, and at the same time in the hour-circle, whether the motion be in the earth or in the heavens. If the earthly globe be turned, the hour-index goes round its hour-circle; but if the sphere be turned, the hour-circle goes round below the index.

And so, by this construction, the machine is equally fitted to shew either the real motion of the earth, or the apparent motion of the heavens.

To rectify the sphere for use, first slacken the screw *r* in the upright stem *R*, and taking hold of the arm *Q*, move it up or down until the given degree of latitude for any place be at the side of the stem *R*; and then the axis of the sphere will be properly elevated, so as to stand parallel to the axis of the world, if the machine be set north and south by a small compass: this done, count the latitude from the north pole, upon the celestial meridian *L L*, down toward the north notch of the horizon, and set the horizon to that latitude; then, turn the nut *b* until the sun *Y* comes to the given day of the year in the ecliptic, and the sun will be at its proper place for that day; find the place of the moon's ascending node, and also the place of the moon, by an Ephemeris, and set them right accordingly; lastly, turn the winch *W*, until either the sun comes to the meridian *L L*, or until the meridian comes to the sun (according as you want the sphere or earth to move) and set the hour-index to the XII, marked noon, and the whole machine will be rectified—Then turn the winch, and observe when the sun or moon rise or set in the horizon, and the hour-index will shew the times thereof for the given day.

As those who understand the use of the globes will be at no loss to work many other problems by this sphere, it is needless to enlarge any farther upon it.

LECT. X.

THE PRINCIPLES AND ART OF DIALLING.

A DIAL is a plane, upon which lines are described in such a manner that the shadow of a wire, or of the upper edge of a plate stile, erected perpendicularly on the plane of the dial, may shew the true time of the day.

The edge of the plate by which the time of the day is found, is called the stile of the dial, which must be parallel to the earth's axis; and the line on which the said plate is erected, is called the substile.

The angle included between the substile and stile, is called the elevation, or height, of the stile.

Those dials whose planes are parallel to the plane of the horizon, are called horizontal dials; and those dials whose planes are perpendicular to the plane of the horizon, are called vertical, or erect, sun-dials.

Those erect dials, whose planes directly front the north or south, are called direct north or south dials; and all other erect dials are called decliners, because their planes are turned away from the north or south.

Those dials, whose planes are neither parallel nor perpendicular to the plane of their horizon, are called inclining or reclining dials, according as their planes make acute or obtuse angles with the horizon; and if their planes are also turned aside from facing the south or north, they are called declining-inclining, or declining-reclining dials.

The intersection of the plane of the dial, with that of the meridian, passing through the stile, is called the meridian of the dial, or the hour-line of XII.

Those meridians, whose planes pass through the stile, and make angles of 15, 30, 45, 60, 75, and 90 degrees with the meridian of the place (which marks the hour-line of XII) are called hour-circles; and their intersections with the plane of the dial, are called hour-lines.

In all declining dials, the substile makes an angle with the hour-line of XII; and this angle is called the distance of the substile from the meridian.

The declining plane's difference of longitude, is the angle formed at the intersection of the stile and plane of the dial, by two meridians; one of which passes through the hour-line of XII, and the other through the substile.

This much being premised concerning dials in general, we shall now proceed to explain the different methods of their construction.

Plate IX. Fig. 10.

The universal principle on which *dialling* depends.

If the whole earth, *a P c p* were transparent, and hollow, like a sphere of glass, and had its equator divided into twenty-four equal parts by so many meridian semi-circles, *a*, *b*, *c*, *d*, *e*, *f*, *g*, &c. one of which is the geographical meridian of any given place, as London, which is supposed to be at the point *a*; and if the hours of XII were marked at the equator, both upon that meridian and the opposite one, and all the rest of the hours in order on the rest of the meridians, those meridians would be the hour-circles of London; then, if the sphere had an opaque axis, as *P E p*, terminating in the poles *P* and *p*, the shadow of the axis would fall upon every particular meridian and hour, when the sun came to the plane of the opposite meridian, and would consequently shew the time at London, and at all other places on the meridian of London.

Horizontal dial.

If this sphere was cut through the middle by a solid plane *A B C D*, in the rational horizon of London, one half of the axis *E P* would be above the plane, and the other half below it; and if straight lines were drawn from the centre of the plane, to those points where the circumference is cut by the hour-circles of the sphere, those lines would be the hour-lines of a horizontal dial for London: for the shadow of the axis would fall upon each particular hour-line of the dial, when it fell upon the like hour-circle of the sphere.

Vertical dials.

If the plane which cuts the sphere be upright, as *A F C G* (Fig. 11), touching the given place (London) at *F*, and directly facing the meridian of London, it will then become the plane of an erect direct south dial; and if right lines be drawn from its centre *E*, to those points of its circumference where the hour-circles of the sphere cut it, these will be the hour-lines of a vertical or direct south dial for London, to which the hours are to be set as in the figure (contrary to those on a horizontal dial), and the lower half $E p$ of the axis will cast a shadow on the hour of the day in this dial, at the same time that it would fall upon the like hour-circle of the sphere, if the dial-plane was not in the way.

Inclining and *reclining dials.*

If the plane (still facing the meridian) be made to incline, or recline, by any given number of degrees, the hour-circles of the sphere will still cut the edge of the plane in those points to which the hour-lines must be drawn straight from the centre; and the axis of the sphere will cast a shadow on these lines at the respective hours. The like will still hold, if the plane be made to decline by any given number of degrees from the meridian, toward the east or west: provided the declination be less than 90 degrees, or the reclination be less than the co-latitude of the place: and the axis of the sphere will be a gnomon, or stile, for the dial. But it cannot be a gnomon, when the declination is quite 90 degrees, nor when the reclination is equal to the co-latitude;[1] because in these two cases, the axis has no elevation above the plane of the dial.

Declining dials.

And thus it appears, that the plane of every dial represents the plane of some great circle upon the earth; and the gnomon the earth's axis, whether it be a small wire, as in the above figures, or the edge of a thin plate, as in the common horizontal dials.

The whole earth, as to its bulk, is but a point, if compared to its distance from the sun; and therefore, if a small sphere of glass be placed upon any part of the earth's surface, so that its axis be parallel to the axis of the earth, and the sphere have such lines upon it, and such planes within it, as above described, it will shew the hours of the day as truly as if it were placed at the earth's centre, and the shell of the earth were as transparent as glass.

Fig. 10, 11.

But because it is impossible to have a hollow sphere of glass perfectly true, blown round a solid plane: or if it was, we could not get at the plane within the glass to set it in any given position; we make use of a wire sphere to explain the principles of dialling, by joining 24 semicircles together at the poles, and putting a thin flat plate of brass within it.

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[1] If the latitude be subtracted from 90 degrees, the remainder is called the co-latitude, or complement of the latitude.

Dialling by the common *terrestrial globe.*

A common globe, of 12 inches diameter, has generally 24 meridian semicircles drawn upon it. If such a globe be elevated to the latitude of any given place, and turned about until any one of these meridians cuts the horizon in the north point, where the hour of XII is supposed to be marked, the rest of the meridians will cut the horizon at the respective distances of all the other hours from XII. Then, if these points of distance be marked on the horizon, and the globe be taken out of the horizon, and a flat board or plate be put into its place, even with the surface of the horizon, and if straight lines be drawn from the centre of the board to those points of distance on the horizon which were cut by the 24 meridian semicircles, these lines will be the hour-lines of a horizontal dial for that latitude, the edge of whose gnomon must be in the very same situation that the axis of the globe was, before it was taken out of the horizon: that is, the gnomon must make an angle with the plane of the dial, equal to the latitude of the place for which the dial is made.

If the pole of the globe be elevated to the co-latitude of the given place, and any meridian be brought to the north point of the horizon, the rest of the meridians will cut the horizon in the respective distances of all the hours from XII, for a direct south dial, whose gnomon must make an angle with the plane of the dial, equal to the co-latitude of the place; and the hours must be set the contrary way on this dial to what they are on the horizontal.

But if your globe have more than twenty-four meridian semicircles upon it, you must take the following method for making *horizontal and south dials by it.*

To construct a *horizontal dial.*

Elevate the pole to the latitude of your place, and turn the globe until any particular meridian (suppose the first) comes to the north point of the horizon, and the opposite meridian will cut the horizon in the south. Then, set the hour-index to the uppermost XII on its circle; which done, turn the globe westward until fifteen degrees of the equator pass under the brazen meridian, and then the hour-index will be at I (for the sun moves fifteen degrees every hour), and the first meridian will cut the horizon in the number of degrees from the north point that I is distant from

XII. Turn on until other 15 degrees of the equator pass under the brazen meridian, and the hour-index will then be at II, and the first meridian will cut the horizon in the number of degrees that II is distant from XII: and so, by making 15 degrees of the equator pass under the brazen meridian for every hour, the first meridian of the globe will cut the horizon in the distances of all the hours from XII to VI, which is just 90 degrees; and then you need go no farther, for the distances of XI, X, IX, VIII, VII, and VI, in the forenoon, are the same from XII, as the distances of I, II, III, IV, V, and VI, in the afternoon; and these hour-lines, continued through the centre, will give the opposite hour-lines on the other half of the dial: but no more of these lines need be drawn than what answer to the sun's continuance above the horizon of your place on the longest day, which may be easily found by the twenty-sixth problem of the foregoing lecture.

Thus, to make a horizontal dial for the latitude of London, which is $51\frac{1}{2}$ degrees north, elevate the north pole of the globe $51\frac{1}{2}$ degrees above the north point of the horizon, and then turn the globe, until the first meridian (which is that of London on the English terrestrial globes) cuts the north point of the horizon, and set the hour-index to XII at noon.

Then, turning the globe westward until the index points successively to I, II, III, IV, V, and VI, in the afternoon; or until 15, 30, 45, 60, 75, and 90 degrees of the equator pass under the brazen meridian, you will find that the first meridian of the globe cuts the horizon in the following number of degrees from the north toward the east, viz. $11\frac{2}{3}$, $24\frac{1}{4}$, $38\frac{1}{12}$, $53\frac{1}{2}$, $71\frac{1}{13}$, and 90; which are the respective distances of the above hours from XII upon the plane of the horizon.

Plate X. Fig. I.

To transfer these, and the rest of the hours, to a horizontal plane, draw the parallel right lines *a c* and *b d* upon that plane, as far from each other as is equal to the intended thickness of the gnomon or stile of the dial, and the space included between them will be the meridian or twelve o'clock line on the dial. Cross this meridian at right angles with the six o'clock line *k h*, and setting one foot of your compasses in the intersection *a*, as a centre, describe the quadrant *k e* with any convenient radius or opening of the compasses: then, setting one foot in the intersection *b*, as a centre, with the same radius describe the quadrant *f h*, and divide each quadrant into 90 equal parts or degrees, as in the figure.

Because the hour-lines are less distant from each other about noon, than in any other part of the dial, it is best to have the centres of these quadrants at a little distance from the centre of the dial-plane, on the side opposite to XII, in order to enlarge the hour-distances thereabout under the same angles on the plane. Thus, the centre of the plane is at *C*, but the centres of the quadrants at *a* and *b*.

Lay a ruler over the point *b* (and keeping it there for the centre of all the afternoon hours in the quadrant *f h*), draw the hour-line of I, through $11\frac{2}{3}$ degrees in the quadrant; the hour-line of II, through $24\frac{1}{4}$ degrees; of III, through $38\frac{1}{12}$ degrees; IIII through $53\frac{1}{2}$; and V, through $71\frac{1}{15}$: and because the sun rises about four in the morning, on the longest days at London, continue the hour-lines of IIII and V, in the afternoon through the centre *b* to the opposite side of the dial. This done, lay the ruler to the centre *a*, of the quadrant *e k*, and through the like divisions or degrees of that quadrant, viz. $11\frac{2}{3}$, $24\frac{1}{4}$, $38\frac{1}{12}$, $53\frac{1}{2}$, and $71\frac{1}{15}$, draw the forenoon hour-lines of XI, X, IX, VIII, and VII; and because the sun does not set before eight in the evening on the longest days, continue the hour-lines of VII and VIII in the forenoon, through the centre *a*, to VII and VIII in the afternoon; and all the hour-lines will be finished on this dial, to which the hours may be set, as in the figure.

Lastly, through $51\frac{1}{2}$ degrees of either quadrant, and from its centre, draw the right line *a g* for the hypothenuse or axis of the gnomon *a g i*; and from *g* let fall the perpendicular *g i*, upon the meridian line *a i*, and there will be a triangle made, whose sides are *a g*, *g i*, and *i a*. If a plate, similar to this triangle, be made as thick as the distance between the lines *a c* and *b d*, and set upright between them, touching at *a* and *b*, its hypothenuse *a g* will be parallel to the axis of the world, when the dial is truly set; and will cast a shadow on the hour of the day.

N.B.—The trouble of dividing the two quadrants may be saved, if you have a scale with a line of chords upon it, such as that on the right hand of Fig. 2; for if you extend the compasses from 0 to 60 degrees of the line of chords, and with that extent, as a radius, describe the two quadrants upon their respective centres, the above distances may be taken with the compasses upon the line, and set off upon the quadrants.

To construct an *erect direct south dial.*

To make an erect direct south dial. Elevate the pole to the co-latitude of your place, and proceed in all respects as above taught for the horizontal dial, from VI in the morning to VI in the afternoon, only the hours must be reversed, as in the figure; and the hypothenuse *a g*, of the gnomon *a g f*, must make an angle with the dial-plane, (Fig 2) equal to the co-latitude of the place. As the sun can shine no longer on this dial than from six in the morning till six in the evening, there is no occasion for having any more than twelve hours upon it.[2]

To construct an *erect declining dial.*

To make an erect dial, declining from the south toward the east or west. Elevate the pole to the latitude of your place, and screw the quadrant of altitude to the zenith. Then, if your dial declines toward the east (which we shall suppose it to do at present) count on the horizon the degrees of declination, from the east point toward the north, and bring the lower end of the quadrant to that degree of declination at which the reckoning ends. This done, bring any particular meridian of your globe (as suppose the first meridian) directly under the graduated edge of the upper part of the brazen meridian, and set the hour-index to XII at noon. Then, keeping the quadrant of altitude at the degree of declination in the horizon, turn the globe eastward on its axis, and observe the degrees cut by the first meridian in the quadrant of altitude (counted from the zenith) as the hour-index comes to XI, X, IX, &c. in the forenoon, or as 15, 30, 45, &c. degrees of the equator pass under the brazen meridian at these hours respectively; and the degrees then cut in the quadrant by the first meridian, are the respective distances of the forenoon hours from XII on the plane of the dial. Then, for the afternoon hours, turn the quadrant of altitude round the zenith until it comes to the degree in the horizon opposite to that where it was placed before; namely, as far from the west point of the horizon toward the south, as it was set at first from the east point toward the north: and turn the globe westward on its axis, until the first meridian comes to the brazen meridian again, and the hour-index to XII: then, continue to turn the globe westward, and as the index points to the afternoon hours I, II, III, &c. or as 15, 30, 45, &c. degrees of the equator pass under the brazen meridian, the first meridian will cut the quadrant of altitude in the respective number of degrees from the zenith, that each of these hours is from XII on the dial. And note, that when the first meridian goes off the quadrant at the horizon, in the forenoon, the hour-index shews the time when the sun will come upon this dial: and when it goes off the quadrant in the afternoon, the index will point to the time when the sun goes off the dial.

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[2] A new and very simple geometrical method of constructing sun-dials may be seen in our author's *Mechanical Exercises*, p. 94.—*Ed.*

Having thus found all the hour-distances from XII, lay them down upon your dial-plate, either by dividing a semicircle into two quadrants of 90 degrees each (beginning at the hour-line of XII), or by the line of chords, as above directed.

In all declining dials, the line on which the stile or gnomon stands (commonly called the *substile-line*) makes an angle with the twelve o'clock line, and falls among the forenoon hour-lines, if the dial declines toward the east; and among the afternoon hour-lines, when the dial declines toward the west; that is, to the left hand from the twelve o'clock line in the former case, and to the right hand from it in the latter.

To find the distance of the substile from the twelve o'clock line: if your dial declines from the south toward the east, count the degrees of that declination in the horizon from the east point toward the north, and bring the lower end of the quadrant of altitude to that degree of declination where the reckoning ends: then turn the globe until the first meridian cuts the horizon in the like number of degrees, counted from the south point toward the east; and the quadrant and first meridian will then cross one another at right angles, and the number of degrees of the quadrant, which are intercepted between the first meridian and the zenith, is equal to the distance of the substile line from the twelve o'clock line; and the number of degrees of the first meridian, which are intercepted between the quadrant and the north pole, is equal to the elevation of the stile above the plane of the dial.

If the dial declines westward from the south, count that declination from the east point of the horizon toward thesouth, and bring the quadrant of altitude to the degree in the horizon at which the reckoning ends; both for finding the forenoon hours, and the distance of the substile from the meridian: and for the afternoon hours, bring the quadrant to the opposite degree in the horizon, namely, as far from the west toward the north, and then proceed in all respects as above.

Thus, we have finished our declining dial; and in so doing, we made four dials, viz.

1, A north dial, declining northward by the same number of degrees. 2, A north dial, declining the same number west. 3, A south dial, declining east. And 4, A south dial, declining west. Only, placing the proper number of hours, and the stile or gnomon respectively, upon each plane. For (as above mentioned) in the south-west plane, the substile-line falls among the afternoon hours; and in the south-east, of the same declination among the forenoon hours, at equal distances from XII. And so, all the morning hours on the west decliner will be like the afternoon hours on the east decliner; the south-east decliner will produce the north-west decliner; and the south-west decliner, the north-east decliner, by only extending the hour-lines, stile and substile, quite through the centre: the axis of the stile (or edge that casts the shadow on the hour of the day) being in all dials whatever parallel to the axis of the world, and consequently pointing toward the north pole of the heaven in north latitudes, and toward the south pole, in south latitudes. *See more of this in the following lecture.*

An easy method for constructing of a *dial.*

But because every one who would like to make a dial, may perhaps not be provided with a globe to assist him, and may probably not understand the method of doing it by logarithmic calculation; we shall shew how to perform it by the plain dialling-lines, or scale of latitudes and hours; such as those on the right hand of Fig. 3, in Plate X, or those shewn in Fig. 15, and which may be had on scales commonly sold by the mathematical instrument makers.

This is the easiest of all mechanical methods, and by much the best, when the lines are truly divided: not only the half hours and quarters may be laid down by all of them, but every fifth minute by most, and every single minute by those where the line of hours is a foot in length.

Having drawn your double meridian line *a b*, *c d*, (Fig. 3) on the plane intended for a horizontal dial, and crossed it at right angles by the six o'clock line *f e* (as in Fig. 1), take the latitude of your place with the compasses, in the scale of latitudes, and set that extent from *c* to *e*, and from *a* to *f*, on the six o'clock line: then, taking the whole six hours between the points of the compasses in the scale of hours, with that extent set one foot in the point *e*, and let the other foot fall where it will upon the meridian-line *c d*, as at *d*. Do the same from *f* to *b*, and draw the right lines *e d* and *f b*, each of which will be equal in length to the whole scale of hours. This done, setting one foot of the compasses in the beginning of the scale at XII, and extending the other to each hour on the scale, lay off these extents from *d* to *e* for the afternoon hours, and from *b* to *f* for those of the forenoon: this will divide the lines *d e* and *b f* in the same manner as the hour-scale is divided, at 1, 2, 3, 4, 5, and 6, on which the quarters may also be laid down, if required. Then, laying a ruler on the point *c*, draw the first five hours in the afternoon, from that point, through the dots at the numeral figures 1, 2, 3, 4, 5, on the line *d e*; and continue the lines of IIII and V through the centre *c* to the other side of the dial, for the like hours of the morning; which done, lay the ruler on the point *a*, and draw the last five hours in the forenoon through the dots 5, 4, 3, 2, 1, on the line *f b*; continuing the hour-lines of VII and VIII through the centre *a* to the other side of the dial, for the like hours of the evening; and set the hours to their respective lines as in the figure. Lastly, make the gnomon the same way as taught above for the horizontal dial, and the whole will be finished.

To make an erect south dial, take the co-latitude of your place from the scale of latitudes, and then proceed in all respects for the hour-lines, as in the horizontal dial; only reversing the hours, as in Fig. 2; and making the angle of the stile's height equal to the co-latitude.

I have drawn out a set of dialling-lines at Fig. 15 of Plate X, large enough for making a dial of nine inches diameter, or more inches, if required; and have drawn them tolerably exact for common practice, to every quarter of an hour. This scale may be cut off from the plate, and pasted upon wood, or upon the inside of one of the boards of this book; and then it will be somewhat more exact than it is on the plate, for being rightly divided upon the copper-plate, and printed off on wet paper, it shrinks as the paper dries; but when it is wetted again, it stretches to the same size as when newly printed; and if pasted on while wet, it will remain of that size afterwards.

But lest the young diallist should have neither globe nor wooden scale, and should tear or otherwise spoil the paper one in pasting, we shall now shew him how he may make a dial without any of these helps. Only, if he has not a line of chords, he must divide a quadrant into 90 equal parts or degrees for taking the proper angle of the stile's elevation, which is easily done.

Horizontal dial. With any opening of the compasses, as *Z L* (Fig. 4), describe the two semicircles *L F k* and *L Q k*, upon the centres *Z* and *z*, where the six o'clock line crosses the double meridian line, and divide each semicircle into 12 equal parts, beginning at *L*; though, strictly speaking, only the quadrants from *L* to the six o'clock line need be divided; then connect the divisions which are equidistant from *L*, by the parallel lines *K M*, *I N*, *H O*, *G P*, and *F Q*. Draw *V Z* for the hypothenuse of the stile, making the angle *V Z E* equal to the latitude of your place: and continue the line *V Z* to *R*. Draw the line *R r* parallel to the six o'clock line, and set off the distance *a K* from *Z* to *Y*, the distance *b I* from *Z* to *X*, *c H* from *Z* to *W*, *d G* from *Z* to *T*, and *e F* from *Z* to *S*. Then draw the lines *S s*, *T t*, *W w*, *X x*, and *Y y*, each parallel to *R r*. Set off the distance *y Y* from *a* to 11, and from *f* to 1: the distance *x X* from *b* to 10, and from *g* to 2; *w W* from *c* to 9, and from *h* to 3; *t T* from *d* to 8, and from *i* to 4; *s S* from *e* to 7, and from *n* to 5. Then, laying a ruler to the centre *Z*, draw the forenoon hour-lines through the points 11, 10, 9, 8, 7; and laying it to the centre *z*, draw the afternoon lines through the points 1, 2, 3, 4, 5; continuing the forenoon lines of VII and VIII through the centre *Z*, to the opposite side of the dial, for the like afternoon hours; and the afternoon lines IIII and V through the centre *z*, to the opposite side, for the like morning hours. Set the hours to these lines as in the figure, and then erect the stile or gnomon, and the horizontal dial will be finished.

South dial. To construct a south dial, draw the lines *V Z*, making an angle with the meridian *Z L* equal to the co-latitude of your place, and proceed in all respects as in the above horizontal dial for the same latitude, reversing the hours as in Fig. 2, and making the elevation of the gnomon equal to the co-latitude.

Perhaps it may not be unacceptable to explain the method of constructing the dialling-lines, and some others, which is as follows.

Dialling-lines, how constructed. Plate X. Fig. 5.

With any opening of the compasses, as *E A*, according to the intended length of the scale, describe the circle *A D C B*, and cross it at right angles by the diameters *C E A* and *D E B*. Divide the quadrant *A B* first into nine equal parts, and then each part into 10; so shall the quadrant be divided into 90 equal parts or degrees. Draw the right line *A F B* for the chord of this quadrant, and setting one foot of the compasses in the point *A*, extend the other to the several divisions of the quadrant, and transfer these divisions to the line *A F B* by the arcs 10, 10, 20, 20, &c. and this will be a line of chords divided into 90 unequal parts; which, if transferred from the line back again to the quadrant, will divide it equally. It is plain by the figure, that the distance from *A* to 60 in the line of chords, is just equal to *A E*, the radius of the circle from which that line is made; for if the arc 60, 60 be continued, of which *A* is the centre, it goes exactly through the centre *E* of the arc *A B*.

And therefore, in laying down any number of degrees on a circle, by the line of chords, you must first open the compasses, so as to take in just 60 degrees upon that line, as from *A* to 60; and then, with that extent, as a radius, describe a circle which will be exactly of the same size with that from which the line was divided: which done, set one foot of the compasses in the beginning of the chord line, as at *A*, and extend the other to the number of degrees you want upon the line, which extent, applied to the circle, will include the like number of degrees upon it.

Divide the quadrant *C D* into 90 equal parts, and from each point of division draw right lines, as *i*, *k*, *l*, &c. to the line *C E*; all perpendicular to that line, and parallel to *D E*, which will divide *E C* into a line of sines; and although these are seldom put among the dialling-lines on a scale, yet they assist in drawing the line of latitudes. For, if a ruler be laid upon the point *D*, and over each division in the line of sines, it will divide the quadrant *C B* into 90 unequal parts, as *B a*, *a b*, &c. shewn by the right lines 10 *a*, 20 *b*, 30 *c*, &c. drawn along the edge of the ruler. If the right line *B C* be drawn, subtending this quadrant, and the nearest distances *B a*, *B b*, *C c*, &c. be taken in the compasses from *B*, and set upon this line in the same manner as directed for the line of chords, it will make a line of latitudes *B C*, equal in length to the line of chords *A B*, and of an equal number of divisions, but very unequal as to their lengths.

Draw the right line *D G A*, subtending the quadrant *D A*, and parallel to it; draw the right line *r s*, touching the quadrant *D A* at the numeral figure 3. Divide this quadrant into six equal parts, as 1, 2, 3, &c. and through these points of division draw right lines from the centre *E* to the line *r s*, which will divide it at the points where the six hours are to be placed, as in the figure. If every sixth part of the quadrant be subdivided into four equal parts, right lines drawn from the centre through these points of division, and continued to the line *r s*, will divide each hour upon it into quarters.

A dial on a card.

In Fig. 6, we have the representation of a portable dial, which may be easily drawn on a card, and carried in a pocket-book. The lines *a d*, *a b*, and *b c* of the gnomon, must be cut quite through the card; and as the end *a b* of the gnomon is raised occasionally above the plane of the dial, it turns upon the uncut line *c d* as on a hinge. The dotted line *A B* must be slit quite through the card, and the thread must be put through the slit, and have a knot tied behind, to keep it from being easily drawn out. On the other end of this thread is a small plummet *D*, and on the middle of it a small bead for shewing the time of the day.

To rectify this dial, set the thread in the slit right against the day of the month, and stretch the thread from the day of the month over the angular point where the curve lines meet at XII; then shift the bead to that point on the thread, and the dial will be rectified.

To find the hour of the day, raise the gnomon (no matter how much or how little) and hold the edge of the dial next the gnomon toward the sun, so that the uppermost edge of the shadow of the gnomon may just cover the *shadow line;* and the bead then playing freely on the face of the dial, by the weight of the plummet, will shew the time of the day among the hour-lines, as it is forenoon or afternoon.

To find the time of sun rising and setting, move the thread among the hour-lines, until it either covers some one of them, or lies parallel betwixt any two; and then it will cut the time of sun-rising among the forenoon hours, and of sun-setting among the afternoon hours, on that day of the year for which the thread is set in the scale of months.

To find the sun's declination, stretch the thread from the day of the month over the angular point at XII, and it will cut the sun's declination, as it is north or south, for that day, in the arched scale of north and south declination.

To find on what day the sun enters the signs: when the bead, as above rectified, moves along any of the curve lines which have the signs of the zodiac marked upon them, the sun enters those signs on the days pointed out by the thread in the scale of months.

The construction of this dial is very easy, especially if the reader compares it all along with Fig. 7, as he reads the following explanation of that figure.

Draw the occult line *A B* (Fig. 7) parallel to the top of the card, and cross it at right angles with the six o'clock line *E C D*; then upon *C*, as a centre, with the radius *C A*, describe the semicircle *A E L*, and divide it into 12 equal parts (beginning at *A*), as *A r*, *r s*, &c. and from these points of division, draw the hour-lines *r*, *s*, *t*, *u*, *v*, *E*, *w*, and *x*, all parallel to the six o'clock line *E C*. If each part of the semicircle be divided into four equal parts, they will give the half-hour lines and quarters, as in Fig. 2. Draw the right line *A S D o*, making the angle *S A B* equal to the latitude of your place. Upon the centre *A* describe the arch *R S T*, and set off upon it the arcs *S R* and *S T*, each equal to 23½ degrees, for the sun's greatest declination; and divide them into 23½ equal parts, as in Fig. 6. Through the intersection *D* of the lines *E C D* and *A D o* draw the right line *F D G* at right angles to *A D o*. Lay a ruler to the points *A* and *R*, and draw the line *A R F* through 23½ degrees of south declination in the arc *S R*; and then laying the ruler to the points *A* and *T*, draw the line *A T G* through 23½ degrees of north declination in the arc *S T*: so shall the lines *A R F* and *A T G* cut the line *F D G* in the proper length for the scale of months. Upon the centre *D*, with the radius *D F*, describe the semicircle *F o G*; and divide it into six equal parts, *F m*, *m n*, *n o*, &c. and from these points of division draw the right lines *m h*, *n i*, *p k*, and *q l*, each parallel to *o D*. Then setting one foot of the compasses in the point *F*, extend the other to *A* and describe the arc *A z H* for the tropic of ♑: with the same extent, setting one foot in *G*, describe the arc *A E O* for the tropic of ♋. Next setting one foot in the point *h*, and extending the other to *A*, describe the arc *A C I* for the beginnings of the signs ♒ and ♐; and with the same extent, setting one foot in the point *l*, describe the arc *A N* for the beginnings of the signs ♊ and ♌. Set one foot in the point *i*, and having extended the other to *A*, describe the arc *A K* for the beginnings of the signs ♓ and ♏; and with the same extent, set one foot in *k*, and describe the arc *A M* for the beginnings of the signs ♉ and ♍. Then, setting one foot in the point *D*, and extending the other to *A*, describe the curve *A L* for the beginnings of ♈ and ♎; and the signs will be finished. This done, lay a ruler from the point *A* over the sun's declination in the arch *R S T* (found by the following Table) for every fifth day of the year, and where the ruler cuts the line *F D G*, make marks; and place the days of the month right against these marks, in the manner shewn by Fig. 6. Lastly, draw the shadow line *P Q* paralle to the occult line *A B*; make the gnomon, and set the hours to their respective lines, as in Fig. 6, and the dial will be finished.

An *universal* dial.

There are several kinds of dials (Fig. 8), which are called *universal*, because they serve for all latitudes. Of these, the best one that I know is Mr. Pardie's, which consists of three principal parts: the first whereof is called the *horizontal plane (A)* because in the practice it must be parallel to the horizon. In this plane is fixed an upright pin, which enters into the edge of the second part *B D*, called the *meridional plane*; which is made of two pieces, the lowest whereof *(B)* is called the *quadrant*, because it contains a quarter of a circle, divided into 90 degrees; and it is only into this part, near *B*, that the pin enters. The other piece is a *semicircle (D)* adjusted to the quadrant, and turning in it by a groove, for raising or depressing the diameter *(E F)* of the semicircle, which diameter is called the *axis* of the instrument. The third piece is a *circle (G)* divided on both sides into 24 equal parts, which are the hours. This circle is put upon the meridional plane, so that the axis *(E F)* may be perpendicular to the circle; and the point *C* be the common centre of the circle, semicircle, and quadrant. The straight edge of the semicircle is chamfered on both sides to a sharp edge, which passes through the centre of the circle. On one side of the chamfered part, the first six months of the year are laid down, according to the sun's declination for their respective days, and on the other side the last six months. And against the days on which the sun enters the signs there are straight lines drawn upon the semicircle, with the characters of the signs marked upon them. There is a black line drawn along the middle of the upright edge of the quadrant, over which hangs a thread *(H)* with its plummet *(I)* for levelling the instrument. *N. B.* From the 22d of September to the 20th of March, the upper surface of the circle must touch both the centre *C* of the semicircle, and the line of ♈ and ♎; and from the 20th of March to the 22d of September, the lower surface of the circle must touch that centre and line.

To find the time of the day by this dial. Having set it on a level place in sun-shine, and adjusted it by the levelling screws *k* and *l*, until the plumb line hangs over the back line upon the edge of the quadrant, and parallel to the said edge; move the semicircle in the quadrant until the line of ♈ and ♎ (where the circle touches) comes to the latitude of your place in the quadrant: then, turn the whole meridional plane *B D*, with its circle *G*, upon the horizontal plane *A* until the edge of the shadow of the circle falls precisely on the day of the month in the semicircle; and then, the meridional plane will be due north and south, the axis *E F* will be parallel to the axis of the world, and will cast a shadow upon the true time of the day, among the hours on the circle.

N. B. As, when the instrument is thus rectified, the quadrant and semicircle are in the plane of the meridian, so the circle is then in the plane of the equinoctial: therefore, as the sun is above the equinoctial in summer (in northern latitudes) and below it in winter; the axis of the semicircle will cast a shadow on the hour of the day, on the upper surface of the circle, from the 20th of March to the 22d of September; and from the 22d of September to the 20th of March, the hour of the day will be determined by the shadow of the semicircle, upon the lower surface of the circle. In the former case, the shadow of the circle falls upon the day of the month, on the lower part of the diameter of the semicircle; and in the latter case on the upper part.

The method of laying down the months and signs upon the semicircle, is as follows. Draw the right line *A C B*, equal to the diameter of the semicircle *E D B* (Fig. 9), and cross it in the middle at right angles with the line *E C D*, equal in length to *A D B*; then *E C* will be the radius of the circle *F C G*, which is the same as that of the semicircle. Upon *E* as a centre, describe the circle *F C G*, on which set off the arcs *C h* and *C i*, each equal to 23½ degrees, and divide them accordingly into that number for the sun's declination. Then, laying the edge of a ruler over the centre *E*, and also over the sun's declination for every fifth day[3] of each month (as in the card-dial), mark the points on the diameter *A B* of the semicircle from *a* to *g*, which are cut by the ruler; and there place the days of the months accordingly, answering the sun's declination. This done, setting one foot of the compasses in *C*, and extending the other to *a* or *g*, describe the semicircle *a b c d e f g*; which divide into six equal parts, and through the points of division draw right lines parallel to *C D*, for the beginning of the signs (of which one half are on one side of the semicircle, and the other half on the other side), and set the characters of the signs to their proper lines, as in the figure.

The following Table shews the sun's place and declination, in degrees and minutes, at the noon of every day of the second year after leap-year; which is a mean between those of leap year itself, and the first and third years after it. It is useful for inscribing the months and their days on sun-dials; and also for finding the latitudes of places, according to the methods prescribed after the table.[4]

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[3] The intermediate days may be drawn in by hand, if the spaces be large enough to contain them.

[4] In this edition, the table of the sun's longitude and declination has been calculated anew, and adapted to the present improved state of the solar tables. The Editor has also added an accurate table of the equation of time, which will be of great use to the practical diallist. The signs + and —, *add* and *subtract*, at the head of the column, denote that the equation of time must be added to, or subtracted from, the apparent time, or that which is deduced from the motion of the sun, in order to obtain the equated or true time, as shewn by a well-regulated clock or watch. The table is calculated for the second after leap year, and is as accurate as the difference between the civil and solar year will permit.—*Ed.*

A Table shewing the Sun's Place and Declination.

JANUARY.					FEBRUARY.					MARCH.				
Days.	Sun's Place.		Sun's Declin.		Days.	Sun's Place.		Sun's Declin.		Days.	Sun's Place.		Sun's Declin.	
	D.	M.	D.	M.		D.	M.	D.	M.		D.	M.	D	M.
1	10♑	27	23 S.	3	1	12♒	0	17 S.	13	1	10♓	15	7 S.	44
2	11	28	22	58	2	13	1	16	56	2	11	15	7	21
3	12	29	22	53	3	14	2	16	38	3	12	15	6	58
4	13	31	22	47	4	15	3	16	20	4	13	15	6	35
5	14	32	22	40	5	16	4	16	2	5	14	15	6	12
6	15	33	22	34	6	17	4	15	44	6	15	15	5	49
7	16	34	22	26	7	18	5	15	26	7	16	15	5	26
8	17	35	22	19	8	19	6	15	7	8	17	15	5	2
9	18	37	22	10	9	20	7	14	48	9	18	15	4	39
10	19	38	22	2	10	21	7	14	28	10	19	15	4	16
11	20	39	21	53	11	22	8	14	9	11	20	15	3	52
12	21	40	21	43	12	23	9	13	49	12	21	15	3	29
13	22	41	21	33	13	24	9	13	29	13	22	14	3	5
14	23	42	21	23	14	25	10	13	9	14	23	14	2	41
15	24	43	21	12	15	26	10	12	49	15	24	14	2	18
16	25	44	21	1	16	27	11	12	28	16	25	13	1	54
17	26	46	20	50	17	28	11	12	7	17	26	13	1	30
18	27	47	20	38	18	29	12	11	46	18	27	12	1	7
19	28	48	20	25	19	0♓	12	11	25	19	28	12	0	43
20	29	49	20	13	20	1	12	11	3	20	29	11	0	19
21	0♒	50	20	0	21	2	13	10	42	21	0♈	11	0 N.	4
22	1	51	19	46	22	3	13	10	20	22	1	10	0	28
23	2	52	19	32	23	4	13	9	58	23	2	10	0	52
24	3	53	19	18	24	5	14	9	36	24	3	9	1	15
25	4	54	19	4	25	6	14	9	14	25	4	9	1	39
26	5	55	18	49	26	7	14	8	52	26	5	8	2	2
27	6	56	18	34	27	8	14	8	29	27	6	7	2	26
28	7	57	18	18	28	9	15	8	7	28	7	6	2	50
29	8	58	18	2	In these Tables N. signifies north, and S. south, declination.					29	8	6	3	13
30	9	58	17	46						30	9	5	3	36
31	10	59	17	30						31	10	4	3	59

A Table shewing the Sun's Place and Declination.

APRIL.					MAY.					JUNE.				
Days.	Sun's Place.		Sun's Declin.		Days.	Sun's Place.		Sun's Declin.		Days.	Sun's Place.		Sun's Declin.	
	D.	M.	D.	M.		D.	M.	D.	M.		D.	M.	D.	M.
1	11 ♈	3	4N.	23	1	10 ♉	21	14N.	57	1	10 ♊	12	22N.	0
2	12	2	4	46	2	11	19	15	15	2	11	10	22	8
3	13	1	5	9	3	12	18	15	33	3	12	7	22	16
4	14	0	5	32	4	13	16	15	50	4	13	4	22	24
5	14	59	5	55	5	14	14	16	8	5	14	2	22	31
6	15	58	6	17	6	15	12	16	25	6	14	59	22	37
7	16	57	6	40	7	16	10	16	42	7	15	57	22	43
8	17	56	7	3	8	17	8	16	58	8	16	54	22	49
9	18	55	7	25	9	18	6	17	14	9	17	51	22	55
10	19	54	7	47	10	19	4	17	30	10	18	49	23	0
11	20	53	8	9	11	20	2	17	46	11	19	46	23	4
12	21	51	8	31	12	20	59	18	1	12	20	43	23	9
13	22	50	8	53	13	21	57	18	17	13	21	40	23	12
14	23	49	9	15	14	22	55	18	31	14	22	38	23	16
15	24	47	9	37	15	23	53	18	46	15	23	35	23	19
16	25	46	9	58	16	24	51	19	0	16	24	32	23	21
17	26	44	10	19	17	25	48	19	14	17	25	29	23	23
18	27	43	20	40	18	26	46	19	27	18	26	27	23	25
19	28	41	11	1	19	27	44	19	41	19	27	24	23	26
20	29	40	11	22	20	28	41	19	53	20	28	21	23	27
21	0 ♉	38	11	43	21	29	39	20	6	21	29	18	23	28
22	1	37	12	3	22	0 ♊	37	20	18	22	0 ♋	16	23	28
23	2	35	12	23	23	1	34	20	30	23	1	13	23	28
24	3	33	12	43	24	2	32	20	41	24	2	10	23	27
25	4	32	13	3	25	3	29	20	53	25	3	7	23	26
26	5	30	13	22	26	4	27	21	3	26	4	5	23	24
27	6	28	13	42	27	5	24	21	14	27	5	2	23	22
28	7	27	14	1	28	6	22	21	24	28	5	59	23	20
29	8	25	14	20	29	7	20	21	33	29	6	56	23	17
30	9	23	14	38	30	8	17	21	43	30	7	53	23	14
					31	9	15	21	52					

A Table shewing the Sun's Place and Declination.

JULY.					AUGUST.					SEPTEMBER.				
Days.	Sun's Place.		Sun's Declin.		Days.	Sun's Place.		Sun's Declin.		Days.	Sun's Place.		Sun's Declin.	
	D.	M.	D.	M.		D.	M.	D.	M.		D.	M.	D.	M.
1	8♋	51	23N.	10	1	8♌	26	18N.	10	1	8♍	17	8N.	29
2	9	48	23	6	2	9	24	17	55	2	9	15	8	7
3	10	45	23	2	3	10	21	17	40	3	10	13	7	45
4	11	42	22	57	4	11	19	17	24	4	11	11	7	23
5	12	40	22	52	5	12	16	17	8	5	12	9	7	1
6	13	37	22	46	6	13	14	16	52	6	13	8	6	38
7	14	34	22	40	7	14	11	16	35	7	14	6	6	16
8	15	31	22	34	8	15	9	16	19	8	15	4	5	53
9	16	28	22	27	9	16	6	16	2	9	16	2	5	31
10	17	26	22	20	10	17	4	15	44	10	17	1	5	8
11	18	23	22	12	11	18	2	15	27	11	17	59	4	45
12	19	20	22	4	12	18	59	15	9	12	18	58	4	22
13	20	17	21	56	13	19	57	14	51	13	19	56	3	59
14	21	14	21	47	14	20	54	14	33	14	20	55	3	36
15	22	12	21	38	15	21	52	14	14	15	21	53	3	13
16	23	9	21	29	16	22	50	13	55	16	22	52	2	50
17	24	6	21	19	17	23	47	13	36	17	23	50	2	27
18	25	3	21	9	18	24	45	13	17	18	24	49	2	4
19	26	1	20	58	19	25	43	12	58	19	25	47	1	40
20	26	58	20	47	20	26	41	12	58	20	26	46	1	17
21	27	55	20	36	21	27	39	12	18	21	27	45	0	54
22	28	52	20	24	22	28	36	11	58	22	28	44	0	30
23	29	50	20	13	23	29	34	11	38	23	29	42	0	7
24	0♌	47	20	0	24	0♍	32	11	18	24	0♎	41	0S.	16
25	1	44	19	48	25	1	30	10	57	25	1	40	0	40
26	2	42	19	35	26	2	28	10	36	26	2	39	1	3
27	3	39	19	22	27	3	26	10	15	27	3	38	1	27
28	4	37	19	8	28	4	24	9	54	28	4	37	1	50
29	5	34	18	54	29	5	22	9	33	29	5	36	2	14
30	6	31	18	40	30	6	20	9	12	30	6	35	2	37
31	7	29	18	25	31	7	18	8	50					

A Table shewing the Sun's Place and Declination.

OCTOBER.					NOVEMBER.					DECEMBER.				
Days.	Sun's Place.		Sun's Declin.		Days.	Sun's Place.		Sun's Declin.		Days.	Sun's Place.		Sun's Declin.	
	D.	M.	D.	M.		D.	M.	D.	M.		D.	M.	D.	M.
1	7♎	34	3S.	0	1	8♏	22	14S.	19	1	8♐	39	21 S.	46
2	8	33	3	24	2	9	22	14	38	2	9	39	21	55
3	9	33	3	47	3	10	23	14	57	3	10	40	22	4
4	10	32	4	10	4	11	23	15	16	4	11	41	22	13
5	11	31	4	34	5	12	23	15	34	5	12	42	22	21
6	12	30	4	57	6	13	23	15	53	6	13	43	22	28
7	13	29	5	20	7	14	24	16	11	7	14	44	22	35
8	14	29	5	43	8	15	24	16	28	8	15	45	32	42
9	15	28	6	6	9	16	24	16	46	9	16	46	22	48
10	16	27	6	29	10	17	24	17	3	10	17	47	22	54
11	17	27	6	51	11	18	25	17	20	11	18	48	23	0
12	18	26	7	14	12	19	25	17	36	12	19	49	23	5
13	19	26	7	37	13	20	26	17	53	13	20	50	23	9
14	20	25	7	59	14	21	26	18	9	14	21	51	23	13
15	21	25	8	22	15	22	27	18	24	15	22	52	23	16
16	22	24	8	44	16	23	27	18	39	16	23	53	23	19
17	23	24	9	6	17	24	28	18	54	17	24	55	23	22
18	24	23	9	28	18	25	28	19	9	18	25	56	23	24
19	25	23	9	50	19	26	29	19	23	19	26	57	23	26
20	26	23	10	11	20	27	30	19	37	20	27	58	23	27
21	27	23	10	33	21	28	30	19	51	21	28	59	23	28
22	28	22	10	54	22	29	31	20	4	22	0♑	0	23	28
23	29	22	11	16	23	0♐	32	20	17	23	1	2	23	28
24	0♏	22	11	37	24	1	33	20	30	24	2	3	23	27
25	1	22	11	58	25	2	33	20	42	25	3	4	23	26
26	2	22	12	18	26	3	34	20	53	26	4	5	23	24
27	3	22	12	39	27	4	35	21	5	27	5	6	23	22
28	4	22	12	59	28	5	36	21	16	28	6	8	23	19
29	5	22	13	14	29	6	37	21	26	29	7	9	23	16
30	6	22	13	39	30	7	38	21	36	30	8	10	23	13
31	7	22	13	59						31	9	11	23	9

Table of the Equation of Time.

Days.	JANUARY.		FEB.		MARCH.		APRIL		MAY.		JUNE.	
	M.	S.	M.	S.	M.	S.	M.	S.	M.	S.	M.	S.
1	3+	48	13+	58	12+	45	4+	7	3—	2	2—	42
2	4	16	14	5	12	33	3	49	3	10	2	34
3	4	44	14	12	12	21	3	31	3	17	2	24
4	5	12	14	19	12	8	3	13	3	23	2	14
5	5	39	14	24	11	54	2	55	3	29	2	4
6	6	6	14	28	11	40	2	37	3	35	1	54
7	6	33	14	32	11	26	2	20	3	40	1	43
8	6	59	14	34	11	14	2	2	3	44	1	32
9	7	24	14	37	10	56	1	45	3	48	1	21
10	7	49	14	38	10	41	1	28	3	51	1	10
11	8	13	14	39	10	25	1	12	3	54	0	58
12	8	37	14	38	10	9	0	55	3	56	0	46
13	9	0	14	37	9	52	0	39	3	57	0	34
14	9	22	14	35	9	35	0	23	3	58	0	22
15	9	44	14	33	9	18	0	8	3	59	0	9
16	10	4	14	29	9	1	0—	7	3	59	0+	3
17	10	25	14	25	8	43	0	22	3	58	0	16
18	10	44	14	20	8	25	0	36	3	57	0	28
19	11	3	14	15	8	7	0	50	3	55	0	41
20	11	21	14	8	7	49	1	4	3	53	0	54
21	11	38	14	2	7	31	1	17	3	50	1	7
22	11	55	13	54	7	12	1	30	3	46	1	20
23	12	11	13	46	6	54	1	42	3	42	1	33
24	12	26	13	37	6	35	1	54	3	38	1	46
25	12	40	13	28	6	17	2	5	3	33	1	59
26	12	53	13	18	5	53	2	16	3	27	2	11
27	13	6	13	8	5	39	2	26	3	21	2	24
28	13	18	12	57	5	21	2	36	3	14	2	37
29	13	29			5	2	2	45	3	7	2	49
30	13	39			4	44	2	53	3	59	3	1
31	13	49			4	24			2	51		

Table of the Equation of Time.

Days.	JULY.		AUGUST.		SEPT.		OCTOBER.		NOVEM.		DECEM.	
	M.	S.	M.	S.	M.	S.	M.	S.	M.	S.	M.	S.
1	3+	13	5+	57	0—	2	10—	10	16—	13	10—	49
2	3	25	5	54	0	20	10	29	16	14	10	27
3	3	36	5	50	0	39	10	47	16	14	10	3
4	3	48	5	46	0	58	11	6	16	14	9	39
5	3	58	5	40	1	18	11	24	16	13	9	15
6	4	9	5	35	1	37	11	42	16	11	8	50
7	4	19	5	28	1	57	11	59	16	8	8	24
8	4	29	5	21	2	18	12	16	16	4	7	58
9	4	38	5	14	2	38	12	32	16	0	7	31
10	4	47	5	5	2	58	12	48	15	54	7	4
11	4	56	4	57	3	19	13	4	15	48	6	37
12	5	4	4	47	3	40	13	19	15	41	6	9
13	5	11	4	37	4	1	13	34	15	33	5	41
14	5	18	4	27	4	22	13	48	15	25	5	13
15	5	25	4	16	4	43	14	2	15	15	4	44
16	5	31	4	4	5	4	14	15	15	5	4	15
17	5	37	3	52	5	25	14	27	14	53	3	45
18	5	42	3	39	5	46	14	39	14	41	3	16
19	5	47	3	26	6	7	14	50	14	28	2	46
20	5	51	3	13	6	28	15	1	14	14	2	16
21	5	54	2	59	6	49	15	11	13	59	1	46
22	5	57	2	45	7	10	15	20	13	44	1	16
23	6	0	2	30	7	30	15	28	13	27	0	45
24	6	2	2	14	7	51	15	36	13	10	0	15
25	6	3	1	59	8	11	15	43	12	52	0+	15
26	6	4	1	43	8	32	15	50	12	33	0	45
27	6	5	1	26	8	52	15	55	12	14	1	15
28	6	4	1	9	9	12	16	0	11	54	1	45
29	6	3	0	52	9	31	16	5	11	33	2	14
30	6	2	0	34	9	51	16	8	11	12	2	43
31	6	0	0	17			16	11			3	13

Explanation of the Table of the Equation of Time.

As our author has already given a familiar explanation of the equation of time, it may be sufficient to observe, that the preceding table contains the difference between true and apparent time, for every day of the year at 12 o'clock noon, when the sun is in the meridian; and is adapted to the second year after leap year. If apparent, or solar, time is to be converted into true time, as shewn by a well regulated clock or watch, the equation of time must be added to the apparent time, if it has the sign +, and subtracted from it if it has the sign —: but if true is to be converted into apparent time, the equation must be applied with contrary signs. If the equation is required for any intermediate hour, take the difference during a day, and say, as 24 hours is to this difference, so is the number of hours which the intermediate hour is from the preceding noon, to a third proportional, which, added to, or subtracted from, the equation of time at noon, according as it is increasing or decreasing, will give the equation of time for the given hour. If the equation of time is wanted, at a time when the signs change from + to —, or from — to +, the difference for 24 hours will be found by *adding* the equations of time for the noon preceding and following the given hour. Thus, if the equation of time is required for the 24th December at 12 o'clock midnight, the equation for the 24th at noon is — 15″, and for the 25th at noon + 15″, the difference of which is 30″. Then, as $24^h : 30'' = 12^h : 15$, which, subtracted from 15 seconds, because the numbers are decreasing, the equation for the 24th noon, leaves 0, so that the hour, as shewn by the sun and clock, is the same on the 24th December at midnight. The equation thus found will be accurate for every second year after leap year, and in other years will vary only a few seconds from the truth, In order, however, to determine the equation of time, with accuracy for any other year, find the difference between the equation of time for the given day, and that which precedes it: then,

1. *For leap year*, take one half of this difference, and add it to the equation for the given time if it increases, but subtract it if it decreases.

2. *For the first after leap year*, take one fourth of the difference, and add it to the equation for the given time if it increases, but subtract it if it decreases.

3. *For the third after leap year*, take one fourth of the difference, and subtract it from the equation for the given time, if it increases, but add it if it decreases. Thus, to find the equa-

tion of time for the 2d May 1805, being the first after leap year, the equation in the table is 3′ 10″, the daily difference is 8″, and the equation increases. Add, therefore, 2″, which is one fourth of the daily difference, to 3′ 10″, and the sum 3′ 12″, will be the true equation of time for the 2d May 1805.—*Ed.*

TO FIND THE LATITUDE OF ANY PLACE BY OBSERVATION.

The latitude of any place is equal to the elevation of the pole above the horizon of that place. Therefore it is plain, that if a star was fixed in the pole, there would be nothing required to find the latitude, but to take the altitude of that star with a good instrument. But although there is no star in the pole, yet the latitude may be found by taking the greatest and least altitude of any star that never sets: for if half the difference between these altitudes be added to the least altitude, or subtracted from the greatest, the sum or remainder will be equal to the altitude of the pole at the place of observation.[5]

But because the length of the night must be more than 12 hours, in order to have two such observations; the sun's meridian altitude and declination are generally made use of for finding the latitude, by means of its complement, which is equal to the elevation of the equinoctial above the horizon; and if this complement be subtracted from 90 degrees, the remainder will be the latitude, concerning which, I think, the following rules take in all the various cases.

1. If the sun has north declination, and is on the meridian, and to the south of your place, subtract the declination from the meridian altitude (taken by a good quadrant) and the remainder will be the height of the equinoctial or complement of the latitude north.

EXAMPLE.

Suppose	The sun's meridian altitude	42°	20′	south.
	And his declination, subtract	10	15	north.
Remains the complement of the latitude,		32	5	
Which subtract from - -		90	0	
And the remainder is the latitude		57	55	north.[6]

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[5] If the altitude of the pole star be taken six hours before, or after, it comes to the meridian, or arrives at its point of greatest and least altitude, the latitude of the place will thus be accurately obtained by only one observation.—*Ed.*

[6] The sun's meridian altitude, as taken by a quadrant, or any other instrument,

2. If the sun has south declination, and is southward of your place at noon, add the declination to the meridian altitude; the sum, if less than 90 degrees, is the complement of the latitude north: but if the sum exceeds 90 degrees, the latitude is south; and if 90 be taken from that sum, the remainder will be the latitude.

EXAMPLES.

The sun's meridian altitude	65°	10′	south
The sun's declination, add	15	30	south
Complement of the latitude	80	40	
Subtract from - -	90	0	
Remains the latitude -	9	20	north.

The sun's meridian altitude	80°	40′	south
The sun's declination, add	20	10	south
The sum is - -	100	50	
From which subtract -	90	0	
Remains the latitude -	10	50	south.

3. If the sun has north declination, and is on the meridian north of your place, and the declination to the north meridian altitude; the sum, if less than 90 degrees, is the complement of the latitude south; but if the sum is more than 90 degrees, subtract 90 from it, and the remainder is the latitude north.

EXAMPLES.

Sun's meridian altitude -	60°	30′	north
Sun's declination, add -	20	10	north
Complement of the latitude	80	40	
Subtract from - -	90	0	
Remains the latitude -	9	20	south.

Sun's meridian altitude -	70°	20′	north
Sun's declination, add -	23	20	north
The sum is - -	93	40	
From which subtract -	90	0	
Remains the latitude -	3	40	north.

must be corrected by the application of parallax and refraction. As the sun is elevated by refraction and depressed by parallax, his apparent meridian altitude must be diminished by the difference between the refraction and parallax.—*Ed.*

4. If the sun has south declination, and is north of your place at noon, subtract the declination from the north meridian altitude, and the remainder is the complement of the latitude south.

EXAMPLE.

Sun's meridian altitude -	52°	30′	north
Sun's declination, subtract	20	10	south
Complement of the latitude	32	20	
Subtract from - -	90	0	
And the remainder is the latitude	57	40	south.

5. If the sun has no declination, and is south of your place at noon, the meridian altitude is the complement of the latitude north: but if the sun be then north of your place, his meridian altitude is the complement of the latitude south.

EXAMPLES.

Sun's meridian altitude -	38°	30′	south
Subtract from - -	90	0	
Remains the latitude -	51	30	north.

Sun's meridian altitude	38°	30′	north
Subtract from - -	90	0	
Remains the latitude -	51	30	south.

6. If you observe the sun beneath the pole, subtract his declination from 90 degrees, and add the remainder to his altitude; and the sum is the latitude.

EXAMPLE.

Sun's declination -	20°	30′	
Subtract from - -	90	0	
Remains - - -	69	30	add
Sun's altitude below the pole	10	20	
The sum is the latitude	79	50	

Which is north or south, according as the sun's declination is north or south; for when the sun has south declination, he is never seen below the north pole; nor is he ever seen below the south pole, when his declination is north.

7. If the sun be in the zenith at noon, and at the same time has no declination, you are then under the equinoctial, and so have no latitude.

If the sun be in the zenith at noon, and has declination, the declination is equal to the latitude, north or south. These two cases are so plain, that they require no examples.[7]

LECTURE XI.

OF DIALLING.

HAVING shewn in the preceding Lecture how to make sun-dials by the assistance of a good globe, or of a dialling-scale, we shall now proceed to the method of constructing dials arithmetically, which will be more agreeable to those who have learned the elements of trigonometry, because globes and scales can never be so accurate as logarithms, in finding the angular distances of the hours. Yet, as a globe may be found exact enough for some other requisites in dialling, we shall take it in occasionally.

The construction of sun-dials on all planes whatever, may be included in one general rule: intelligible, if that of a horizontal dial for any given latitude be well understood. For there is no plane, however obliquely situated with respect to any given place, but what is parallel to the horizon of some other place; and therefore, if we can find that other place by a problem on the terrestrial globe, or by a trigonometrical calculation, and construct a horizontal dial for it; that dial, applied to the plane where it is to serve, will be a true dial for that place.—Thus, an erect direct south dial in 51½ degrees north latitude, would be a horizontal dial on the same meridian, 90 degrees southward of 51½ degrees north latitude; which falls in with 38½ degrees of south latitude: but if the upright plane declines from facing the south at the given place, it would still be a horizontal plane 90 degrees from that place, but for a different longitude; which would alter the reckoning of the hours accordingly.

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[7] The latitude of a place may be found with equal facility and accuracy, by taking the meridian altitude of the planets and fixed stars, and observing the same directions which are given by our author in the case of the sun. When fixed stars, however, are employed, their altitude must be corrected by refraction only, as their parallax is not sensible.—*Ed.*

CASE I.

1. Let us suppose that an upright plane at London declines 36 degrees westward from facing the south; and that it is required to find a place on the globe, to whose horizon the said plane is parallel; and also the difference of longitude between London and that place.

Rectify the globe to the latitude of London, and bring London to the zenith under the brass meridian, then that point of the globe which lies in the horizon at the given degree of declination (counted westward from the south point of the horizon) is the place at which the above mentioned plane would be horizontal. Now, to find the latitude and longitude of that place, keep your eye upon the place, and turn the globe eastward until it comes under the graduated edge of the brass meridian; then the degree of the brass meridian that stands directly over the place, is its latitude; and the number of degrees in the equator, which are intercepted between the meridian of London and the brass meridian, is the place's difference of longitude.

Thus, as the latitude of London is 51½ degrees north, and the declination of the place is 36 degrees west; I elevate the north pole 51½ degrees above the horizon, and turn the globe until London comes to the zenith, or under the graduated edge of the meridian; then, I count 36 degrees on the horizon westward from the south point, and make a mark on that place of the globe over which the reckoning ends, and bringing the mark under the graduated edge of the brass meridian, I find it to be under 30¼ degrees in south latitude: keeping it there, I count in the equator the number of degrees between the meridian of London and the brazen meridian (which now becomes the meridian of the required place) and find it to be 42¾. Therefore an upright plane at London, declining 36 degrees westward from the south, would be a horizontal plane at that place; whose latitude is 30¼ degrees south of the equator, and longitude 42¾ degrees west of the meridian of London.

Which difference of longitude being converted into time, is 2 hours 51 minutes.

The vertical dial declining westward 36 degrees at London, is therefore to be drawn in all respects as a horizontal dial for south latitude 30¼ degrees; save only, that the reckoning of the hours is to anticipate the reckoning on the horizontal dial, by 2 hours 51 minutes: for so much sooner will the sun come to the meridian of London, than to the meridian of any place whose longitude is 42¾ degrees west from London.

2. But to be more exact than the globe will shew us, we shall use a little trigonometry. Plate X. Fig. 10.

Let *N E S W* be the horizon of London, whose zenith is *Z*, and *P* the north pole of the sphere; and let *Z h* be the position of a vertical plane at *Z*, declining westward from *S* (the south) by an angle of 36 degrees; on which plane an erect dial for London at *Z* is to be described. Make the semidiameter *Z D* perpendicular to *Z h*, and it will cut the horizon in *D*, 36 degrees west of the south *S*. Then, a plane in the tangent *H D*, touching the sphere in *D*, will be parallel to the plane *Z h*; and the axis of the sphere will be equally inclined to both these planes.

Let *W Q E* be the equinoctial, whose elevation above the horizon of *Z* (London) is 38½ degrees; and *P R D* be the meridian of the place *D*, cutting the equinoctial in *R*. Then, it is evident, that the arc *R D* is the latitude of the place *D* (where the plane *Z h* would be horizontal) and the arc *R Q* is the difference of longitude of the planes *Z h* and *D H*.

In the spherical triangle *W D R*, the arc *W D* is given, for it is the complement of the plane's declination from *S* the south; which complement is 54° (viz. 90°—36°): the angle at *R*, in which the meridian of the place *D* cuts the equator, is a right angle; and the angle *R W D* measures the elevation of the equinoctial above the horizon of *Z*, namely 38½ degrees. Say, therefore, as radius is to the co-sine of the plane's declination from the south, so is the co-sine of the latitude of *Z* to the sign of *R D* the latitude of *D*;[1] which is of a different denomination from the latitude of *Z*, because *Z* and *D* are on different sides of the equator.

As radius	- -	-	- -	10.00000
To co-sine	36° 0′	=	*R Q*	9.90796
So co-sine	51° 30′	=	*Q Z*	9.79415
To Sine	30° 14′	=	*D R*	9.70211=

the latitude of *D*, whose horizon is parallel to the vertical plane *Z h* at *Z*.

N. B. When radius is made the first term, it may be omitted, and then, by subtracting it, mentally, from the sum of the other two, the operation will be shortened. Thus, in the present case,

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[1] See Playfair's *Elements of Geometry*, *Spher. Trig.* Prop. XIX.—*Ed.*

To the logarithmic sine of $WR =$ [2] $54^\circ\ 0'$ 9.90796
Add the logarithmic sine of $RD =$ [3] $38^\circ\ 30'$ 9.79415

Their sum *minus* radius - - 9.70211

gives the same solution as above. And we shall keep to this method in the following part of the work.

To find the difference of longitude of the places D and Z, say, as radius is to the co-sine of $38\frac{1}{2}$ degrees, the height of the equinoctial at Z, so is the co-tangent of 36 degrees, the plane's declination, to the co-tangent of the difference of longitudes.[4] Thus,

To the logarithmic sine of[5] $51^\circ\ 30'$ 9.89364
Add the logarithmic tangent of[6] $54^\circ\ 0'$ 10.13874

Their sum *minus* radius - - 10.03238

is the nearest tangent of $47^\circ\ 8' = WR$; which is the co-tangent of $42^\circ\ 52' = RQ$, the difference of longitude sought. Which difference being reduced to time, is 2 hours $51\frac{1}{2}$ minutes.

3. And thus having found the exact latitude and longitude of the place D, to whose horizon the vertical plane at Z is parallel, we shall proceed to the construction of a horizontal dial for the place D, whose latitude is 30° 14′ south; but anticipating the time at D by 2 hours 51 minutes (neglecting the $\frac{1}{2}$ minute in practice) because D is so far westward in longitude from the meridian of London; and this will be a true vertical dial at London, declining westward 36 degrees.

Assume any right line CSL (Fig. 11.) for the substile of the dial, and make the angle KCP equal to the latitude of the place (viz. 30° 14′) to whose horizon the plane of the dial is parallel; then CRP will be the axis of the stile, or edge that casts the shadow on the hours of the day, in the dial. This done, draw the contingent line EQ, cutting the substilar line at right angles in K; and from K make KR perpendicular to the axis CRP. Then KG ($= KR$) being made radius, that is, equal to the chord of 60° or tangent of 45°, on a good sector, take 42° 52′ (the difference of longitude of the places *Z* and *D*) from the tangents, and having set it from *K* to *M*, draw *C M* for the hour-line of XII. Take *K N* equal to the tangent of an angle less by 15 degrees than *K M*; that is, the tangent 27° 52′; and through the point *N* draw *C N* for the hour-line of I. The tangent of 12° 52′ (which is 15° less than 27° 52′) set off the same way, will give a point between *K* and *N*, through which the hour-line of II is to be drawn. The tangent of 2° 8′ (the difference between 45° and 42° 52′) placed on the other side of *C L*, will determine the point through which the hour-line of III is to drawn: to which 2° 8′, if the tangent of 15° be added, it will make 17° 8′; and this set off from *K* toward *Q* on the line *E Q* will give the point for the hour-line of IV; and so of the rest. The forenoon hour-lines are drawn the same way, by the continual addition of the tangents 15°, 30°, 45°, &c. to 42° 52′ (= the tangent of *K M*) for the hours of XI, X, IX, &c. as far as necessary; that is, until there be five hours on each side of the substile. The sixth hour, accounted from that hour or part of the hour on which the substile falls, will be always in a line perpendicular to the substile, and drawn through the centre *C*.

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[2] The co-sine of 36° 0′, or of RQ.

[3] The co-sine of 51° 30′, or of QZ.

[4] Playfair's *Geom. Spher. Trig.* Prop. XVIII.—*Ed.*

[5] The co-sine of 38° 30′, or of WDR.

[6] The co-tangent of 36°, or of DW.

4. In all erect dials, *C M*, the hour-line of XII is perpendicular to the horizon of the place for which the dial is to serve: for that line is the intersection of a vertical plane with the plane of the meridian of the place, both which are perpendicular to the plane of the horizon: and any line *H O*, or *h o*, perpendicular to *C M*, will be a horizontal line on the plane of the dial, along which line the hours may be numbered: and *C M* being set perpendicular to the horizon, the dial will have its true position.

5. If the plane of the dial had declined by an equal angle toward the east, its description would have differed only in this, that the hour-line of XII would have fallen on the other side of the substile *C L*, and the line *H O* would have a sub-contrary position to what it has in this figure.

6. And these two dials, with the upper points of their stiles turned toward the north pole, will serve for the other two planes parallel to them; the one declining from the north toward the east, and the other from the north toward the west, by the same quantity of angle. The like holds true of all dials in general, whatever be their declination and obliquity of their planes to the horizon.

CASE II.

7. If the plane of the dial not only *declines*, (Fig. 12) but also *reclines*, or *inclines*. Suppose its declination from fronting the south *S* be equal to the arc *S D* on the horizon; and its reclination be equal to the arc *D d* of the vertical circle *D Z*; then it is plain, that if the quadrant of altitude *Z d D*, on the globe, cuts the point *D* in the horizon, and the reclination is counted upon the quadrant from *D* to *d*; the intersection of the hour-circle *P R d*, with the equinoctial *W Q E*, will determine *R d*, the latitude of the place *d*, whose horizon is parallel to the given plane *Z h* at *Z*; and *R Q* will be the difference in longitude of the planes at *d* and *Z*.

Trigonometrically thus: let a great circle pass through the three points *W*, *d*, *E*; and in the triangle *W D d*, right-angled at *D*, the sides *W D* and *D d* are given; and thence the angle *D W d* is found, and so is the hypothenuse *W d*. Again, the difference, or the sum, of *D W d* and *D W R*, the elevation of the equinoctial above the horizon of *Z*, gives the angle *d W R*; and the hypothenuse of the triangle *W R d* was just now found; whence the sides *R d* and *W R* are found, the former being the latitude of the place *d*, and the latter the complement of *R Q*, the difference of longitude sought.

Thus, if the latitude of the place *Z* be 52° 10′ north; the declination *S D* of the plane *Z h* (which would be horizontal at *d*) be 36°, and the reclination be 15°, or equal to the arc *D d*; the south latitude of the place *d*, that is, the arc *R d*, will be 15° 9′; and *R Q* the difference of the longitude, 36° 2′. From these data, therefore, let the dial (Fig. 13) be described, as in the former example.

8. Only it is to be observed, that in the reclining or inclining dials, the horizontal line will not stand at right angles to the hour-line of XII, as in erect dials; but its position may be found as follows.

To the common substilar line *C K L* (Fig. 13), on which the dial for the place *d* was described, draw the dial *C r p m* 12 for the place *D*, whose declination is the same as that of *d*, viz. the arc *S D*; and *H O*, perpendicular to *C m*, the hour-line of XII on this dial, will be a horizontal line on the dial *C P R M* XII. For the declination of both dials being the same, the horizontal line remains parallel to itself, while the erect position of one dial is reclined or inclined with respect to the position of the other.

Or, the position of the dial may be found by applying it to its plane, so as to mark the true hour of the day by the sun, as shewn by another dial; or, by a clock regulated by a true meridian line and equation table.

9. There are several other things requisite in the practice of dialling; the chief of which I shall give in the form of arithmetical rules, simple and easy to those who have learned the elements of trigonometry. For, in practical arts of this kind, arithmetic should be used as far as it can go; and scales never trusted to, except in the final construction, where they are absolutely necessary in laying down the calculated hour distances on the plane of the dial. And although the inimitable artists of this metropolis have no occasion for such instructions, yet they may be of some use to students, and to private gentlemen, who amuse themselves this way.

RULE I.—*To find the angles which the hour-lines on any dial make with the substile.*

To the logarithmic sine of the given latitude, or of the stile's elevation above the plane of the dial, add the logarithmic tangent of the hour distance[7] from the meridian, or from the substile;[8] and the sum *minus* radius will be the logarithmic tangent of the angle sought.

For, in Fig. 11, KC is to KM in the ratio compounded of the ratio of KC to KG ($=KR$) and of KG to KM; which making CK the radius, 10,000,000, or 100,000, or 10, or 1, are the ratio of 10,000,000, or of 100,000, or of 10, or of 1, to $KG \times KM$.

Thus, in a horizontal dial, for latitude 51° 30′, to find the angular distance of XI in the forenoon, or I in the afternoon, from XII.

To the logarithmic sine of	51° 30′ - -	9.89354[9]
Add the logarithmic tangent of	51° 0′ - -	9.42805
The sum *minus* radius is	- - - -	9.32159=

the logarithmic tangent of 11° 50′, or of the angle which the hour-line of XI or I makes with the hour of XII.

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[7] That is, of 15, 30, 45, 60, 75°, for the hours of I, II, III, IV, V, in the afternoon; and XI, X, IX, VIII, VII, in the forenoon.

[8] In all horizontal dials, and erect north or south dials, the substile and meridian are the same; but in all declining dials, the substile line makes an angle with the meridian.

[9] In which case, the radius CK is supposed to be divided into 1,000,000 equal parts.

And by computing in this manner, with the sine of the latitude, and the tangents of 30, 45, 60, and 75°, for the hours of II, III, IV, and V, in the afternoon; or of X, IX, VIII, and VII, in the forenoon; you will find their angular distances from XII to be 24° 18′, 38° 3′, 53° 35′, and 71° 6′: which are all that there is occasion to compute for. And these distances may be set off from XII by a line of chords; or rather, by taking 1,000 from a scale of equal parts, and setting that extent as a radius from *C* to XII: and then, taking 209 of the same parts, (which, in the tables, are the natural tangent of 11° 50′), and setting them from XII to XI and to I, on the line *h o*, which is perpendicular to *C* XII (Fig. 11): and so for the rest of the hour-lines, which in the table of natural tangents, against the above distances, are 451, 782, 1,355, and 2,920 of such equal parts from XII, as the radius *C* XII contains 1,000. And lastly, set off 1,257 (the natural tangent of 51° 30′) for the angle of the stile's height, which is equal to the latitude of the place.

The reason why I prefer the use of the tabular numbers, and of a scale decimally divided, to that of the line of chords is because there is the least chance of mistake and error in this way; and likewise, because in some cases it gives us the advantage of a *nonius* division.[1]

In the universal ring-dial, for instance, the divisions on the axis are the tangents of the angles of the sun's declination placed on either side of the centre. But instead of laying them down from a line of tangents, I would make a scale of equal parts, whereof 1,000 should answer exactly to the length of the semi-axis, from the centre to the inside of the equinoctial ring; and then lay down 434 of these parts toward each end from the centre, which would limit all the divisions on the axis, because 434 is the natural tangent of 23° 29′. And thus, by a *nonius* affixed to the sliding piece, and taking the sun's declination from an ephemeris, and the tangent of that declination from the table of natural tangents, the slider might be always set true to within two minutes of a degree.

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[1] This scale, for subdividing the limbs of quadrants, and the divisions of other mathematical instruments, is improperly called *Nonius*, from one Nonius, who is supposed to be its inventor. The honour of the invention is due to Peter Vernier, a French gentleman, from whom it frequently receives its name. The Vernier scale consists of a piece of brass or ivory, which moves along the limb of the quadrant. A space, equal to any number of degrees in the circular arch, 10°, for example, is transferred to this piece of brass, and divided into 10 parts, so that each division of the vernier will exceed each division of the limb by $\frac{1}{10}$ of a degree. Suppose the plumb-line of the quadrant to fall between the 25th and 26th degree, and that the degrees run from right to left. Then, in order to find the number of minutes above 25°, move the vernier till the plumb-line falls on the beginning of its scale, and find what division of the vernier coincides with any division on the limb; and by so many 10ths of a degree will the angle exceed 25°. If the 7th division of the vernier, for instance, coincides with a division on the limb, then, $\frac{7}{10}$ths of a degree, or 42 minutes, must be added to 25 degrees, and the angle will be 25° 42′.—Nonius's method consisted in drawing a number of concentric circles, the outermost of which was divided into 90 parts; the next into 89; the next into 88, &c. so that the plumb-line was sure to coincide with some division in one of these circles, and the angle could be easily deduced, from the number of parts into which that circle was divided. See Ferguson's *Astronomy*. Appendix, Vol. II, p. 319.—*Ed.*

And this scale of 434 equal parts might be placed right against the $23\frac{1}{2}$ degrees of the sun's declination, on the axis, instead of the sun's place, which is there of very little use. For then, the slider might be set in the usual way, to the day of the month, for common use; but to the natural tangent of the declination, when great accuracy is required.

The like may be done wherever a scale of sines or tangents is required on any instrument.

RULE II.—*The latitude of the place, the sun's declination, and his hour-distance from the meridian, being given, to find,* 1, *his altitude;* 2, *his azimuth.*

1. Let d (Plate X, Fig. 12) be the sun's place, dR his declination: and in the triangle PZd, Pd the sum, or the difference, of dR, and the quadrant PR being given by the supposition, as also the complement of the latitude PZ, and the angle dPZ, which measures the horary distance of d from the meridian; we shall (by case 4, of Keill's *Oblique Spheric Trigonometry*) find the base Zd, which is the sun's distance from the zenith, or the complement of his altitude.

And, 2, as sine Zd : sine Pd :: sine dPZ : dZP, or of its supplement DZS, the azimuthal distance from the south.

Or, the practical rule may be as follows:

Write A for the sine of the sun's altitude, L and l for the sine and co-sine of the latitude, D and d for the sine and co-sine of the sun's declination, and H for the sine of the horary distance from VI.

Then the relation of H to A will have three varieties.

1. When the declination is toward the elevated pole, and the hour of the day is between XII and VI; it is

$$A = L\,D + H\,l\,d, \text{ and } H = \frac{A - L\,D.}{l\,d}$$

2. When the hour is after VI, it is $A = L\,D - H\,l\,d$, and

$$H = \frac{L\,D - A.}{l\,d}$$

3. When the declination is toward the depressed pole, we have $A = H\,l\,d - L\,D$, and $H = \frac{A + L\,D.}{l\,d}$

Which theorems will be found useful, and expeditious enough for solving those problems in geography and dialling, which depend on the relation of the sun's altitude to the hour of the day.

EXAMPLE I.

Suppose the latitude of the place to be 51½ degrees north; the time five hours distant from XII, that is, an hour after VI in the morning, or before VI in the evening; and the sun's declination 20° north. *Required the sun's altitude?*

Then, to log. L = log. sine 51° 38′ 1.89354[2]
Add log. D = log. sine 20° 0′ 1.53405

Their sum - - - 1.42759

gives $L\,D$ = logarithm of 0.267664, in the natural sines.

And, to log. H = log. sine[3] 15° 0′ 1.41300
add { log. l. = log. sine[4] 38° 0′ 1.79414
add { log. d. = log. sine[5] 70° 0′ 1.97300

Their sum - - - 1.18014

gives $H\,l\,d$ = logarithm of 0.151408, in the natural sines.

And these two numbers of (0.267664 and 0.151408) make 0.419072 = A; which, in the table, is the nearest natural sine of 24° 47′, the sun's altitude sought.

The same hour-distance being assumed on the other side of VI, then $L\,D - H\,l\,d$ is 0.116256, the sine of 6° 40½′; which is the sun's altitude at V in the morning, or VII in the evening, when his north declination is 20°.

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[2] Here we consider the radius as unity, and not 1,000,000, by which, instead of the index 9, we have —1, as above: which is of no further use, than making the work a little easier.

[3] The distance of one hour from VI.

[4] The co-latitude of the place.

[5] The co-declination of the sun.

But when the declination is 20° south (or toward the depressed pole), the difference $H\,l\,d - L\,D$ becomes negative, and thereby shews that an hour before VI in the morning, or past VI in the evening, the sun's centre is $6\frac{1}{2}°\ 40'$ below the horizon.

EXAMPLE II.—*In the same latitude and north declination from the given altitude, to find the hour.*

Let the altitude be 48°; and because, in this case $H = \frac{A - L\,D}{l\,d}$ and A (the natural sine of 48°) $= .743145$, and $L\,D = 267664$, $A - L\,D$ will be 0.475481, whose logarithmic sine is - - - - 1.6771331
from which taking the logarithmic sine of $l + d =$ 1.7671354

Remains - - - - - 1.9099977
the logarithmic sine of the hour-distance sought, viz. of 54° 22′; which, reduced to time is $3^h\ 37\frac{1}{2}^m$; that is, $IX^h\ 37\frac{1}{2}^m$ in the forenoon, or $II^h\ 22\frac{1}{2}^m$ in the afternoon.

Put the altitude = 18°, whose natural sine is .3090170; and thence $A - L\,D$ will be $= .0491953$; which divided by $l + d$, gives .0717179, the sine of $4°\ 6\frac{1}{2}'$, in time $16\frac{1}{2}$ minutes nearly, before VI in the morning, or after VI in the evening, when the sun's altitude is 18°.

And, if the declination 20° had been toward the south pole, the sun would have been depressed 18° below the horizon at $16\frac{1}{2}$ minutes after VI in the evening; at which time the twilight would end; which happens about the 22d of November, and 19th of January, in the latitude of $51\frac{1}{2}°$ north. The same way may the end of twilight, or beginning of dawn, be found for any time of the year.

Note 1. If in theorem 2 and 3 (page 274) A is put $= 0$, and the value of H is computed, we have the hour of sun-rising and setting for any latitude and time of the year. And if we put $H = 0$, and compute A, we have the sun's altitude or depression at the hour of VI. And lastly, if H, A, and D are given, the latitude may be found by the resolution of a quadratic equation; for $l = \sqrt{1 - L^2}$.

Note 2. When A is equal 0, H is equal $\frac{L\,D}{l\,d} = T\,L \times T\,D$, the tangent of the latitude multiplied by the tangent of the declination.

As, if it was required, *what is the greatest length of day in latitude* 51° 30′?

To the log. tangent of 51° 30′ - 0.0993948
Add the log. tangent of 23° 39′ - 1.6379563

Their sum - - - 1.7373511

is the log. sine of the hour-distance 33° 7′; in time $2^{h}\ 12\frac{1}{2}^{m}$. The longest day therefore is $12^{h} + 4^{h}\ 25^{m} = 16^{h}\ 25^{m}$. And the shortest day is $12^{h} - 4^{h}\ 25^{m} = 7^{h}\ 35^{m}$.

And if the longest day is given, the latitude of the place is found; $\frac{H}{T\,D}$ being equal to $T\,L$. Thus, if the longest day is $13\frac{1}{2}^{h} = 2 \times 6^{h} + 45^{m}$, and 45^{m} in time being equal to $11\frac{1}{4}$ degrees,

From the log. sine of 11° 15′ - 1.2902357
Take the log. tang. of 23° 29′ - 1.6379562

Remains - - - - 1.6522795

= the logarithmic tangent of lat. 24° 11′.

And the same way, the latitudes, where the several geographical *climates* and parallels begin, may be found; and the latitudes of places, that are assigned in authors from the length of their days, may be examined and corrected.

Note 3. The same rule for finding the longest day, in a given latitude, distinguishes the hour-lines that are necessary to be drawn on any dial from those which would be superfluous.

In lat. 52° 10′ the longest day is $16^{h}\ 32^{m}$, and the hour-lines are to be marked from 44^{m} after III in the morning, to 16^{m} after VIII in the evening.

In the same latitude, let the dial of Art. 7, Fig. 13, be proposed; and the elevation of its stile (or the latitude of the place *d*, whose horizon is parallel to the plane of the dial) being 15° 9′; the longest day at *d*, that is, the longest time that the sun can illuminate the plane of the dial, will (by the rule $H = T\,L \times T\,D$) be twice $6^{h}\ 27^{m} = 12^{h}\ 54^{m}$. The difference of longitude of the planes *d* and *Z* was found in the same example to be 36° 2′; in time, $2^{h}\ 24^{m}$; and the declination of the plane was from the south toward the west. Adding therefore $2^{h}\ 24^{m}$ to $5^{h}\ 33^{m}$, the earliest sun-rising on a horizontal dial at *d*, the sum $7^{h}\ 57^{m}$ shews that the morning hours, or the parallel dial at *Z*, ought to begin at 3^{m} before VIII. And to the latest sun-setting at *d*, which is $6^h\ 27^m$, adding the same $2^h\ 24^m$, the sum $8^h\ 51^m$ exceeding $6^h\ 16^m$, the latest sun-setting at *Z*, by 35^m, shews that none of the afternoon hour-lines are superfluous. And the $4^h\ 13^m$ from $III^h\ 44^m$, the sun-rising at *Z*, to $VII^h\ 57^m$; the sun-rising at *d*, belong to the other face of the dial; that is, to a dial declining 36° from north to east, and inclining 15°.

EXAMPLE III.—*From the same data to find the sun's azimuth.*

If *H*, *L*, and *D*, are given, then (by Art. 2, of Rule II) from *H* having found the altitude and its complement *Z d*; and the arc *P D* (the distance from the pole) being given, say, as the co-sine of the altitude is to the sine of the distance from the pole, so is the sine of the hour-distance from the meridian to the sine of the azimuth distance from the meridian.

Let the latitude be 51° 30′ north, the declination 15° 9′ south, and the time $II^h\ 24^m$ in the afternoon, when the sun begins to illuminate a vertical wall, and it is required to find the position of the wall.

Then, by the foregoing theorems, the complement of the altitude will be 81° 32½′, and *P d* the distance from the pole being 109° 5′, and the horary distance from the meridian, or the angle *d P Z*, 36°,

To log. sine 74° 51′ - - -	1.98464
Add log. sine 36° 0′ - -	1.76922
And from the sum - - -	1.75386
Take the log. sine 81° 32½′ -	1.99525
Remains - - - -	1.75861

= log. sine 35°, the azimuth distance south.

When the altitude is given, find from thence the hour, and proceed as above.

This praxis is of singular use on many occasions: in finding the declination of vertical planes more exactly than in the common way, especially if the transit of the sun's centre is observed by applying a ruler with sights, either plane or telescopical, to the wall or plane, whose declination is required;—in drawing a meridian-line, and finding the magnetic variation;—in finding the bearings of places in terrestrial surveys, the transit of the sun over any place, or his horizontal distance from it being observed, together with the altitude and hour; and thence determining small differences of longitude ;—in observing the variation at sea, &c.

The learned Mr. Andrew Reid invented an instrument several years ago, for finding the latitude at sea from two altitudes of the sun, observed on the same day, and the interval of the observations, measured by a common watch. And this instrument, whose only fault was that of its being somewhat expensive, was made by Mr. Jackson. Tables have been lately computed for that purpose.

But we may often, from the foregoing rules, resolve the same problem without much trouble ; especially if we suppose the master of the ship to know within 2 or 3 degrees what his latitude is,—Thus, assume the two nearest probable limits of the latitude, and by the theorem $H = \frac{A + LD}{l\,d}$, compute the hours of observation for both suppositions. If one interval of those computed hours coincides with the interval observed, the question is solved. If not, the two distances of the intervals computed, from the true interval, will give a proportional part to be added to, or substracted from, one of the latitudes assumed. And if more exactness is required, the operation may be repeated with the latitude already found.

But whichever way the question is solved, a proper allowance is to be made for the difference of latitude arising from the ship's course in the time between the two observations.

Of the double horizontal Dial, and the Babylonian and Italian Dials.

To the *gnomonic* projection, there is sometimes added a *stereographic* projection of the hour-circles, and the parallels of the sun's declination, on the same horizontal plane ; the upright side of the gnomon being sloped into an edge, standing perpendicularly over the centre of the projection : so that the dial being in its due position, the shadow of *that* perpendicular edge is a vertical circle passing through the sun, in the stereographic projection.

The months being duly marked on the dial, the sun's declination, and the length of the day at any time, are had by inspection ; as also his altitude, by means of a scale of tangents. But its chief property is, that it may be placed true whenever the sun shines, without the help of any other instrument.

Let *d* (Fig. 12), be the sun's place in the stereographic projection, *x d y z* the parallel of the sun's declination, *Z d* a vertical circle through the sun's centre, *P d* the hour-circle; and it is evident, that the diameter *N S* of this projection being placed duly north and south, these three circles will pass through the point *d*. And therefore, to give the dial its due position, we have only to turn its gnomon toward the sun, on a horizontal plane, until the hour on the common gnomonic projection coincides with that marked by the hour-circle *P d*, which passes through the intersection of the shadow *Z d* with the circle of the sun's present declination.

The Babylonian and Italian dials reckon the hours, not from the meridian, as with us, but from the sun's rising and setting. Thus, in Italy, one hour before sun-set is reckoned the 23d hour, two hours before sun-set the 22d hour, and so of the rest. And the shadow that marks them on the hour-lines, is *that* of the point of a stile. This occasions a perpetual variation between their dials and clocks, which they must correct from time to time, before it arises to any sensible quantity, by setting their clocks so much faster or slower. And in Italy they begin their day, and regulate their clocks, not from sun-set, but from about mid-twilight, when the *Ave Maria* is said; which corrects the difference that would otherwise be between the clock and the dial.

The improvements which have been made in all sorts of instruments and machines for measuring time, have rendered such dials of little account. Yet, as the theory of them is ingenious, and they are really, in some respects, the best contrived of any for vulgar use, a general idea of their description may not be unacceptable.

Let Fig. 14 represent an erect direct south wall, on which a *Babylonian dial* is to be drawn, shewing the hours from sun-rising; the latitude of the place, whose horizon is parallel to the wall, being equal to the angle *K C R*. Make, as for a common dial, *K G* = *K R*, (which is perpendicular to *C R*) the radius of the equinoctial *Æ Q*, and draw *R S* perpendicular to *C K* for the stile of the dial; the shadow of whose point *R* is to mark the hours, when *S R* is set upright on the plane of the dial.

Then it is evident, that in the contingent line *Æ Q*, the spaces

K 1, *K* 2, *K* 3, &c. being taken equal to the tangents of the hour-distances from the meridian, to the radius *K G*, one, two three, &c. hours after sun-rising, on the equinoctial day; the shadow of the point *R* will be found at these times respectively in the points 1, 2, 3, &c.

Draw, for the like hours after sun-rising, when the sun is in the tropic of Capricorn ♑ *v*, the like common lines *C D*, *C E*, *C F*, &c. and at these hours the shadow of the point *R* will be found in those lines respectively. Find the sun's altitudes above the plane of the dial at these hours, and with their co-tangents *S d*, *S e*, *S f*, &c. to radius *S R*, describe arcs intersecting the hour-lines in the points *d*, *e*, *f*, &c. so shall the right lines 1 *d*, 2 *e*, 3 *f*, &c. be the lines of I, II, III, &c. hours after sun-rising.

The construction is the same in every other case, due regard being had to the difference of longitude of the place at which the dial would be horizontal, and the place for which it is to serve.. And likewise, taking care to draw no lines but what are necessary; which may be done, partly by the rules already given for determining the time that the sun shines on any plane, and partly from this, that on the tropical days the hyperbola described by the shadow of the point *R* limits the extent of all the hour-lines.

The most useful, however, as well as the simplest, of such dials, is that which is described on the two sides of the meridian plane.

That the Babylonian and Italic hours are truly enough marked by right lines, is easily shewn. Mark the three points on a globe, where the horizon cuts the equinoctial, and the two tropics, toward the east or west: and turn the globe on its axis 15°, or 1 hour; and it is plain that the three points which were in a great circle (viz. the horizon) will be in a great circle still; which will be projected geometrically into a straight line. But these three points are universally the sun's places one hour after sun-set (or one hour before sun-rise) on the equinoctial and solstitial days. The like is true of all other circles of declination, beside the tropics; and therefore, the hours on such dials are truly marked by straight lines limited by the projections of the tropics.; and which are rightly drawn, as in the foregoing example.

Note 1.—The same dials may be delineated without the hour-lines, *C D*, *C E*, *C F*, &c. by setting off the sun's azimuths on the plane of the dial, from the centre *S*, on either side of the substile *C S K*, and the corresponding co-tangents of altitude from the same centre *S*, for I, II, III, &c. hours before or after the sun is in the horizon of the place for which the dial is to serve, on the equinoctial and solstitial days.

2. One of these dials has its name from the hours being reckoned from sun-rising, the beginning of the Babylonian day. But we are not thence to imagine that the *equal* hours, which it shews, were those in which the astronomers of that country marked their observations. These, we know with certainty, were unequal, like the Jewish, as being twelfth parts of the natural day: and an hour of the night was, in like manner, a twelfth part of the night; longer or shorter, according to the season of the year. So that an hour of the day, and an hour of the night, at the same place, would always make $\frac{1}{12}$ of 24, or 2 equinoctial hours. In Palestine, among the Romans, and in several other countries, 3 of these unequal nocturnal hours were a *vigilia*, or *watch*. And the reduction of equal and unequal hours into one another is extremely easy. If, for instance, it is found, by a foregoing rule, that in a certain latitude, at a given time of the year, the length of a day is 14 equinoctial hours, the unequal hour is then $\frac{14}{12}$ or $\frac{7}{6}$ of an hour, that is, 70 minutes; and the nocturnal hour is 50 minutes. The first watch begins at VII (sun-set); the second at three times 50 minutes after, viz. $\text{IX}^{h}\ 30^{m}$; the third always at midnight; the morning watch at half an hour past II.

If it were required to draw a dial for shewing these unequal hours, or twelfth parts of the day, we must take as many declinations of the sun as are thought necessary, from the equator toward each tropic: and having computed the sun's altitude and azimuth for $\frac{1}{12}$, $\frac{2}{12}$, $\frac{3}{12}$th parts, &c. of each of the diurnal arcs belonging to the declinations assumed: by these, the several points in the circles of declination, where the shadow of the stile's point falls, are determined; and curve lines drawn through the points of a homologous division will be the hour-lines required.[6]

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[6] For the description of a new dial, invented by Lambert, and of a curious Analemmatic dial, which can be properly placed without a mariner's needle, or a meridian line, and which can be drawn in a garden, the spectator being its stile, see *Appendix.—Ed.*

Of the right placing of dials, and having a true meridian line for the regulation of clocks and watches.[7]

The plane on which the dial is to rest, being duly prepared, and every thing necessary for fixing it, you may find the hour tolerably exact by a large equinoctial ring-dial, and set your watch to it. And then the dial may be fixed by the watch at your leisure.

If you would be more exact, take the sun's altitude by a good quadrant, noting the precise time of observation by a clock or watch. Then, compute the time for the altitude observed (by the rule, page 275), and set the watch to agree with that time, according to the sun. A Hadley's quadrant is very convenient for this purpose; for, by it you may take the angle between the sun and his image, reflected from a bason of water: the half of which angle, subtracting the refraction, is the altitude required. This is best done in summer, and the nearer the sun is to the prime vertical (the east or west azimuth) when the observation is made, so much the better.

Or, in summer, take two equal altitudes of the sun in the same day; one any time between seven and ten in the morning, the other between two and five in the afternoon; noting the moments of these two observations by a clock or watch: and if the watch shews the observations to be at equal distances from noon, it agrees exactly with the sun; if not, the watch must be corrected by half the difference of the forenoon and afternoon intervals; and then the dial may be set true by the watch.

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[7] In another work, when speaking upon the placing of sun-dials, our author observes, "that if the dial be made according to the strictest rules of calculation, and be truly set at the instant when the sun's centre is on the meridian, it will be a minute too fast in the forenoon, and a minute too slow in the afternoon, by the shadow of the stile; for the edge of the shadow that shews the time is even with the sun's foremost edge all the time before noon, and even with his hindermost all the afternoon on the dial. And it is the sun's centre that determines the time in the supposed hour-circles of the heavens. And as the sun is half a degree in breadth, he takes two minutes to move through a space equal to his breadth, so that there will be two minutes at noon in which the shadow will have no motion at all on the dial; consequently, if the dial be set true by the sun in the forenoon, it will be two minutes too slow in the afternoon; and, if it be set true in the afternoon, it will be two minutes too fast in the forenoon. The only way that I know of to remedy this is, to set every hour and minute division on the dial one minute nearer 12 than the calculation makes it to be." *Tables and Tracts*, 2d edit. p. 73. These observations are new, and just enough in themselves; but the evil which the author points out may be remedied by observing the middle of the shadow's penumbra, which corresponds with the sun's centre, instead of the border of the real shadow; and I believe it will be found, that every person naturally does this when he determines the hour of the day upon a sun-dial.—*Ed.*

Thus, for example, suppose you have taken the sun's altitude when it was twenty minutes past VIII in the morning by the watch, and found, by observing in the afternoon, that the sun had the same altitude ten minutes before IV, then it is plain, that the watch was five minutes too fast for the sun: for five minutes after XII is the middle time between VIII^h 20^m in the morning, and III^h 50^m in the afternoon; and therefore, to make the watch agree with the sun, it must be set back five minutes.[8]

A good *meridian line*, for regulating clocks or watches, may be had by the following method. A *meridian line.*

Make a round hole, almost a quarter of an inch diameter, in a thin plate of metal; and fix the plate in the top of a south window, in such a manner, that it may recline from the zenith at an angle equal to the co-latitude of your place, as nearly as you can guess; for then the plate will face the sun directly at noon on the equinoctial days. Let the sun shine freely through the hole into the room; and hang a plumb-line to the ceiling of the room, at least five or six feet from the window, in such a place as that the sun's rays, transmitted through the hole, may fall upon the line when it is noon by the clock; and having marked the said place on the ceiling, take away the line.

Having adjusted a sliding bar to a dove-tail groove, in a piece of wood about eighteen inches long, and fixed a hook in the middle of the bar, nail the wood to the above-mentioned place on the ceiling, parallel to the side of the room in which the window is; the groove and bar being toward the window. Then hang the plumb-line upon the hook of the bar, the weight or plummet reaching almost to the floor; and the whole will be prepared for farther and proper adjustment.

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[8] The above method of finding the hour of the day by corresponding altitudes of the sun or stars, is the easiest and most correct that can be employed. Owing, however, to the change that takes place in the sun's declination before the afternoon altitude is taken, it is liable to an error, which, at a maximum, amounts to 30″ in the time of the equinoxes. A table containing this correction, which depends upon the interval between the altitudes, and upon the declination of the sun, may be seen in the *Astronomie de la Lande*, edit. 3d, tom. i, *Tables*, p. 37, and in the *Tables de Berlin*, tom. i, p. 291.—*Ed.*

This done, find the true solar time by either of the two last methods, and thereby regulate your clock. Then, at the moment of next noon by the clock, when the sun shines, move the sliding bar in the groove until the shadow of the plumb-line bisects the image of the sun (made by his rays transmitted through the hole) on the floor, wall, or on a white screen placed on the north side of the line; the plummet or weight at the end of the line hanging freely in a pail of water placed below it on the floor. But because this may not be quite correct for the first time, on account of the plummet not settling immediately, even in water; it may be farther corrected on the following days, by the above method, with the sun and clock, and so brought to a very great exactness.

N. B. The rays transmitted through the hole will cast but a faint image of the sun, even on a white screen, unless the room be so darkened that no sun-shine may be allowed to enter but what comes through the small hole in the plate. And always, for some time before the observation is made, the plummet ought to be immersed in a jar of water, where it may hang freely; by which means the line will soon become steady, which otherwise would be apt to continue swinging.

As this meridian line will not only be sufficient for regulating clocks and watches to the true time by equation tables, but also for most astronomical purposes, I shall say nothing of the magnificent and expensive meridian lines at Bologna and Rome, nor of the better methods by which astronomers observe precisely the transits of the heavenly bodies over the meridian.[9]

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[9] For farther information upon dialling, the reader may consult Orontii Finei Opera, fol. lib. iii.—De Horologiis Sciothericis a Joanne Voello, Turoni 1608.—Horologiographia per Sebastianum Munsterum 1533.—Christ. Clavii Bambergensis Horologiorum Nova Descriptio.—Demonstratio et Constructio Horologiorum Novorum, auctore Georgio Schombergero.—Gnomonice Schoner, 4to.—The *Gnomonique* of De La Hire.—Kæstner's *Gnomonica Universalis.*—Horsley *On the Projection of the Sphere.*—Wolfii Oper. Mathemat. tom. ii, p. 787, Ferguson's Select Exercises, Leybourn's Dialling, Leadbetter's Dialling, and an excellent treatise by the celebrated Deparcieux, published at the end of his *Traité de Trigonometrie rectiligne et spherique.* This subject is treated more profoundly by M. Sejour, in his *Recherches sur la Gnomonique*, 1761, and in his *Traité Analytique*, tom. i, p. 705.—See also The *Edinburgh Encyclopædia*, vol. vii, p. 691.

LECTURE XII.

SHEWING HOW TO CALCULATE THE MEAN TIME OF ANY NEW OR FULL MOON, OR ECLIPSE, FROM THE CREATION OF THE WORLD TO THE YEAR OF CHRIST 5800.

Calculation of new and full moons.

In the following tables, the mean lunation is about a 20th part of a second of time longer than its measure, as now printed in the last edition of my *Astronomy;* which makes the difference of an hour and thirty minutes in 3000 years. But this is not material, when only the mean times are required.

PRECEPTS.—*To find the mean time of any new or full moon in any given year and month after the Christian era.*

1. If the given year be found in the third column of the *table of the moon's mean motion from the sun,* under the title *years before and after Christ:* write out that year, with the mean motions belonging to it, and thereto join the given month with its mean motions. But, if the given year be not in the table, take out the next lesser one to it that you find, in the same column; and thereto add as many *complete years,* as will make up the given year: then, join the month and all the respective mean motions.

2. Collect these mean motions into one sum of signs, degrees, minutes, and seconds; remembering that 60 seconds (″) make a minute, 60 minutes (′) a degree, 30 degrees (°) a sign, and 12 signs (^{s}) a circle. When the signs exceed 12, or 24, or 36 (which are whole circles), reject them, and set down only the remainder; which, together with the odd degrees, minutes, and seconds, already set down, must be reckoned the whole sum of the collection.

3. Subtract the result, or sum of this collection, from 12 signs; and write down the remainder. Then look in the table under *days,* for the next less mean motions to this remainder, and subtract them from it, writing down their remainder.

This done, look in the table under *hours* (marked H) for the next less mean motions to this last remainder, and subtract them from it, writing down their remainder.

Then look in the table under *minutes* (marked M) for the next less mean motions to this remainder, and subtract them from it, writing down their remainder.

Lastly, look in the table under *seconds* (marked S) for the next less mean motion to this remainder, either greater or less; and against it you have the seconds answering thereto.

4. And these times collected, will give the mean time of the *required new moon;* which will be right in common years; and also in January and February in leap years; but always one day too late in leap years after February.

EXAMPLE I.—*Required the time of new moon in September 1764?*

(A year not inserted in the table.)

		Moon from sun.			
		s	°	′	″
To the year after Christ's birth	1753	10	9	24	56
Add complete years	11	0	10	14	20
	(sum 1764)				
And join September		2	22	21	8
The sum of these mean motions is		1	12	0	24
Which, being subtracted from a circle, or		12	0	0	0
Leaves remaining		10	27	59	36
Next less mean motion for twenty-six days, subtract		10	16	57	34
And there remains			1	2	2
Next less mean motion for two hours, subtract			1	0	57
And the remainder will be				1	5
Next less mean motion for two minutes, subtract				1	1
Remains the mean motion of twelve seconds					4

These times, being collected, would shew the mean time of the required new moon in September 1764, to be on the 29th day at 2^h 2^m 12^s past noon. But, as it is in a leap year, and after February, the time is one day too late. So, the true mean time is September the 25th, at 2^m 12^s past II in the afternoon.

N. B. The tables always begin the day at noon, and reckon thenceforward, to the noon of the day following.

To find the mean time of full moon in any given year and month after the Christian era.

Having collected the moon's mean motion from the sun for the beginning of the given year and month, and subtracted their sum from twelve signs (as in the former example), add six signs to the remainder, and then proceed in all respects as above.

Ex. II.—*Required the mean time of full moon in September 1764?*

		Moon from sun.			
		s	°	′	″
To the year after Christ's birth	1753	10	9	24	56
Add complete years	11	0	10	14	20
	(sum 1764)				
And join September		2	22	21	8
The sum of these mean motions is		1	12	0	24
Which, being subtracted from a circle, or		12	0	0	0
Leaves remaining		10	17	59	36
To which remainder add		6	0	0	0
And the sum will be		4	17	59	36
Next less mean motion for eleven days, subtract		4	14	5	54
And there remains			3	53	42
Next less mean motion for seven hours, subtract			3	33	20
And the remainder will be				20	22
Next less mean motion for forty minutes, subtract				20	19
Remains the mean motion for eight seconds					3

So, the mean time, according to the tables, is the 11th of September, at 7h 40m 8s past noon. One day too late, being after February in a leap year.

And thus may the mean time of any new or full moon be found, in any year after the Christian era.

To find the mean time of new or full moon in any given year and month before the Christian era.

If the given year before the year of Christ 1 be found in the third column of the table, under the title of *years before and after Christ*, write it out, together with the given month, and join the mean motions. But, if the given year be not in the table, take out the next greater one to it that you find; which being still farther back than the given year, add as many complete years to it as will bring the time forward to the given year; then join the month, and proceed in all respects as above.

Ex. III.—*Required the mean time of new moon in May, the year before Christ* 585?

The next greater year in the table is 600; which being 15 years before the given year, add the mean motions for 15 years to those of 600, together with those for the beginning of May.

	Moon from sun.			
	s	°	′	″
To the year before Christ 600 - - -	5	11	6	16
Add complete years motion 15 - - -	6	0	55	24
And the mean motion for May - - -	0	22	53	23
The whole sum is - - - -	0	4	55	3
Which, being subtracted from a circle, or -	12	0	0	0
Leaves remaining - - - -	11	25	4	57
Next less mean motion for twenty-nine days, subtract - - - - -	11	23	31	54
And there remains - - - -		1	33	3
Next less mean motion for three hours, subtract		1	31	26
And the remainder will be - - -			1	37
Next less mean motion for three minutes, subtract			1	31
Remains the mean motion of fourteen seconds				6

So the mean time, by the tables, was the 29th of May, at $3^h\ 3^m\ 14^s$ past noon: a day later than the truth, on account of its being in a leap year. For, as the year of Christ 1 was the first after a leap year, the year 585 before the year 1 was a leap year of course.

If the given year be after the Christian era, divide its date by 4, and if nothing remains, it is a leap year in the old stile. But if the given year was before the Christian era (or year of Christ 1), subtract one from its date, and divide the remainder by 4; then, if nothing remains, it was a leap year; otherwise not.

To find whether the sun is eclipsed at the time of any given change, or the moon at any given full.

Of eclipses. From the *table of the sun's mean motion* (or distance) *from the moon's ascending node*, collect the mean motions answering to the given time; and if the result shews the sun to be within 18° of either of the nodes at the time of new moon, the sun will be eclipsed at that time. Or, if the result shews the time to be within 12° of either of the nodes at the time of full moon, the moon will be eclipsed at that time, in or near the contrary node; otherwise not.

Ex. IV.—*The moon changed on the 26th of September* 1764, *at* 2^h 2^m *(neglecting the seconds) afternoon.* (See Ex. I). Qu. *Whether the sun was eclipsed at that time?*

		Sun from node.			
		s	°	′	″
To the year after Christ's birth	1753	1	28	0	19
Add complete years,	11	7	2	3	56
	(sum 1764)				
And September		8	12	22	49
26 days			27	0	13
2 hours				5	12
2 minutes					5
Sun's distance from the ascending node		6	9	32	34

Now, as the descending node is just opposite to the ascending (viz. six signs distant from it), and the tables shew only how far the sun has gone from the ascending node, which, by this example, appears to be 6ˢ 9° 32′ 34″, it is plain that he must have then been eclipsed; as he was then only 9° 32′ 34″ short of the descending node.

Ex. V.—*The moon was full on the* 11*th of September* 1764, *at* 7^h 40^m *past noon.* (See Example II). Qu. *Whether she was eclipsed at that time?*

		Sun from node.			
		s	°	′	″
To the year after Christ's birth	1753	1	28	0	19
Add complete years	11	7	2	3	56
	(sum 1764)				
And September		8	12	22	49
11 days			11	25	29
7 hours				18	11
40 minutes				1	44
Sun's distance from the ascending node		5	24	12	28

Which being subtracted from six signs, leaves only 5° 47′ 32″

remaining; and this being all the space that the sun was short of the descending node, it is plain that the moon must then have been eclipsed, because she was just as near the contrary node.

Ex. VI.—Qu. *Whether the sun was eclipsed in May, the year before Christ* 585? (See Example III.)

		Sun from node. s	°	′	″
To the year before Christ 600		9	9	23	51
Add the mean motion of 15 complete years		9	19	27	49
And	May	4	4	37	57
	29 days	1	0	7	10
	3 hours			7	48
	3 minutes (neglecting the seconds)				8
Sun's distance from the ascending node		0	3	44	43

Which being less than 18°, shews that the sun was eclipsed at that time.

Thales's eclipse.

This eclipse was foretold by Thales, and is thought to be the eclipse which put an end to the war between the Medes and Lydians.

When *eclipses must happen.*

The times of the *sun's conjunction with the nodes,* and consequently the *eclipse months* of any given year, are easily found by the *Tables of the sun's mean motion from the moon's ascending node;* and much in the same way as the mean conjunctions of the sun and moon are found by the table of the moon's mean motions from the sun. For, collect the sun's mean motion from the node (which is the same as his distance gone from it) for the beginning of any given *year,* and subtract it from 12 signs; then, from the remainder, subtract the next less mean motions belonging to whatever *month* you find them in the table; and from the remainder subtract the next less mean motion for *days,* and so on for *hours* and *minutes;* the result of all which will shew the time of the sun's mean conjunction with the *ascending node* of the moon's orbit.

Ex. VII.—*Required the time of the sun's conjunction with the ascending node in the year* 1764?

		Sun from node. s	°	′	″
To the year after Christ's birth	1753	1	28	0	19
Add complete years	11	7	2	3	56
Mean distance at beginning of A. D.	1764	9	0	4	15

	Sun from node.			
	s	°	′	″
Mean distance at beginning of 1764 (brought over)	9	0	4	15
Subtract this distance from a circle, or - -	12	0	0	0
And there remains - - - - -	2	29	55	45
Next less mean motion for March, subtract -	2	1	16	39
And the remainder will be - - - -		28	39	6
Next less mean motion for 27 days, subtract -		28	2	32
And there remains - - - - -			36	34
Next less mean motion for 14 hours, subtract -			36	21
Remains, nearly, the mean motion of 5 minutes -				13

Hence it appears, that the sun will pass by the moon's *ascending node* on the 27th of March, at 14ʰ 5ᵐ past noon, viz. on the 28th day, at 5ᵐ past II in the morning, according to the tables; but this being in a leap year, and after February, the time is one day too late. Consequently, the true time is at 5ᵐ past II in the morning on the 27th day; at which time the descending node will be directly opposite to the sun.

If 6 signs be added to the remainder arising from the first subtraction (viz. from 12 signs) and then the work carried on as in the last example, the result will give the mean time of the sun's conjunction with the descending node. Thus, in

Ex. VIII.—*To find when the sun will be in conjunction with the descending node in the year* 1764?

		Sun from node.			
		s	°	′	″
To the year after Christ's birth -	1753	1	28	0	19
Add complete years - - -	11	7	2	3	56
Mean distance from ascending node at beginning of - - - - - -	1764	9	0	4	15
Subtract this distance from a circle, or - -		12	0	0	0
And the remainder will be - - - -		2	29	55	45
To which add half a circle, or - - -		6	0	0	0
And the sum will be - - - - -		8	29	55	45
Next less mean motion for September subtracted		8	12	22	49
And there remains - - - - -			17	32	56

A Table of mean Lunations.

This table is made by the continual addition of a mean lunation, viz. 29^{d} 12^{h} 44^{m} 3^{s} 6^{th} 12^{iv} 14^{v} 24^{vi} 0^{vii}.

Lun.		Days.	H.	M.	S.	Th.
1	Contain	29	12	44	3	6
2		59	1	28	6	13
3		88	14	12	9	19
4		118	2	56	12	25
5		147	15	40	15	32
6		177	4	24	18	38
7		206	17	8	21	44
8		236	5	52	24	51
9		265	18	36	27	57
10		295	7	20	31	3
20		590	14	41	2	7
30		885	22	1	33	11
40		1181	5	22	4	14
50		1476	12	42	35	18
100		2953	1	25	10	35
200		5906	2	50	21	11
300		8859	4	15	31	46
400		11812	5	40	42	22
500		14756	7	5	52	57
1000		29530	14	11	45	54
2000		59661	4	23	31	48
3000		88591	18	35	17	42
4000		118122	8	47	3	36
5000		147652	22	58	49	30
10000		295305	21	57	39	0
20000		590611	19	55	18	0
30000		885917	17	52	57	0
40000		1181223	15	50	36	0
50000		1476529	13	48	15	0
100000		2953059	3	36	39	0

In 100000 mean lunations there are 8085 Julian years 12 days 21 hours 36 minutes 30 seconds
2953059 days 3 hours 36 minutes 30 seconds.

Proof of the Table.

In		Moon from sun. s	°	′	″
Jul. years	4000	1	14	22	12
	4000	1	14	22	12
	80	5	23	41	15
	5	10	0	18	28
Days	12	4	26	17	20
Hours	21		10	40	1
Min.	36			18	7
Sec.	20				15
M. fr. sun,		0	0	0	0

Having by the former precepts computed the mean time of new moon in January, for any given year, it is easy, by this table, to find the mean time of new moon in January for any number of years afterwards; and by means of a small table of lunations for 12 or 13 months, to make a general table for finding the mean time of new or full moon in any given year and month whatever.

	D.	H.	M.	S.	Th.
In 11 lunations there are	324	20	4	34	10
In 12 lunations	354	8	48	37	16
In 13 lunations	383	21	32	40	23

But then it would be best to begin the year with March, to avoid the inconvenience of losing a day by mistake in leap year.

A Table of the Moon's mean Motion from the Sun.

Years of the Julian period.	Years of the World.	Years before and after Christ.		Moon from sun.				Complete years.	Moon from sun.			
				s	°	'	"		s	°	'	"
706	0	Before the year of Christ 1.	4008	5	28	1	17	11	0	10	14	20
714	8		4000	5	9	23	24	12	5	2	3	11
1714	1008		3000	11	20	28	57	13	9	11	40	35
2714	2008		2000	6	1	34	30	14	1	21	18	0
3714	3008		1000	0	12	40	3	15	6	0	55	24
3814	3108		900	10	19	46	36	16	10	22	44	15
3914	3208		800	8	26	53	9	17	3	2	21	39
4014	3308		700	7	3	59	43	18	7	11	59	4
4114	3408		600	5	11	6	16	19	11	21	36	27
4214	3508		500	3	18	12	49	20	4	13	25	19
4314	3608		400	1	25	19	23	40	8	26	50	37
4414	3708		300	0	2	25	56	60	1	10	15	56
4514	3808		200	10	9	32	29	80	5	23	41	15
4614	3908		100	8	16	39	3	100	10	7	6	33
4714	4008		1	6	23	45	36	200	8	14	13	7
4814	4108	After Christ.	101	5	0	52	9	300	6	21	19	40
4914	4208		201	3	7	58	43	400	4	28	26	13
5014	4308		301	1	15	5	16	500	3	5	32	47
5114	4408		401	11	22	11	49	1000	6	11	5	33
5214	5508		501	9	29	18	23	2000	0	22	11	6
5714	5008		1001	1	4	51	9	3000	7	3	16	39
6414	5708		1701	0	24	37	2	4000	1	14	22	12
6466	5760		1753	10	9	24	56					
6514	5808		1801	6	5	26	15					

	Complete years.						Months.				
The 4008th year before the year of Christ 1, was the 4007th year before the year of his birth; and is supposed to have been the year of the creation.							Jan.	0	0	0	0
							Feb.	0	17	54	48
	Julian years, 3 of which have 365 days, and the 4th 366.	1	4	9	37	24	Mar.	11	29	15	16
		2	4	19	14	8	April	0	17	10	3
		3	0	28	52	13	May	0	22	53	23
		4	5	20	41	4	June	1	10	40	11
		5	10	0	18	28	July	1	16	31	32
		6	2	9	55	52	Aug.	2	4	6	20
		7	6	19	33	17	Sept.	2	22	21	8
		8	11	11	22	7	Oct.	2	28	4	29
		9	3	20	59	32	Nov.	3	15	59	17
		10	8	0	36	55	Dec.	3	21	42	7

This table agrees with the *old style* until the year 1753; and after that with the *new*.

	Sun from node. °	′	″
Remainder (brought over) - - - -	17	32	56
Next less mean motion for 16 days subtracted	16	37	4
And the remainder will be - - - -		55	52
Next less mean motion for 21 hours subtracted		54	32
Remains, nearly, the mean motion of 31 minutes		1	20

So that, according to the tables, the sun will be in conjunction with the *descending node* on the 16th of September, at 21 hours 31 minutes past noon: one day later than the truth, on account of the leap year.

The limits of *eclipses.*

When the moon changes within 18 days before or after the sun's conjunction with either of the nodes, the sun will be eclipsed at that change: and when the moon is full within 12 days before or after the time of the sun's conjunction with either of the nodes, she will be eclipsed at the full: otherwise not.

Their period and restitution.

If to the mean time of any eclipse, either of the sun or moon, we add 557 Julian years 21 days 18 hours 11 minutes and 51 seconds (in which there are exactly 6890 mean lunations) we shall have the mean time of another eclipse.[1] For at the end of that time the moon will be either new or full, according as we add it to the time of new

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[1] Dr. Halley's period of eclipses contains only 18 years 11 days 7 hours 43 minutes 20 seconds; in which time, according to his tables, there are just 223 mean lunations; but as in that time, the sun's mean motion from the node is no more than $11^s\ 29°\ 31'\ 49''$, which wants 28′ 11″ of being as nearly in conjunction with the same node at the end of the period as it was at the beginning, this period cannot be of constant duration for finding eclipses, because it will in time fall quite without their limits. The following tables make this period 31″ shorter, as appears by the calculation annexed.*

The period.		Moon from the sun. s	°	′	″	Sun from node. s	°	′	″
Complete years,	18	7	11	59	4	11	17	46	18
——— days,	11	4	14	5	54		11	25	29
——— hours,	7		3	33	20			18	11
——— minutes,	42			21	20			1	49
——— seconds,	44				22				2
Mean motions,		0	0	0	0	11	29	31	49

* By computing from the new solar tables of De Lambre, and the lunar tables of Mayer, as improved by Mason, this short period of eclipses, which is generally called the period of Pliny, or the Chaldaic period will amount only to 18 years 11 days 7 hours 42 minutes and 31 seconds; and the sun's distance from the moon's node to 28′ 10″.—*Ed.*

or full moon; and the sun will be only 45″ farther from the same node, at the end of the said time, than he was at the beginning of it; as appears by the following example.[2]

The period.		Moon from sun.				Sun from node.			
		s	°	′	″	s	°	′	″
Complete years	500	3	5	32	47	10	14	45	8
	40	8	26	50	37	1	23	58	49
	17	3	2	21	39	10	28	40	55
days - -	21	8	16	0	21		21	48	38
hours - -	18		9	8	35			46	44
minutes -	11			5	35				29
seconds -	51				26				2
Mean motions		0	0	0	0	0	0	0	45

And this period is so very near, that in 6000 years it will vary no more from the truth as to the restitution of eclipses, than 8¼ minutes of a degree; which may be reckoned next to nothing. It is the shortest in which, after many trials, I can find so near a conjunction of the sun, moon, and the same node.

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[2] The period here mentioned by Mr. Ferguson amounts only to 557 years 21 days 18 hours 4 minutes 47 seconds; and the sun's distance from the moon's node is fully 1′ 41″.—*Ed.*

A Table of the Moon's mean Motion from the Sun.

Days.	Moon from sun.				Moon from sun.				Moon from sun.			
	s	°	′	″	H. M. S.	° ′ ″	′ ″ ‴	″ ‴ ⁗	M. S. Th.	′ ″ ‴	″ ‴ ⁗	‴ ⁗ v
1	0	12	11	27								
2	0	24	22	53								
3	1	6	34	20	1	0	30	29	31	15	44	47
4	1	18	45	47	2	1	0	57	32	16	15	16
5	2	0	57	13	3	1	31	26	33	16	45	44
6	2	13	8	40	4	2	1	54	34	17	16	13
7	2	25	20	7	5	2	32	23	35	17	46	42
8	3	7	31	34	6	3	2	52	36	18	17	10
9	3	19	43	0	7	3	33	20	37	18	47	39
10	4	1	54	27	8	4	3	49	38	19	18	7
11	4	14	5	54	9	4	34	18	39	19	48	36
12	4	26	17	20	10	5	4	46	40	20	19	5
13	5	8	28	47	11	5	35	15	41	20	49	33
14	5	20	40	14	12	6	5	43	42	21	20	2
15	6	2	51	40	13	6	36	12	43	21	50	31
16	6	15	3	7	14	7	6	41	44	22	20	59
17	6	27	14	34	15	7	37	9	45	22	51	28
18	7	9	26	0	16	8	7	38	46	23	21	56
19	7	21	37	27	17	8	38	6	47	23	52	25
20	8	3	48	54	18	9	8	35	48	24	22	54
21	8	16	0	21	19	9	39	4	49	24	53	22
22	8	28	11	47	20	10	9	32	50	25	23	51
23	9	10	23	14	21	10	40	1	51	25	54	19
24	9	22	34	41	22	11	10	30	52	26	24	48
25	10	4	46	7	23	11	40	58	53	26	55	17
26	10	16	57	34	24	12	11	27	54	27	25	45
27	10	29	9	1	25	12	41	55	55	27	56	14
28	11	11	20	27	26	13	12	24	56	28	26	43
29	11	23	31	54	27	13	42	53	57	28	57	11
30	0	5	43	21	28	14	13	21	58	29	27	40
31	0	17	54	48	29	14	43	50	59	29	58	8
32	1	0	6	14	30	15	14	18	60	30	28	37

1 Lunation = $29^{d}\ 12^{h}\ 44^{m}\ 3^{s}\ 6^{th}\ 21^{iv}\ 14^{v}\ 24^{vi}\ 0^{vii}$.

In leap years, after February, a day and its motion must be added to the time for which the moon's mean distance from the sun is given. But when the mean time of any new or full moon is required in leap year after February, a day must be subtracted from the mean time thereof, as found by the tables. In common years they give the day right.

A Table of the Sun's mean Motion from the Moon's ascending node.

Before the year of Christ 1.——After Christ.

Years of the Julian Period.	Years of the World.	Years before and after Christ.	Sun from node. s	°	′	″	Complete years.	Sun from node. s	°	′	″
706	0	4008	7	6	17	9	11	7	2	3	56
714	8	4000	0	11	4	55	12	7	22	11	39
1714	1008	3000	9	10	35	11	13	8	11	17	2
2714	2008	2000	6	10	5	28	14	9	0	22	25
3714	3008	1000	3	9	35	44	15	9	19	27	49
3814	3108	900	7	24	32	46	16	10	9	35	31
3914	3208	800	0	9	29	48	17	10	28	40	55
4014	3308	700	4	24	26	49	18	11	17	46	18
4114	3408	600	9	9	23	51	19	0	6	51	43
4214	3508	500	1	24	20	53	20	0	26	59	24
4314	3608	400	6	9	17	54	40	1	23	58	49
4414	3708	300	10	24	14	56	60	2	20	58	13
4514	3808	200	3	9	11	58	80	3	17	57	37
4614	3908	100	7	24	8	59	100	4	14	57	2
4714	4008	1	0	9	6	1	200	8	29	54	3
4814	4108	101	4	24	3	3	300	1	14	51	5
4914	4208	201	9	9	0	4	400	5	29	48	7
5014	4308	301	1	23	57	6	500	10	14	45	8
5114	4408	401	6	8	54	8	1000	8	29	30	17
5214	4508	501	10	23	51	9	2000	5	29	0	33
5714	5008	1001	9	8	36	18	3000	2	28	30	50
6414	5708	1701	4	23	15	30	4000	11	28	1	6
6466	5760	1753	1	28	0	19					
6514	5808	1801	8	25	44	44					

The 4008th year before the year of Christ 1, was the 4007th year before the year of his birth; and is supposed to have been the year of the creation.

Julian years, 3 of which have 365 days, and the 4th 366.

Complete years.	s	°	′	″	Months.	s	°	′	″
					Jan.	0	0	0	0
					Feb.	1	2	11	48
1	0	19	5	23	Mar.	2	1	16	39
2	1	8	10	47	April	3	3	28	27
3	1	27	16	10	May	4	4	37	57
4	2	17	23	53	June	5	6	49	45
5	3	6	29	16	July	6	7	59	14
6	3	25	34	40	Aug.	7	9	11	1
7	4	14	40	3	Sept.	8	12	22	49
8	5	4	47	46	Oct.	9	13	32	18
9	5	23	53	9	Nov.	10	15	44	5
10	6	12	58	33	Dec.	11	16	53	34

This table agrees with the *old style* until the year 1753; and after that, with the *new*.

A Table of the Sun's mean Motion from the Moon's ascending node.

Days.	Sun from node. s ° ′ ″	H. M. S.	Sun from node. ° ′ ″ ′ ″ ‴ ″ ‴ ⁗	M. S. Th.	Sun from node. ′ ″ ‴ ″ ‴ ⁗ ‴ ⁗ v
1	0 1 2 19				
2	0 2 4 38				
3	0 3 6 57	1	0 2 36	31	1 20 31
4	0 4 9 16	2	0 5 12	32	1 23 7
5	0 5 11 36	3	0 7 48	33	1 25 43
6	0 6 13 54	4	0 10 23	34	1 28 9
7	0 7 16 13	5	0 12 59	35	1 31 55
8	0 8 18 32	6	0 15 35	36	1 33 31
9	0 9 20 51	7	0 18 11	37	1 36 6
10	0 10 23 10	8	0 20 47	38	1 38 42
11	0 11 25 29	9	0 23 23	39	1 41 18
12	0 12 27 48	10	0 25 58	40	1 43 54
13	0 13 30 7	11	0 28 33	41	1 46 36
14	0 14 32 26	12	0 31 9	42	1 49 5
15	0 15 34 15	13	0 33 45	43	1 51 41
16	0 16 37 4	14	0 36 21	44	1 54 17
17	0 17 39 23	15	0 38 57	45	1 56 53
18	0 18 41 41	16	0 41 32	46	1 59 29
19	0 19 44 0	17	0 44 8	47	2 2 5
20	0 20 46 19	18	0 46 44	48	2 4 41
21	0 21 48 38	19	0 49 20	49	2 7 17
22	0 22 50 57	20	0 51 56	50	2 9 53
23	0 23 53 16	21	0 54 32	51	2 12 29
24	0 24 55 35	22	0 57 8	52	2 15 5
25	0 25 57 54	23	0 59 43	53	2 17 41
26	0 27 0 13	24	1 2 19	54	2 20 17
27	0 28 2 32	25	1 4 55	55	2 22 53
28	0 29 4 51	26	1 7 31	56	2 25 29
29	1 0 7 10	27	1 10 7	57	2 28 4
30	1 1 9 29	28	1 12 43	58	2 30 40
31	1 2 11 48	29	1 15 9	59	2 33 16
32	1 3 14 47	30	1 17 55	60	2 35 52

In leap years, after February, add one day and one day's motion to the time at which the sun's mean distance from the ascending node is required.

A SUPPLEMENT

TO THE

PRECEDING LECTURES,

BY THE AUTHOR.

MECHANICS.

THE DESCRIPTION OF A NEW AND SAFE CRANE, WHICH HAS FOUR DIFFERENT POWERS, ADAPTED TO DIFFERENT WEIGHTS.[1]

Description of a new crane.

THE common crane consists only of a large wheel and axle; and the rope, by which goods are drawn up from ships, or let down from the quay to them, winds or coils round the axle, as the axle is turned by men walking in the wheel. But, as these engines have nothing to stop the weight from running down, if any of the men happen to trip or fall in the wheel, the weight descends, and turns the wheel rapidly backward, and tosses the men violently about within it; which has produced melancholy instances, not only of limbs broke, but even of lives lost, by the ill-judged construction of cranes. And besides, they have but one power for all sorts of weights; so that they generally spend as much time in raising a small weight as in raising a great one.

These imperfections and dangers induced me to think of a method for remedying them. And for that purpose, I contrived a crane with a proper stop to prevent the danger, and with different powers suited to different weights; so that there might be as little loss of time as possible: and also, that when heavy goods are let down into ships, the descent may be regular and deliberate.

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[1] Our author received a reward of fifty pounds for the invention of this crane, from the Society for the encouragement of Arts; and a description of it was honoured with a place in the *Transactions of the Royal Society of London*. See vol. xlv, p. 42.—*Ed.*

This crane has four different powers: and, I believe it might be built in a room eight feet in width: the gib being on the outside of the room.

Three trundles, with different numbers of staves, are applied to the cogs of a horizontal wheel with an upright axle; and the rope that draws up the weight coils round the axle. The wheel has ninety-six cogs, the largest trundle twenty-four staves, the next largest has twelve, and the smallest has six. So that the largest trundle makes four revolutions for one revolution of the wheel: the next makes eight, and the smallest makes sixteen. A winch is occasionally put upon the axis of either of these trundles, for turning it; the trundle being then used that gives a power best suited to the weight: and the handle of the winch describes a circle in every revolution equal to twice the circumference of the axle of the wheel. So that the length of the winch doubles the power gained by each trundle.

As the power gained by any machine, or engine whatever, is, in direct proportion, as the velocity of the power is to the velocity of the weight; the powers of this crane are easily estimated, and they are as follows.

If the winch be put upon the axle of the largest trundle, and turned four times round, the wheel and axle will be turned once round: and the circle described by the power that turns the winch, being, in each revolution, double the circumference of the axle, when the thickness of the rope is added thereto; the power goes through eight times as much space as the weight rises through: and therefore (making some allowance for friction) a man will raise eight times as much weight by the crane as he would by his natural strength without it: the power, in this case, being as eight to one.

If the winch be put upon the axis of the next trundle, the power will be as sixteen to one, because it moves sixteen times as fast as the weight moves.

If the winch be put upon the axis of the smallest trundle, and turned round, the power will be as thirty-two to one.

But if the weight should be too great, even for this power to raise, the power may be doubled by drawing up the weight by one of the parts of a double rope, going under a pulley in the moveable block, which is hooked to the weight below the arm of the gib; and then the power will be as sixty-four to one. That is, a man could then raise sixty-four times as much weight by the crane as he could raise by his natural strength without it; because, for every inch that the weight rises, the working power will move through sixty-four inches.

By hanging a block with two pullies to the arm of the gib, and having two pullies in the moveable block that rises with the weight, the rope being doubled over and under these pullies, the power of the crane will be as 128 to one. And so, by increasing the number of pullies, the power may be increased as much as you please: always remembering, that the larger the pullies are, the less is their friction.

While the weight is drawing up, the ratch-teeth of a wheel slip round below a catch or click that falls successively into them, and so hinders the crane from turning backward, and detains the weight in any part of its ascent, if the man who works at the winch should accidentally happen to quit his hold, or choose to rest himself before the weight be quite drawn up.

In order to let down the weight, a man pulls down one end of a lever of the second kind, which lifts the catch of the ratchet-wheel, and gives the weight liberty to descend. But, if the descent be too quick, he pulls the lever a little farther down, so as to make it rub against the outer edge of a round wheel; by which means he lets down the weight as slowly as he pleases: and, by pulling a little harder, he may stop the weight, if needful, in any part of its descent. If he accidentally quits hold of the lever, the catch immediately falls, and stops both the weight and the whole machine.

This crane is represented in Plate XI, Fig. 1, where *A* is the great wheel, and *B* its axle on which the rope *C* winds. This rope goes over a pulley *D* in the end of the arm of the gib *E*, and draws up the weight *F*, as the winch *G* is turned round. *H* is the largest trundle, *I* the next, and *K* is the axis of the smallest trundle, which is supposed to be hid from view by the upright supporter *L*. A trundle *M* is turned by the great wheel, and on the axis of this trundle is fixed the ratchet-wheel *N*, into the teeth of which the catch *O* falls. *P* is the lever, from which goes a rope *Q Q*, over a pulley *R* to the catch; one end of the rope being fixed to the lever, and the other end to the catch. *S* is an elastic bar of wood, one end of which is screwed to the floor: and from the other end goes a rope (out of sight in the figure) to the further end of the lever, beyond the pin or axis on which it turns in the upright supporter *T*. The use of this bar is to keep up the lever from rubbing against the edge of the wheel *U*, and to let the catch keep in the teeth of the ratchet-wheel: but a weight hung to the farther end of the lever would do full as well as the elastic bar and rope.

When the lever is pulled down, it lifts the catch out of the ratchet-wheel, by means of the rope *Q Q*, and gives the weight *F* liberty to descend: but if the lever *P* be pulled a little farther down than what is sufficient to lift the catch *O* out of the ratchet wheel *N*, it will rub against the edge of the wheel *U*, and thereby hinder the too quick descent of the weight; and will quite stop the weight if pulled hard. And if the man who pulls the lever, should happen inadvertently to let it go, the elastic bar will suddenly pull it up, and the catch will fall down and stop the machine.

W W are two upright rollers above the axis or upper gudgeon of the gib *E*; their use is to let the rope *C* bend upon them, as the gib is turned to either side, in order to bring the weight over the place where it is intended to be let down.

N. B. The rollers ought to be so placed, that if the rope *C* be stretched close by their utmost sides, the half thickness of the rope may be perpendicularly over the centre of the upper gudgeon of the gib. For then, and in no other position of the rollers, the length of the rope between the pulley in the gib and the axle of the great wheel will be always the same, in all positions of the gib: and the gib will remain in any position to which it is turned.

When either of the trundles is not turned by the winch in working the crane, it may be drawn off from the wheel, after the pin near the axis of the trundle is drawn out, and the thick piece of wood is raised a little behind the outward supporter of the axis of the trundle. But this is not material; for, as the trundle has no friction on its axis but what is occasioned by its weight, it will be turned by the wheel without any sensible resistance in working the crane.[2]

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[2] A full account of various new and ingenious cranes, particularly Bramah's Hydrostatic Crane, will be found in the *Edinburgh Encyclopædia*, Art. *Crane*, vol. vii, p. 312.

A PYROMETER, THAT MAKES THE EXPANSION OF METALS BY HEAT VISIBLE TO THE FIVE-AND-FORTY THOUSANDTH PART OF AN INCH.

Description of a new pyrometer.

The upper surface of this machine is represented by Fig. 2 of Plate XI. Its frame *A B C D* is made of mahogany wood, on which is a circle divided into 360 equal parts; and within that circle is another, divided into eight equal parts. If the short bar *E* be pushed one inch forward (or toward the centre of the circle) the index *e* will be turned 125 times round the circle of 360 parts or degrees. As 125 times 360 is 45,000, it is evident, that if the bar *E* be moved only the 45,000th part of an inch, the index will move one degree of the circle. But as in my pyrometer the circle is nine inches in diameter, the motion of the index is visible to half a degree, which answers to the ninety thousandth part of an inch in the motion or pushing of the short bar *E*.

One end of a long bar of metal *F* is laid into a hollow place in a piece of iron *G*, which is fixed to the frame of the machine; and the other end of this bar is laid against the end of the short bar *E*, over the supporting cross bar *H I*: and, as the end *f* of the long bar is placed close against the end of the short bar, it is plain, that if *F* expands, it will push *F* forward, and turn the index *e*.

The machine stands on four short pillars, high enough from a table to let a spirit-lamp be put on the table under the bar *F*; and when that is done, the heat of the flame of the lamp expands the bar, and turns the index.

There are bars of different metals, as silver, brass, and iron, all of the same length as the bar *F*, for trying experiments on the different expansions of different metals, by equal degrees of heat applied to them for equal lengths of time; which may be measured by a pendulum, that swings seconds. Thus, put on the brass bar *F*, and set the index to the 360th degree: then put the lighted lamp under the bar, and count the number of seconds in which the index goes round the plate, from 360 to 360 again; and then blow out the lamp, and take away the bar.

Method of using it.

This done, put on an iron bar *F* where the brass one was before, and then set the index to the 360th degree again. Light the lamp and put it under the iron bar, and let it remain just as many seconds as it did under the brass one; and then blow it out, and you will see how many degrees the index has moved in the circle; and by that means you will know in what proportion the expansion of iron is to the expansion of brass; which I find to be as 210 is to 360, or as seven is to twelve. By this method, the relative expansions of different metals may be found.

The bars ought to be exactly of equal size; and to have them so, they should be drawn, like wire, through a hole.

When the lamp is blown out, you will see the index turn backward: which shews that the metal contracts as it cools.

The inside of this pyrometer is constructed as follows:—

In Fig. 3, Plate XI, *A a* is the short bar which moves between rollers; and, on the side *a* it has fifteen teeth in an inch, which take into the leaves of a pinion *B* (twelve in number) on whose axis is the wheel *C* of 100 teeth, which take into the ten leaves of the pinion *D*, on whose axis is the wheel *E* of 100 teeth, which take into the ten leaves of the pinion *F*, on the top of whose axis is the index above mentioned.

Now, as the wheels *C* and *E* have 100 teeth each; and the pinions *D* and *F* have ten leaves each, it is plain, that if the wheel *C* turns once round, the pinion *F* and the index on its axis will turn 100 times round. But, as the first pinion *B* has only twelve leaves, and the bar *A a* that turns it has fifteen teeth in an inch, which is twelve and a fourth part more; one inch motion of the bar will cause the last pinion *F* to turn a hundred times round, and a fourth part of a hundred over and above, which is twenty-five. So that if *A a* be pushed one inch, *F* will be turned 125 times round.

A silk thread *b* is tied to the axis of the pinion *D*, and wound several times round it; and the other end of the thread is tied to a piece of slender watch-spring *G*, which is fixed into the stud *H*. So that as the bar *f* expands, and pushes the bar *A a* forward, the thread winds round the axle, and draws out the spring: and as the bar contracts, the spring pulls back the thread, and turns the work the contrary way, which pushes back the short bar *A a* against the long bar *f*. This spring always keeps the teeth of the wheels in contact with the leaves of the pinions, and so prevents any shake in the teeth.

In Fig. 1, the eight divisions of the inner circle are so many thousandth parts of an inch in the expansion or contraction of the bars; which is just one thousandth part of an inch for each division moved over by the index.

A WATER-MILL INVENTED BY DR. BARKER, THAT HAS NEITHER WHEEL NOR TRUNDLE.

Barker's water-mill.

This machine is represented by Fig. 4 of Plate XI, in which *A* is a pipe or channel that brings water to the upright tube *B*. The water runs down the tube, and thence into the horizontal trunk *C*, and runs out through holes at *d* and *e* near the ends of the trunk on the contrary sides thereof.

The upright spindle *D* is fixed in the bottom of the trunk, and screwed to it below by the nut *g*; and is fixed into the trunk by two cross bars at *f*: so that, if the tube *B* and trunk *C* be turned round, the spindle *D* will be turned also.

The top of the spindle goes square into the rynd of the upper mill-stone *H*, as in common mills; and, as the trunk, tube, and spindle, turn round, the mill-stone is turned round thereby. The lower, or quiescent, mill-stone is represented by *I*; and *K* is the floor on which it rests, and wherein is the hole *L* for letting the meal run through, and fall down into a trough, which may be about *M*. The hoop or case that goes round the mill-stone rests on the floor *K*, and supports the hopper, in the common way. The lower end of the spindle turns in a hole in the bridge-tree *G F*, which supports the millstone, tube, spindle, and trunk. This tree is moveable on a pin at *b*, and its other end is supported by an iron rod *N* fixed into it, the top of the rod going through the fixed bracket *O*, and having a screw nut *o* upon it, above the bracket. By turning this nut forward or backward, the mill-stone is raised or lowered at pleasure.

While the tube *B* is kept full of water from the pipe *A*, and the water continues to run out from the ends of the trunk; the upper mill-stone *H*, together with the trunk, tube, and spindle, turns round. But, if the holes in the trunk were stopped, no motion would ensue; even though the tube and trunk were full of water. For, if there were no hole in the trunk, the pressure of the water would be equal against all parts of its sides within. But, when the water has free egress through the holes, its pressure there is entirely removed: and the pressure against the parts of the sides which are opposite to the holes, turns the machine.[2]

HYDROSTATICS.

A MACHINE FOR DEMONSTRATING THAT, ON EQUAL BOTTOMS, THE PRESSURE OF FLUIDS IS IN PROPORTION TO THEIR PERPENDICULAR HEIGHTS, WITHOUT ANY REGARD TO THEIR QUANTITIES.

Hydrostatical Paradox.

THIS is termed *the Hydrostatical Paradox:* and the machine for shewing it is represented in Fig. 1 of Plate XII. In which *A* is a box that holds about a pound of water, *a b c d e* a glass tube fixed in the top of the box, having a small wire within it; one end of the wire being hooked to the end *F* of the beam of a balance, and the other end of the wire fixed to a moveable bottom, on which the water lies, within the box; the bottom and wire being of equal weight with an empty scale (out of sight in the figure) hanging at the other end of the balance. If this scale be pulled down, the bottom will be drawn up within the box, and that motion will cause the water to rise in the glass tube.

Put one pound weight into the scale, which will move the bottom a little, and cause the water to appear just in the lower end of the tube at *a;* which shews that the water presses with the force of one pound on the bottom; put another pound into the scale, and the water will rise from *a* to *b* in the tube, just twice as high above the bottom as it was when at *a;* and then, as its pressure on the bottom supports two pound weight in the scale, it is plain that the pressure on the bottom is then equal to two pounds. Put a third pound weight in the scale, and the water will be raised from *b* to *c* in the tube, three times as high above the bottom as when it began to appear in the tube at *a;* which shews, that the same quantity of water that pressed but with the force of one pound on the bottom, when raised no higher than *a*, presses with the force of three pounds on the bottom when raised three times as high to *c* in the tube. Put a fourth pound weight into the scale, and it will cause the water to rise in the tube from *c* to *d*, four times as high as when it was all contained in the box, which shews that its pressure then upon the bottom is four times as great as when it lay all within the box. Put a fifth pound weight into the scale, and the water will rise in the tube from *d* to *e*, five times as high as it was above the bottom, before it rose in the tube; which shews that its pressure on the bottom is then equal to five pounds, seeing that it supports so much weight in the scale. And so on, if the tube was still longer; for it would still require an additional pound put into the scale to raise the water in the tube to an additional height equal to the space *d e;* even if the bore of the tube was so small as only to let the wire move freely within it, and leave room for any water to get round the wire.

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[2] See the *Appendix* for farther information on the construction of Dr. Barker's mill.—*Ed.*

Hence we infer, that if a long narrow pipe or tube was fixed in the top of a cask full of liquor, and if as much liquor was poured into the tube as would fill it, even though it were so small as not to hold an ounce weight of liquor; the pressure arising from the liquor in the tube would be as great upon the bottom, and be in as much danger of bursting it out, as if the cask was continued up in its full size, to the height of the tube, and filled with liquor.

Solution of the paradox.

In order to account for this surprising affair, we must consider that fluids press equally in all manner of directions: and consequently that they press just as strongly upward as they do downward. For, if another tube, as *f*, be put into a hole made into the top of the box, and the box be filled with water; and then, if water be poured in at the top of the tube *a b c d e*, it will rise in the tube *f* to the same height as it does in the other tube: and if you leave off pouring, when the water is at *c*, or any other place in the tube *a b c d e*, you will find it just as high in the tube *f:* and if you pour in water to fill the first tube, the second will be filled also.

Now, it is evident, that the water rises in the tube *f*, from the downward pressure of the water in the tube *a b c d e*, on the surface of the water, contiguous to the inside of the top of the box; and as it will stand at equal heights in both tubes, the upward pressure in the tube *f* is equal to the downward pressure in the other tube. But, if the tube *f* were put in any other part of the top of the box, the rising of the water in it would still be the same: or, if the top was full of holes, and a tube put into each of them, the water would rise as high in each tube as it was poured into the tube *a b c d e*; and then the moveable bottom would have the weight of the water in all the tubes to bear, beside the weight of all the water in the box.

And seeing that the water is pressed upward into each tube, it is evident that, if they be all taken away, excepting the tube *a b c d e*, and the holes in which they stood be stopped up; each part, thus stopped, will be pressed as much upward, as was equal to the weight of water in each tube. So that, the upward pressure against the inside of the top of the box, on every part equal in breadth to the width of the tube *a b c d e*, will be pressed upward with a force equal to the whole weight of water in the tube. And consequently, the whole upward pressure against the top of the box, arising from the weight or downward pressure of the water in the tube, will be equal to the weight of a column of water of the same height with that in the tube, and of the same thickness as the width of the inside of the box: and this upward pressure against the top will re-act downward against the bottom, and be as great thereon, as would be equal to the weight of a column of water as thick as the moveable bottom is broad, and as high as the water stands in the tube. And thus, the paradox is solved.

The moveable bottom has no friction against the inside of the box, nor can any water get between it and the box. The method of making it so, is as follows:—

Construction of the moveable bottom.

In Fig. 2, *A B C D* represents a section of the box, and *a b c d* is the lid or top thereof, which goes on tight, like the lid of a common paper snuff-box. *E* is the moveable bottom, with a groove around its edge, and it is put into a bladder *f g*, which is tied close around it in the groove by a strong waxed thread; the bladder coming up like a purse within the box, and put over the top of it at *a* and *d* all round, and then the lid pressed on. So that, if water be poured in through the hole *l l* of the lid, it will lie upon the bottom *E*, and be contained in the space *f E g h* within the bladder; and the bottom may be raised by pulling the wire *i*, which is fixed to it at *E*; and by thus pulling the wire, the water will be lifted up in the tube *k*, and as the bottom does not touch the inside of the box, it moves without friction.

Now, suppose the diameter of this round bottom to be three inches (in which case, the area thereof will be nine circular inches), and the diameter of the bore of the tube to be a quarter of an inch; the whole area of the bottom will be 144 times as great as the area of the top of a pin that would fill the tube like a cork.

And hence it is plain, that if the moveable bottom be raised only the 144th part of an inch, the water will thereby be raised a whole inch in the tube; and consequently, that if the bottom be raised one inch, it would raise the water to the top of a tube 144 inches, or twelve feet in height.

N. B. The box must be open below the moveable bottom, to let in the air. Otherwise, the pressure of the atmosphere would be so great upon the moveable bottom, if it be three inches in diameter, as to require 108 pounds in the scale, to balance that pressure, before the bottom could begin to move.

A MACHINE, TO BE SUBSTITUTED IN PLACE OF THE COMMON HYDROSTATICAL BELLOWS.

Substitute for the hydrostatical bellows.

In Fig. 3 of Plate XII, *A B C D* is an oblong square box, in one end of which is a round groove, as at *a*, from top to bottom, for receiving the upright glass tube *I*, which is bent to a right angle at the lower end (as at *i* in Fig. 4), and to that part is tied the neck of a large bladder *K* (Fig. 4), which lies in the bottom of the box. Over this bladder is laid the moveable board *L* (Fig. 3 and 5), in which is fixed an upright wire *M;* and leaden weights *N N,* to the amount of sixteen pounds, with holes in their middle, which are put upon the wire, over the board, and press upon it with all their force.

The cross bar *p* is then put on, to secure the tube from falling, and keep it in an upright position: and then the piece *E F G* is to be put on, the part *G* sliding tight into the dovetailed groove *H*, to keep the weights *N N* horizontal, and the wire *M* upright; there being a round hole *e* in the part *E F* for receiving the wire.

There are four upright pins in the four corners of the box within, each almost an inch long, for the board *L* to rest upon: to keep it from pressing the sides of the bladder below it close together at first.

The whole machine being thus put together, pour water into the tube at top; and the water will run down the tube into the bladder below the board; and after the bladder has been filled up to the board, continue pouring water into the tube, and the upward pressure which it will excite in the bladder, will raise the board with all the weight upon it, even though the bore of the tube should be so small, that less than an ounce of water would fill it.[3]

This machine acts upon the same principle as the one last described, concerning the *Hydrostatical paradox.* For, the upward pressure against every part of the board (which the bladder touches), equal in area to the area of the bore of the tube, will be pressed upward with a force equal to the weight of the water in the tube; and the sum of all these pressures against so many areas of the board, will be sufficient to raise it with all the weights upon it.

In my opinion, nothing can exceed this simple machine, in making the upward pressure of fluids evident to sight.

THE CAUSE OF RECIPROCATING SPRINGS, AND OF EBBING AND FLOWING WELLS, EXPLAINED.[4]

Cause of reciprocating springs.

In Fig. 7 of Plate XII, let *a b c d* be a hill, within which is a large cavern *A A* near the top, filled or fed by rains and melted snow on the top *a*, making their way through chinks and crannies into the said cavern, from which proceeds a small stream *C C* within the body of the hill, and issues out in a spring at *G* on the side of the hill, which will run constantly while the cavern is fed with water.

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[3] Upon this principle, it has been justly affirmed by some writers on natural philosophy, that a certain quantity of water, however small, may be rendered capable of exerting a force equal to any assignable one, by increasing the height of the column, and diminishing the base on which it presses. Dr. Goldsmith observes, that he has seen a strong hogshead split in this manner. A small, though strong tube of tin, twenty feet high, was inserted in the bung-hole of the hogshead. Water was then poured into the tube till the hogshead was filled, and the water had reached within a foot of the top of the tin tube. By the pressure of this column of water, the hogshead burst with incredible force, and the water was scattered in every direction. By diminishing the area of the tube one half, or doubling its height, the same quantity of water would have a double force.—*Ed.*

[4] Dr. Atwell of Oxford seems to have been the first person that pointed out the cause of reciprocating springs. The theory of this gentleman, of which the article in the text is an abridgment, was published in Number 424 of the *Philosophical Transactions*, and was suggested by the phenomena of *Laywell* spring, at *Brixam*, in Devonshire. See Desagulier's *Experimental Philosophy*, vol. ii, p. 173, and p. 92 of this work.—*Ed.*

From the same cavern *A A*, let there be a small channel *D*, to carry water into the cavern *B*; and from that cavern let there be a bended channel *E e F*, larger than *D*, joining with the former channel *C C*, as at *f* before it comes to the side of the hill; and let the joining at *f* be below the level of the bottom of both these caverns.

As the water rises in the cavern *B*, it will rise as high in the channel *E e F*: and when it rises to the top of that channel at *e*, it will run down the part *e F G*, and make a swell in the spring *G*, which will continue till all the water is drawn off from the cavern *B*, by the natural syphon *E e F* (which carries off the water faster from *B* than the channel *D* brings water to it), and then the swell will stop, and only the small channel *C C* will carry water to the spring *G*, till the cavern *B* is filled to *B* again by the rill *D*; and then the water being at the top *e* of the channel *E e F*, that channel will act again as a syphon, and carry off all the water from *B* to the spring *G*, and so make a swelling flow of water at *G* as before.

Illustrated by a machine.

To illustrate this by a machine (Fig. 8), let *A* be a large wooden box, filled with water; and let a small pipe *C C* (the upper end of which is fixed into the bottom of the box) carry water from the box to *G*, where it will run off constantly, like a small spring. Let another small pipe *D* carry water from the same box to the box or well *B*, from which let a syphon *E e F* proceed, and join with the pipe *C C* at *f*: the bore of the syphon being larger than the bore of the feeding-pipe *D*. As the water from this pipe rises in the well *B*, it will also rise as high in the syphon *E e F*; and when the syphon is full to the top *e*, the water will run over the bend *e*, down the part *e F*, and go off at the mouth *G*; which will make a great stream at *G*: and that stream will continue, till the syphon has carried off all the water from the well *B*; the syphon carrying off the water faster from *B* than the pipe *D* brings water to it: and then the swell at *G* will cease, and only the water from the small pipe *C C* will run off at *G*, till the pipe *D* fills the well *B* again; and then the syphon will run, and make a swell at *G* as before.

And thus, we have an artificial representation of an ebbing and flowing well, and of a reciprocating spring, in a very natural and simple manner.

HYDRAULICS.

AN ACCOUNT OF THE PRINCIPLES BY WHICH MR. BLAKEY PROPOSES TO RAISE WATER FROM MINES, OR FROM RIVERS, TO SUPPLY TOWNS, AND GENTLEMEN'S SEATS, BY HIS NEW-INVENTED FIRE-ENGINE, FOR WHICH HE HAS RECEIVED HIS MAJESTY'S LETTERS PATENT.

Blakey's fire-engine.

ALTHOUGH I am not at liberty to describe the whole of this simple engine, yet I have the patentee's leave to describe such a one as will shew the principles by which it acts.

In Fig. 6 of Plate XII, let *A* be a large, strong, close vessel, immersed in water up to the cock *b*, and having a hole in the bottom, with a valve *a* upon it, opening upward within the vessel. A pipe *B C* rises from the bottom of this vessel, and has a cock *c* in it near the top, which is small there, for playing a very high jet *d*. *E* is the little boiler (not so big as a common tea-kettle), which is connected with the vessel *A* by the steam-pipe *F*; and *G* is a funnel, through which a little water must be occasionally poured into the boiler, to yield a proper quantity of steam; and a small quantity of water will do for that purpose, because steam possesses upward of 14,000 times as much space or bulk as the water does from which it proceeds.

The vessel *A* being immersed in water up to the cock *b*, open that cock, and the water will rush in through the bottom of the vessel at *a*, and fill it as high up as the water stands on its outside: and the water, coming into the vessel, will drive the air out of it (as high as the water rises within it) through the cock *b*. When the water has done rushing into the vessel, shut the cock *b*, and the valve *a* will fall down, and hinder the water from being pushed out that way, by any force that presses on its surface. All the part of the vessel above *b* will be full of common air when the water rises to *b*.

Shut the cock *c*, and open the cocks *d* and *e*; then pour as much water into the boiler *E* (through the funnel *G*) as will about half fill the boiler; and then shut the cock *d*, and leave the cock *e* open.

This done, make a fire under the boiler *E*, and the heat thereof will raise a steam from the water in the boiler; and the steam will make its way thence, through the pipe *F*, into the vessel *A*; and the steam will compress the air (above *b*) with a very great force upon the surface of the water in *A*.

When the top of the vessel *A* feels very hot by the steam under it, open the cock *c* in the pipe *C*; and the air being strongly compressed in *A*, between the steam and the water therein, will drive all the water out of the vessel *A*, up the pipe *B C*, from which it will fly up in a jet to a very great height. In my fountain, which is made in this manner after Mr. Blakey's, three tea-cup-fulls of water in the boiler will afford steam enough to play a jet thirty feet high.

When all the water is out of the vessel *A*, and the compressed air begins to follow the jet, open the cocks *b* and *d* to let the steam out of the boiler *E* and vessel *A*, and shut the cock *e* to prevent any more steam from getting into *A*; and the air will rush into the vessel *A* through the cock *b*, and the water through the valve *a*: and so the vessel will be filled with water, up to the cock *b* as before. Then shut the cock *b*, and the cocks *c* and *d*, and open the cock *e*; and then the next steam that rises in the boiler will make its way into the vessel *A* again; and the operation will go on, as above.

When all the water in the boiler is evaporated, and gone off into steam, pour a little more into the boiler, through the funnel *G*.

In order to make this engine raise water to any gentleman's house, if the house be on the bank of a river, the pipe *B C* may be continued up to the intended height, in the direction *H I*. Or, if the house be on the side or top of a hill, at a distance from the river, the pipe, through which the water is forced up, may be laid along on the hill, from the river or spring to the house.

The boiler may be fed by a small pipe *K*, from the water that rises in the main pipe *B C H I*: the pipe *K* being of a very small bore, so as to fill the funnel *G* with water in the time that the boiler *E* will require a fresh supply. And then, by turning the cock *d*, the water will fall from the funnel into the boiler. The funnel should hold as much water as will about half fill the boiler.

When either of these methods of raising water, perpendicularly or obliquely, is used, there will be no occasion for having the cock *c* in the main pipe *B C H I*: for such a cock is requisite only when the engine is used as a fountain.

A contrivance may be very easily made, from a lever to the cocks *b*, *d*, and *e*; so that, by pulling the lever, the cocks *b* and *d* may be opened when the cock *e* must be shut; and the cock *e* be opened when *b* and *d* must be shut.

Boiler.

The boiler *E* should be inclosed in a brick wall, at a little distance from it, all around; to give liberty for the flames of the fire under the boiler to ascend round about it. By which means (the wall not covering the funnel *G*) the force of the steam will be prodigiously increased by the heat round the boiler; and the funnel and water in it will be heated from the boiler; so that the boiler will not be chilled by letting cold water into it; and the rising of the steam will be so much the quicker.

Mr. Blakey is the only person who ever thought of making use of air as an intermediate body between steam and water: by which means, the steam is always kept from touching the water, and consequently from being condensed by it. And on this new principle he has obtained a patent: so that no one (vary the engine how he will) can make use of the air between steam and water, without infringing on the patent, and being subject to the penalties of the law.

This engine may be built for a trifling expense, in comparison of the common fire-engine now in use. It will seldom need repairs, and will not consume half so much fuel. As it has no pumps with pistons, it is clear of all their friction: and the effect is equal to the whole strength or compressive force of the steam; which the effect of the common fire-engine never is, on account of the great friction of the pistons in their pumps.[5]

ARCHIMEDES'S SCREW-ENGINE FOR RAISING WATER.

Archimedes's screw-engine.

In Fig. 1 of Plate XIII, *A B C D* is a wheel, which is turned round, according to the order of the letters, by the fall of water *E F*, which need not be more than three feet. The axle *G* of the wheel is elevated so as to make an angle of about 44° with the horizon; and on the top of that axle is a wheel *H*, which turns such another wheel *I* of the same number of teeth: the axle *K* of this last wheel being parallel to the axle *G* of the two former wheels.

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[5] A full account of steam-engines on the newest construction will be found in the *Appendix*, Vol. II.—*Ed.*

The axle *G* is cut into a double-threaded screw (Fig. 2), exactly resembling the screw on the axis of the fly of a common jack, which must be (what is called) a right-handed screw, like the wood-screws, if the first wheel turns in the direction *A B C D;* but must be a left-handed screw, if the stream turns the wheel the contrary way. And, whichever way the screw on the axle *G* be cut, the screw on the axle *K* must be cut the contrary way; because these axles turn in contrary directions.

The screws being thus cut, they must be covered close over with boards, like those of a cylindrical cask; and then they will be spiral tubes. Or, they may be made of tubes of stiff leather, and wrapt round the axles in shallow grooves cut therein, as in Fig. 3.

The lower end of the axle *G* (Fig. 1) turns constantly in the stream that turns the wheel, and the lower ends of the spiral tubes are open into the water; so that, as the wheel and axle are turned round, the water rises in the spiral tubes, and runs out at *L*, through the holes *M N*, as they come about below the axle. These holes (of which there may be any number, as four or six) are in a broad close ring on the top of the axle, into which ring the water is delivered from the upper open ends of the screw-tubes, and falls into the open box *O*.

The lower end of the axle *K* turns on a gudgeon, in the water in *O;* and the spiral tubes in that axle take up the water from *O*, and deliver it into such another box under the top of *K;* on which there may be such another wheel as *I*, to turn a third axle by such a wheel upon it. And in this manner, water may be raised to any given height, when there is a stream sufficient for that purpose to act on the broad float-boards of the first wheel.[6]

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[6] As Mr. Ferguson has not explained the reason why the water rises in the spirals of the screw engine, we hope the reader will understand it from the following remarks. When the screw *B F*, in Fig. 3, Plate XIII, is in a vertical position, the spiral excavations will be inclined to the horizon, and if a portion of water be introduced at the top *A*, it will descend to *F*, the bottom of the tube. If the screw be in a horizontal position, and the water introduced at *B*, it will fall to *C*, and remain there. But if the screw be turned upon its axis from *B* towards *A*, so that the lowest point *C* of the tube may ascend to *D*, while the point *B* is depressed to

A QUADRUPLE PUMP-MILL FOR RAISING WATER.

Quadruple pump-mill.

This engine is represented in Plate XIII, Fig. 4, in which *A B C D* is a wheel, turned by water according to the order of the letters. On the horizontal axis are four small wheels, toothed almost half round: and the parts of their edges on which there are no teeth are cut down so as to be even with the bottoms of the teeth where they stand.

The teeth of these four wheels take alternately into the teeth of four racks, which hang by two chains over the pullies *Q* and *L*; and to the lower ends of these racks there are four iron rods fixed, which go down into the four forcing pumps, *S*, *R*, *M*, and *N*. And, as the wheels turn, the rack and pump-rods are alternately moved up and down.

Thus, suppose the wheel *G* has pulled down the rack *I*, and drawn up the rack *K* by the chain; as the last tooth of *G* just leaves the uppermost tooth of *I*, the first tooth of *H* is ready to take into the lowermost tooth of the rack *K*, and pull it down as far as the teeth go; and then the rack *I* is pulled upward through the whole space of its teeth, and the wheel *G* is ready to take hold of it, and pull it down again, and so draw up the

C, the water will, by its own gravity, move from *C* to *B*, where it will be discharged: so that water introduced into one extremity of the screw-engine, in a horizontal position, will be discharged at the other. Now, let the end *B* of the screw *B F*, be elevated so as to be inclined to the horizon, and the same effect will be produced: the water at *C* will rise towards *B*, till the angle of inclination which the machine makes with the horizon is equal to the angle formed by the spirals with the axis of the engine. At this particular angle the water will have as great a tendency to flow towards *D* as towards *C*, because the surface of the tube between these two points is parallel to the horizon; but at a greater angle, the fluid will descend towards *D*, and flow out at the extremity *F*. The ascension of the water, therefore, in the Archimedean screw-engine arises from its tendency to occupy the lowest parts of the spiral, while the rotatory motion withdraws this part of the spiral from the fluid, and causes it to ascend to the top of the tube. By wrapping a right-angled triangle round a cylindrical pin, so that the hypothenuse may form a spiral upon its surface, and by attending to the position of the spirals at different angles of inclination, the preceding observations will be easily understood. In practice, the angle of inclination should be about 50°, and the angle which the spirals form with the axis should exceed the angle of the engine's inclination by about 15°. The theory of this engine is treated at great length by *Hennert*, in his *Dissertation sur la vis D'Archimede*, Berlin, 1767; and by *Euler*, in the *Nov. Comment Petrop.* tom. v. See also *Gregory's Mechanics*, vol. ii, p. 343, and the *Edinburgh Encyclopædia*, Art. *Hydrodynamics*, vol. xi, p. 562.—*Ed.*

the other. In the same manner, the wheels *E* and *F* work the racks *O* and *P*.[7]

These four wheels are fixed on the axle of the great wheel in such a manner, with respect to the positions of their teeth, that while they continue turning round, there is never one instant of time in which one or other of the pump-rods is not going down, and forcing the water. So that, in this engine, there is no occasion for having a general air-vessel to all the pumps, to procure a constant stream of water flowing from the upper end of the main pipe.

The pistons of these pumps are solid plungers, the same as described in Lecture fifth. See Plate 4, Fig. V, *with the description of the figure.*

From each of these pumps, near the lowest end, in the water, there goes off a pipe, with a valve on its farthest end from the pump; and these ends of the pipes all enter one close box, into which they deliver the water: and into this box, the lower end of the main conduit-pipe is fixed; so that, as the water is forced or pushed into this box, it is also pushed up the main pipe to the height that it is intended to be raised.

There is an engine of this sort, described in Ramelli's work: but I can truly say, that I never saw it till some time after I had made this model.

The said model is not above twice as big as the figure of it, here described. I turn it by a winch fixed on the gudgeon of the axle behind the water wheel; and when it was newly made, and the pistons had valves in good order, I put tin pipes 15 feet high upon it, when they were joined together, to see what it could do; and I found, that in turning it moderately by the winch, it would raise a hogshead of water in an hour to the height of 15 feet.

DIALLING.

THE UNIVERSAL DIALLING CYLINDER.

Universal dialling cylinder.

In Fig. 1, of Plate XIV, *A B C D* represents a cylindrical glass tube, closed at both ends with brass plates, and having a wire or axis *E F G* fixed in the centres of the brass plates at top and bottom. This tube is fixed to a horizontal board *H*, and its axis makes an angle with the board equal to the angle of the earth's axis with the horizon of any given place, for which the cylinder is to serve as a dial. And it must be set with its axis parallel to the axis of the world in that place; the end *E* pointing to the elevated pole. Or, it may be made to move upon a joint; and then it may be elevated for any particular latitude.

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[7] For the proper form which must be given to the teeth of the wheels and racks,

There are 24 straight lines, drawn with a diamond, on the outside of the glass, equi-distant from each other, and all of them parallel to the axis. These are the hour-lines; and the hours are set to them as in the figure: the XII next *B* stands for midnight, and the opposite XII, next the board *H*, stands for mid-day or noon.

The axis being elevated to the latitude of the place, and the foot-board set truly level, with the black line along its middle in the plane of the meridian, and the end *N* toward the north; the axis *E F G* will serve as a stile or gnomon, and cast a shadow on the hour of the day among the parallel hour-lines when the sun shines on the machine. For, as the sun's apparent diurnal motion is equable in the heavens, the shadow of the axis will move equably in the tube; and will always fall upon *that* hour-line which is opposite to the sun at any given time.

The brass plate *A D*, at the top, is parallel to the equator, and the axis *E F G* is perpendicular to it. If right lines be drawn from the centre of this plate to the upper ends of the equi-distant parallel lines on the outside of the tube; these right lines will be the hour-lines on the equinoctial dial *A D*, at 15° distance from each other: and the hour letters may be set to them, as in the figure. Then, as the shadow of the axis within the tube comes on the hour-lines of that tube, it will cover the like hour-lines on the equinoctial plate *A D*.

If a thin horizontal plate *e f* be put within the tube, so that its edge may touch the tube all around; and right lines be drawn from the centre of the plate to these points of its edge which are cut by the parallel hour-lines on the tube; these right lines will be the hour-lines of a horizontal dial, for the

in order to produce an equable and uniform motion, see *Appendix*. This method of moving the pistons is preferable to the crank motion employed in the engine which is represented in Plate V, Fig. 5.—*Ed.*

latitude to which the tube is elevated. For, as the shadow of the axis comes successively to the hour-lines of the tube, and covers them, it will then cover the like hour-lines on the horizontal plate *e f*, to which the hours may be set, as in the figure.

If a thin vertical plate *g C*, be put within the tube, so as to front the meridian, or 12 o'clock line thereof, and the edge of this plate touch the tube all around: and then, if right lines be drawn from the centre of the plate to those points of its edge which are cut by the parallel hour-lines on the tube; these right lines will be the hour-lines of a vertical south dial; and the shadow of the axis will cover them at the same times when it covers those of the tube.

If a thin plate be put within the tube so as to decline, or incline, or recline, by any given number of degrees; and right lines be drawn from its centre to the hour-lines of the tube; these right lines will be the hour-lines of a declining, inclining, or reclining, dial, answering to the like number of degrees, for the latitude to which the tube is elevated.

And thus, by this simple machine, all the principles of dialling are made very plain and evident to the sight. And the axis of the tube (which is parallel to the axis of the world in every latitude to which it is elevated) is the stile or gnomon for all the different kinds of sun-dials.

And, lastly, if the axis of the tube be drawn out, with the plates *A D*, *e f*, and *g C* upon it; and set it up in sunshine, in the same position as they were in the tube; you will have an equinoctial dial *A D*, a horizontal dial *e f*, and a vertical south dial *g C*; on all which the time of the day will be shewn by the shadow of the axis or gnomon *E F G*.

Let us now suppose that, instead of a glass tube, *A B C D* is a cylinder of wood, on which the 24 parallel hour-lines are drawn all around, at equal distances from each other; and that, from the points at top, where these lines end, right lines are drawn toward the centre, on the flat surface *A D*: these right lines will be the hour-lines on an equinoctial dial, for the latitude of the place to which the cylinder is elevated above the horizontal foot or pedestal *H*; and they are equi-distant from each other, as in Fig. 2, which is a full view of the flat surface or top *A D* of the cylinder, seen obliquely in Fig. 1. And the axis of the cylinder (which is a straight wire *E F G* all down its middle) is the stile or gnomon, which is perpendicular to the plane of the equinoctial dial, as the earth's axis is perpendicular to the plane of the equator.

To make a horizontal dial, by the cylinder, for any latitude to which its axis is elevated; draw out the axis and cut the cylinder quite through, as at *e h f g*, parallel to the horizontal board *H*, and take off the top part *e A D f e;* and the section *e h f g e* will be of an elliptical form, as in Fig. 3. Then, from the points of this section (on the remaining part *e B C f*), where the parallel lines on the outside of the cylinder meet it, draw right lines to the centre of the section; and they will be the true hour-lines for a horizontal dial, as *a b c d a* in Fig. 3, which may be included in a circle drawn on that section. Then put the wire into its place again, and it will be a stile for casting a shadow on the time of the day on that dial. So *E* (Fig. 3) is the stile of the horizontal dial, parallel to the axis of the cylinder.

To make a vertical south dial by the cylinder, draw out the axis, and cut the cylinder perpendicularly to the horizontal board *H*, as at *g i C k g*, beginning at the hour-line (*B g e A*) of XII, and making the section at right angles to the line *S H N* on the horizontal board. Then, take off the upper part *g A D C*, and the face of the section thereon will be elliptical, as shewn in Fig. 4. From the points in the edge of this section, where the parallel hour-lines on the round surface of the cylinder meet it, draw right lines to the centre of the section; and they will be the true hour-lines on a vertical direct south dial, for the latitude to which the cylinder was elevated; and will appear as in Fig. 4, on which the vertical dial may be made of a circular shape, or of a square shape, as represented in the figure; and *F* will be its stile parallel to the axis of the cylinder.

And thus, by cutting the cylinder any way, so that its section may either incline, or decline, or recline, by any given number of degrees; and from those points in the edge of the section, where the outside parallel hour-lines meet it, draw right lines to the centre of the section; and they will be the true hour-lines for the like declining, reclining, or inclining, dial: and the axis of the cylinder will always be the gnomon or stile of the dial; for, whichever way the plane of the dial lies, its stile (or the edge thereof that casts the shadow on the hours of the day) must be parallel to the earth's axis, and point toward the elevated pole of the heaven.

TO DELINEATE A SUN-DIAL ON PAPER, WHICH, WHEN PASTED ROUND A CYLINDER OF WOOD, SHALL SHEW THE TIME OF THE DAY, THE SUN'S PLACE IN THE ECLIPTIC, AND HIS ALTITUDE, AT ANY TIME OF OBSERVATION.

Plate XIV. Fig. 5.

Draw the right line *a A B*, parallel to the top of the paper; and with any convenient opening of the compasses set one foot in the end of the line at *a*, as a centre, and with the other foot describe the quadrantal arc *A E*, and divide it into 90 equal parts or degrees. Draw the right line *A C*, at right angles to *a A B*, and touching the quadrant *A E* at the point *A*. Then, from the centre *a*, draw right lines through as many degrees of the quadrant as are equal to the sun's altitude at noon, on the longest day of the year, at the place for which the dial is to serve; which altitude at London is 62 degrees: and continue these right lines till they meet the tangent line *A C*, and from these points of meeting, draw straight lines across the paper, parallel to the first right line *A B*, and they will be the parallels of the sun's altitude, in whole degrees, from sun-rise till sun-set, on all the days of the year. These parallels of altitude must be drawn out to the right line *B D*, which must be parallel to *A C*, and as far from it as is equal to the intended circumference of the cylinder on which the paper is to be pasted, when the dial is drawn upon it. Divide the space between the right lines *A C* and *B D* (at top and bottom) into twelve equal parts, for the twelve signs of the ecliptic; and, from mark to mark of these divisions at top and bottom, draw right lines parallel to *A C* and *B D*; and place the characters of the twelve signs in these twelve spaces, at the bottom, as in the figure; beginning with ♑ or Capricorn, and ending with ♓ or Pisces. The spaces including the signs should be divided by parallel lines into halves; and if the breadth will admit of it without confusion, into quarters also.

At the top of the dial, make a scale of the months and days of the year, so that the days may stand over the sun's place for each of them in the signs of the ecliptic. The sun's place, for every day of the year, may be found by any common ephemeris: and here it will be best to make use of an ephemeris for the second year after leap year; as the nearest means for the sun's place on the days of the leap year, and on those of the first, second, and third year after.

Compute the sun's altitude for every hour (in the latitude of your place), when he is in the beginning, middle, and end, of each sign of the ecliptic; his altitude at the end of each sign being the same as at the beginning of the next. And, in the upright parallel lines, at the beginning and middle of each sign, make marks for those computed altitudes among the horizontal parallels of altitude, reckoning them downward, according to the order of the numeral figures set to them at the right hand, answering to the like division of the quadrant at the left; and, through these marks, draw the curve hour-lines, and set the hours to them, as in the figure, reckoning the forenoon hours downward, and the afternoon hours upward. The sun's altitude should also be computed for the half hours; and the quarter-lines may be drawn, very nearly in their proper places, by estimation and accuracy of the eye. Then, cut off the paper at the left hand, on which the quadrant was drawn, close by the right line *A C*, and all the paper at the right hand close by the right line *B D;* and cut it also close by the top and bottom horizontal lines; and it will be fit for pasting round the cylinder.

This cylinder is represented in miniature by Fig. 1, Plate XV. It should be hollow, to hold the stile *D E* when it is not used. The crooked end of the stile is put into a hole in the top *A D* of the cylinder; and the top goes on tightish, but must be made to turn round on the cylinder, like the lid of a paper snuff-box. The stile must stand straight out, perpendicular to the side of the cylinder, just over the right line *A B* in Plate IX, where the parallels of the sun's altitude begin: and the length of the stile, or distance of its point *e* from the cylinder, must be equal to the radius *a A* of the quadrant *A E* in Plate XIV, Fig. 5.

Plate XV. Fig. 1. The method of using this dial is as follows. Place the horizontal foot *B C* of the cylinder on a level table where the sun shines, and turn the top *A D* till the stile stands just over the day of the then present month. Then turn the cylinder about on the table, till the shadow of the stile

falls upon it, parallel to those upright lines, which divide the signs, that is, till the shadow be parallel to a supposed axis in the middle of the cylinder: and then, the point, or lowest end of the shadow, will fall upon the time of the day, as it is before or after noon, among the curve hour-lines; and will shew the sun's altitude at that time, among the cross parallels of his altitude, which go round the cylinder: and, at the same time, it will shew in what sign of the ecliptic the sun then is, and you may very nearly guess at the degree of the sign, by estimation of the eye.

When a level table cannot be had, the dial may be hung by the ring *F* at the top; and when it is not used, the wire that serves for a stile may be drawn out, and put up within the cylinder; and the machine carried in the pocket.

TO MAKE THREE SUN-DIALS UPON THREE DIFFERENT PLANES, SO THAT THEY MAY ALL SHEW THE TIME OF THE DAY BY ONE GNOMON.

Plate XV. Fig. 2.

On the flat board *A B C*, describe a horizontal dial, according to any of the rules laid down in the Lecture on Dialling: and to it fix its gnomon *F G H*, the edge of the shadow from the side *F G* being that which shews the time of the day

To this horizontal or flat board, join the upright board *E D C*, touching the edge *G H* of the gnomon. Then, making the top of the gnomon at *G* the centre of the vertical south dial, describe a south dial on the board *E D C*.

Lastly, on a circular plate *I K* describe an equinoctial dial, all the hours of which dial are equidistant from each other; and making a slit *c d* in that dial, from its edge to its centre, in the XII o'clock line, put the said dial perpendicularly on the gnomon *F G*, as far as the slit will admit of; and the triple dial will be finished; the same gnomon serving all the three, and shewing the same time of the day on each of them.[8]

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[8] This dial may be converted into a portable and universal one by a very simple contrivance. Remove the stile *F H G* and the dial *K*, and make *E D C* turn upon *H C* as a hinge, so that it may fold down upon *A B*, and thus go into very small compass when not used. Fix a silk thread at *F*, and having divided the line *G H* continued, into a line of tangents for the radius *F H*, make a small hole through the board at every degree of the line of tangents. Extend the silk thread from *F* towards

AN UNIVERSAL DIAL ON A PLAIN CROSS.

Universal dial. This dial is represented by Fig. 3, of Plate XV, and is moveable on a joint *C*, for elevating it to any given latitude, on the quadrant *C* 0 90, as it stands upon the horizontal board *A*. The arms of the cross stand at right angles to the middle part; and the top of it from *a* to *n*, is of equal length with either of the arms *n e* or *m k*.

Having set the middle line *t u* to the latitude of your place, on the quadrant, the board *A* level, and the point *N* northward by the needle; the plane of the cross will be parallel to the plane of the equator; and the machine will be rectified.

Then, from III o'clock in the morning till VI, the upper edge *k l* of the arm *i o* will cast a shadow on the time of the day on the side of the arm *c m*: from VI till IX the lower edge *i* of the arm *i o* will cast a shadow on the hours on the side *o q*. From IX in the morning till XII at noon, the edge *a b* of the top part *a n* will cast a shadow on the hours on the arm *n e f*: from XII to III in the afternoon, the edge *c d* of the top part will cast a shadow on the hours on the arm *k l m*; from III to VI in the evening, the edge *g h* will cast a shadow on the hours on the part *p s*; and from VI till IX, the shadow of the edge *e f* will shew the time on the top part *a n*.

The breadth of each part *a b*, *e f*, &c. must be so great as never to let the shadow fall quite without the part or arm on which the hours are marked, when the sun is at his greatest declination from the equator.

To determine the breadth of the sides of the arms which contain the hours, so as to be in just proportion to their length, make an angle *A B C* (Fig. 4) of 23½°, which is equal to the sun's greatest declination: and suppose the length of each arm, from the side of the long middle part, and also the length of the top part above the arms, to be equal to *B d*.

G, making it pass through the hole at the degree of the line of tangents answering to the latitude of the place. The thread will then be the gnomon of the horizontal dial *A B C*, which is set due south, by means of a small mariner's compass placed between *F* and *H*, allowance being made for the variation. The vertical south dial *E C* serves only for a place, the tangent of whose latitude is equal to *H G*. This dial is not altogether correct, but is remarkably convenient for carrying in the pocket.—*Ed.*

Then, as the edges of the shadow from each of the arms, will be parallel to *B o*, making an angle of 23½° with the side *B n* of the arm when the sun's declination is 23½°; it is plain, that if the length of the arm be *B n*, the least breadth that it can have, to keep the edge *B o* of the shadow *B o g d* from going off the side of the arm *n o* before it comes to the end *o n* thereof, must be equal to *o n* or *d B*. But in order to keep the shadow within the quarter divisions of the hours, when it comes near the end of the arm, the breadth thereof should be still greater, so as to be almost doubled, on account of the distance between the tips of the arms.

To place the hours right on the arms, take the following method:—Lay down the cross *a b c d* (Fig. 5) on a sheet of paper; and with a black lead pencil, held close to it, draw its shape and size on the paper. Then, taking the length *a e* in your compasses, and setting one foot in the corner *a*, with the other foot describe the quadrantal arc *e f*. Divide this arc into six equal parts, and through the division marks draw right lines *a g*, *a h*, &c. continuing three of them to the arm *c e*, which are all that can fall upon it; and they will meet the arm in these points through which the lines that divide the hours from each other (as in Fig. 3.) are to be drawn right across it.

Divide each arm, for the three hours it contains in the same manner; and set the hours to their proper places (on the sides of the arms), as they are marked in Fig. 3. Each of the hour spaces should be divided into four equal parts, for the half hours and quarters, in the quadrant *e f;* and right lines should be drawn through these division marks in the quadrant, to the arms of the cross, in order to determine the places thereon where the sub-divisions of the hours must be marked.

This is a very simple kind of universal dial; it is very easily made, and will have a pretty uncommon appearance in a garden. I have seen a dial of this sort, but never saw one of the kind that follows.

AN UNIVERSAL DIAL, SHEWING THE HOURS OF THE DAY BY A TERRESTRIAL GLOBE, AND BY THE SHADOWS OF SEVERAL GNOMONS AT THE SAME TIME: TOGETHER WITH ALL THE PLACES OF THE EARTH WHICH ARE THEN ENLIGHTENED BY THE SUN; AND THOSE TO WHICH THE SUN IS THEN RISING, OR ON THE MERIDIAN, OR SETTING.

Universal dial by a terrestrial globe and several gnomons. Plate XVI. Fig. 1.

This dial is made of a thick square piece of wood, or hollow metal. The sides are cut into semicircular hollows, in which the hours are placed; the stile of each hollow coming out from the bottom thereof, as far as the ends of the hollows project. The corners are cut out into angles, in the insides of which, the hours are also marked; and the edge of the end of each side of the angle serves as a stile for casting a shadow on the hours marked on the other side.

In the middle of the uppermost side or plane, there is an equinoctial dial: in the centre whereof an upright wire is fixed, for casting a shadow on the hours of that dial, and supporting a small terestrial globe on its top.

The whole dial stands on a pillar, in the middle of a round horizontal board, in which there is a compass and magnetic needle, for placing the *meridian* stile toward the south. The pillar has a joint with a quadrant upon it divided into 90 degrees (supposed to be hid from sight under the dial in the figure), for setting it to the latitude of any given place, the same way as already described in the dial on the cross.

The equator of the globe is divided into 24 equal parts, and the hours are laid down upon it at these parts. The time of the day may be known by these hours, when the sun shines upon the globe.

To rectify and use this dial, set it on a level table, or sole of a window, where the sun shines, placing the meridian stile due south, by means of the needle; which will be, when the needle points as far from the north fleur-de-lis toward the west, as it declines westward, at your place.[9] Then bend the pillar in the joint, till the black line on the pillar comes to the latitude of your place in the quadrant.

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[9] As the declination of the needle is very uncertain, and varies even at the same place, the dial should be rectified by means of a meridian line, drawn upon the side of the window.—*Ed.*

The machine being thus rectified, the plane of its dial-part will be parallel to the equator, the wire or axis that supports the globe will be parallel to the earth's axis, and the north pole of the globe will point toward the north pole of the heaven.

The same hour will then be shewn in several of the hollows, by the ends of the shadows of their respective stiles. The axis of the globe will cast a shadow on the same hour of the day, in the equinoctial dial, in the centre of which it is placed, from the 20th of March to the 22d of September; and, if the meridian of your place on the globe be set even with the meridian stile, all the parts of the globe that the sun shines upon, will answer to those places of the real earth which are then enlightened by the sun. The places where the shade is just coming upon the globe, answer all to those places of the earth to which the sun is then setting; as the places where it is going off, and the light coming on, answer to all those places of the earth where the sun is then rising. And, lastly, if the hour of VI be marked on the equator in the meridian of your place (as it is marked on the meridian of London in the figure), the division of the light and shade on the globe will shew the time of the day.

The northern stile of the dial (opposite to the southern or meridian one) is hid from sight in the figure, by the axis of the globe. The hours in the hollow to which that stile belongs, are also supposed to be hid by the oblique view of the figure; but they are the same as the hours in the front hollow. Those also in the right and left hand semicircular hollows are mostly hid from sight; and so also are all those on the sides next the eye of the four acute angles.

The construction of this dial is as follows. (See Plate XVI, Fig. 2.)

On a thick square piece of wood, or metal, draw the lines *a c* and *b d*, as far from each other as you intend for the thickness of the stile *a b c d*, and in the same manner, draw the like thickness of the other three stiles, *e f g h*, *i k l m*, and *n o p q*, all standing outright as from the centre.

With any convenient opening of the compasses, as *a A* (so as to leave proper strength of stuff when *K I* is equal to *a A*) set one foot in *a*, as a centre, and with the other foot describe the quadrantal arc *A c*. Then, without altering the compasses, set one foot in *b* as a centre, and with the other foot describe the quadrant *d B*. All the other quadrants in the figure must be described in the same manner, and with the same opening of the compasses, on their centres *e*, *f*; *i*, *k*; and *n*, *o*: and each quadrant divided into six equal parts, for so many hours, as in the figure; each of which parts must be subdivided into four, for the half hours and quarters.

At equal distances from each corner, draw the right lines *I p* and *K p*, *L q* and *M q*, *N r* and *O r*, *P s* and *Q s*; to form the four angular hollows *I p K*, *L q M*, *N r O*, and *P s Q*: making the distances between the tips of the hollows, as *I K*, *L M*, *N O*, and *P Q*, each equal to the radius of the quadrants; and leaving sufficient room within the angular points, *p*, *q*, *r*, and *s*, for the equinoctial circle in the middle.

To divide the insides of these angles properly for the hour-spaces thereon, take the following method.

Set one foot of the compasses in the point *I*, as a centre: and open the other to *K*, and with that opening describe the arc *K t*: then, without altering the compasses, set one foot in *K*, and with the other foot describe the arc *I t*. Divide each of these arcs, from *I* and *K* to their intersection at *t*, into four equal parts; and from their centres *I* and *K*, through the points of division, draw the right lines *I* 3, *I* 4, *I* 5, *I* 6, *I* 7; and *K* 2, *K* 1, *K* 12, *K* 11; and they will meet the sides *K p* and *I p* of the angle *I p K* where the hours thereon must be placed. And these hour-spaces in the arcs must be subdivided into four equal parts, for the half hours and quarters. Do the like for the other three angles, and draw the dotted lines, and set the hours in the insides where those lines meet them, as in the figure: and the like hour-lines will be parallel to each other in all the quadrants and in the angles.

Mark points for all these hours, on the upper side, and cut out all the angular hollows, and the quadrantal ones, quite through the places where their four gnomons must stand; and lay down the hours on their insides, as in Plate XII, and then set in their four gnomons, which must be as broad as the dial is thick; and this breadth and thickness must be large enough to keep the shadows of the gnomons from ever falling quite out at the sides of the hollows, even when the sun's declination is at the greatest.

Lastly, draw the equinoctial dial in the middle, all the hours of which are equidistant from each other; and the dial will be finished.

As the sun goes round, the broad end of the shadow of the stile *a b c d* will shew the hours in the quadrant *A c*, from sun-rise till VI in the morning; the shadow from the end *M* will shew the hours on the side *L q* from V to IX in the morning; the shadow of the stile *e f g h* in the quadrant *D g* (in the long days) will shew the hours from sun-rise till VI in the morning; and the shadow of the end *N* will shew the morning hours, on the side *O r*, from III to VII.

Just as the shadow of the northern stile *a b c d* goes off the quadrant *A c*, the shadow of the southern stile *i k l m* begins to fall within the quadrant *F l*, at VI in the morning; and shews the time, in that quadrant, from VI till XII at noon; and from noon till six in the evening in the quadrant *m E*. And the shadow of the end *O* shews the time from XI in the forenoon till III in the afternoon, on the side *r N;* as the shadow of the end *P* shews the time from IX in the morning till I o'clock in the afternoon, on the sides *Q s*.

At noon, when the shadow of the eastern stile *e f g h* goes off the quadrant *h C* (in which it shewed the time from six in the morning till noon, as it did in the quadrant *g D* from sun-rise till VI in the morning) the shadow of the western stile *n o p q* begins to enter the quadrant *H p;* and shews the hours thereon from XII at noon till VI in the evening; and after *that* till sun-set, in the quadrant *q G;* and the end *Q* casts a shadow on the side *P s* from V in the evening till IX at night, if the sun be not set before that time.

The shadow of the end *I* shews the time on the side *K p* from III till VII in the afternoon; and the shadow of the stile *a b c d* shews the time from VI in the evening till the sun sets.

The shadow of the upright central wire, that supports the globe at top, shews the time of the day, in the middle or equinoctial dial, all the summer half year, when the sun is on the north side of the equator.

In this Supplement to my Book of Lectures, all the machines that I have added to my apparatus, since that book was printed, are described, excepting two; one of which is a model of a mill for sawing timber, and the other is a model of the great engine at London Bridge, for raising water. And my reasons for leaving them out are as follows.

First, I found it impossible to make such a drawing of the saw-mill as could be understood; because in whatever view it be taken, a great many parts of it hid others from sight. And, in order to shew it in my lectures, I am obliged to turn it into all manner of positions.[10]

Secondly, because any person who looks on Fig. 5 of Plate V. in the book, and reads the account of it in the fifth Lecture therein, will be able to form a very good idea of the London Bridge engine, which has only two wheels and two trundles more than there are in Mr. Aldersea's engine, from which the said figure was taken.

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[10] For the plan and elevation of a Saw-mill, see Gray's *Experienced Millwright*, p. 68. For the method of constructing one, see Wolfii *Opera Mathematica*, tom. i, p. 694, and Boecklerus's *Theatrum Machinarum*. An excellent saw-mill was invented by Mr. James Stanfield, in the year 1765, for which he received a reward of one hundred pounds from the Society for the Encouragement of Arts. The original mill which Mr. Stanfield constructed, was worked for five successive years, in consequence of successive premiums offered and paid by the society, amounting, in all, to two hundred and twenty pounds. A description of this machine, illustrated by five folio plates, will be found in *Bailey's Designs of Machines, &c. approved and adopted by the Society for the Encouragement of Arts*, vol. i, p. 137. Saw-mills of a very superior kind have been erected by that celebrated mechanic Mr Brunel, in different parts of the kingdom. See the *Edinburgh Encyclopædia*, Arts. *Block Machinery*, and *Saw-mills.—Ed.*

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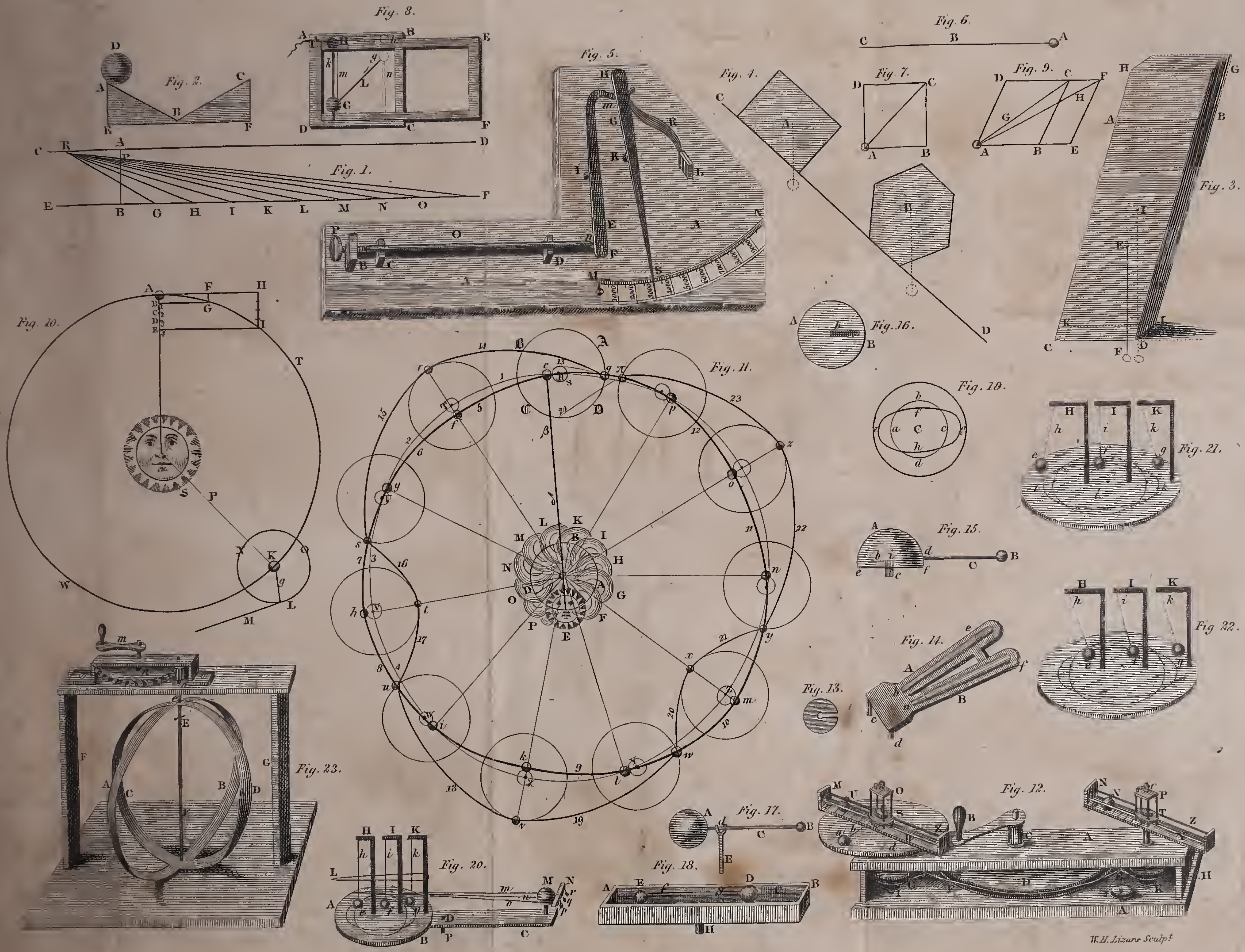

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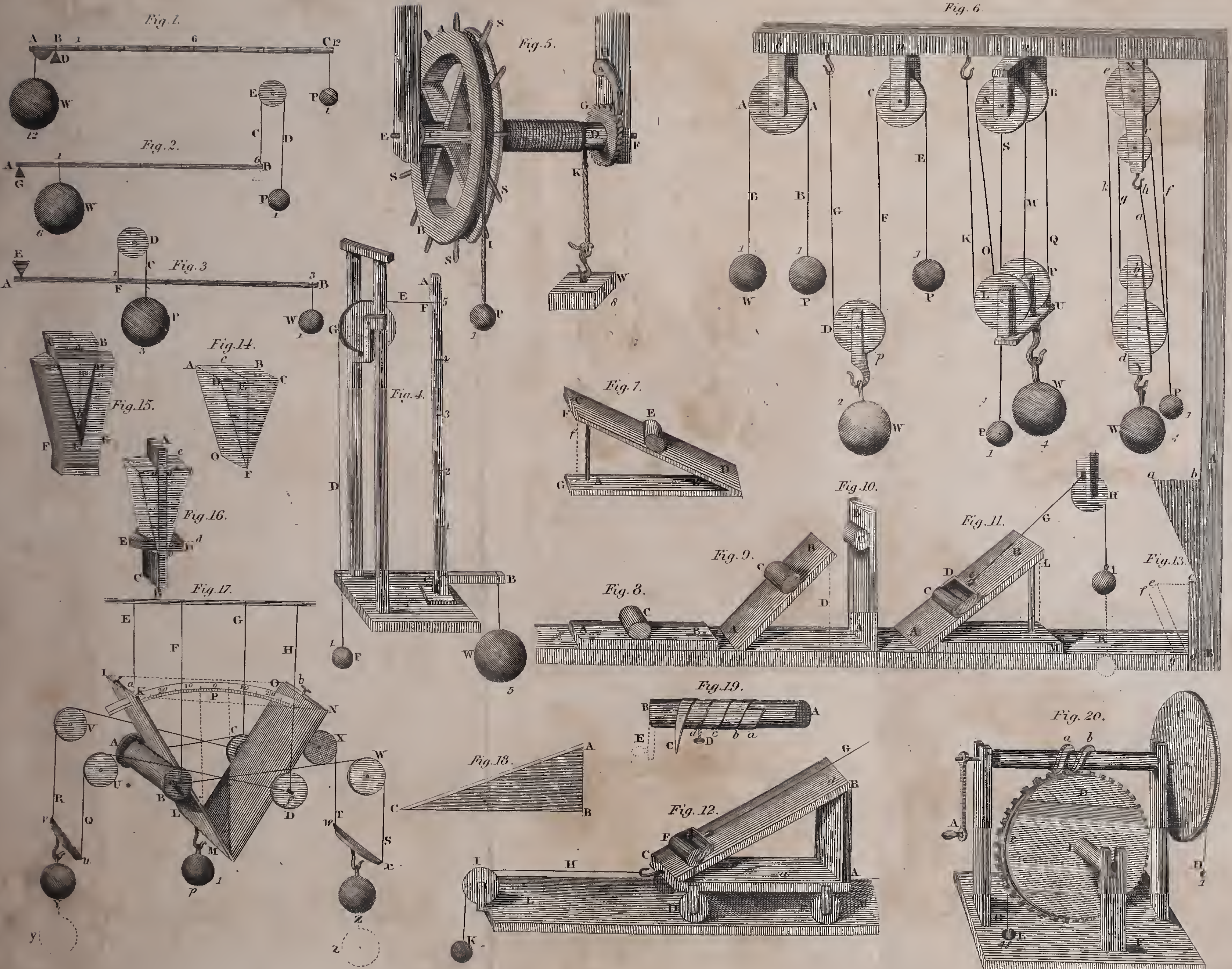

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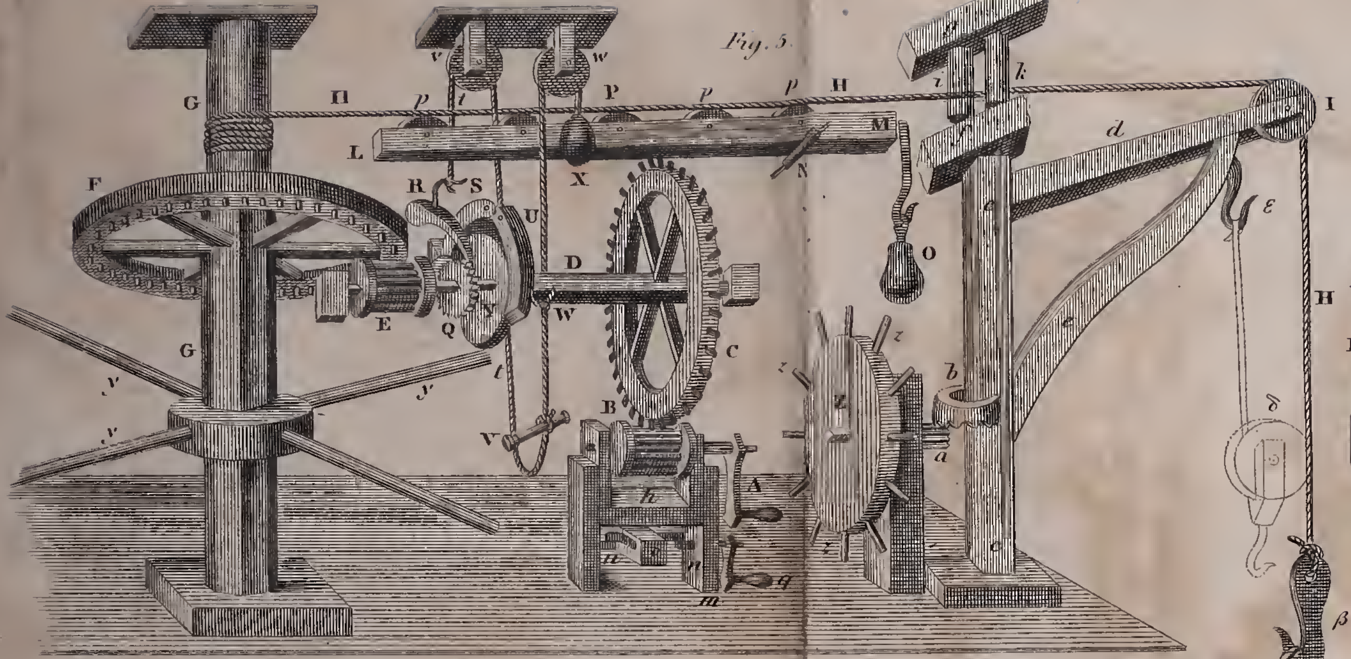

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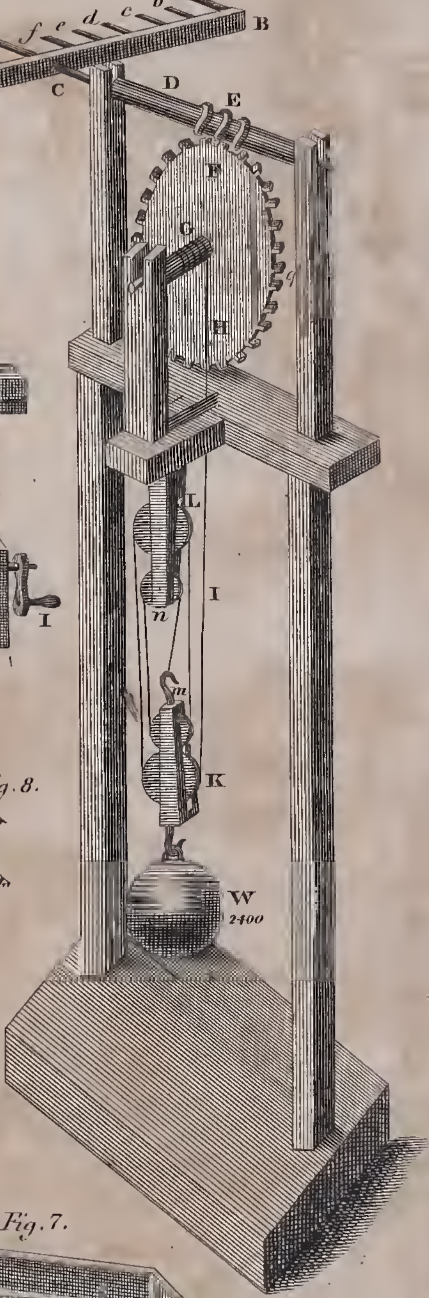

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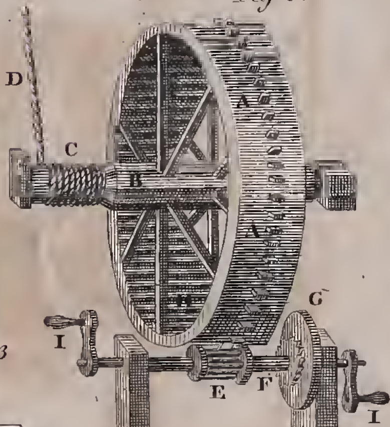

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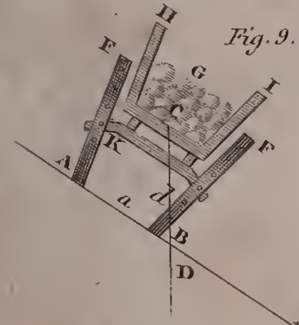

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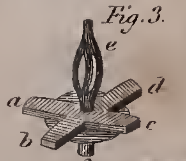

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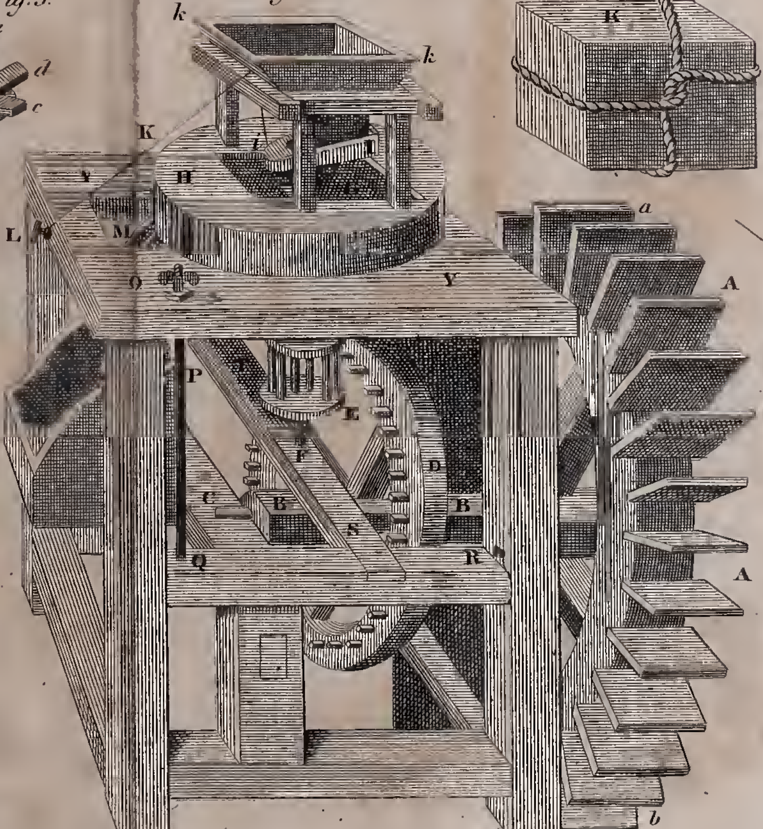

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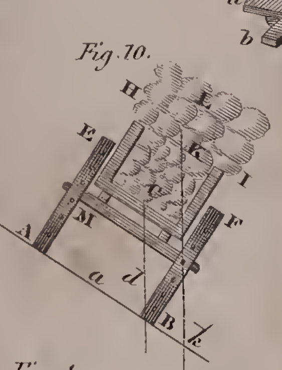

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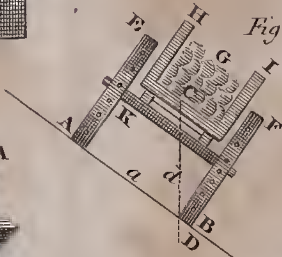

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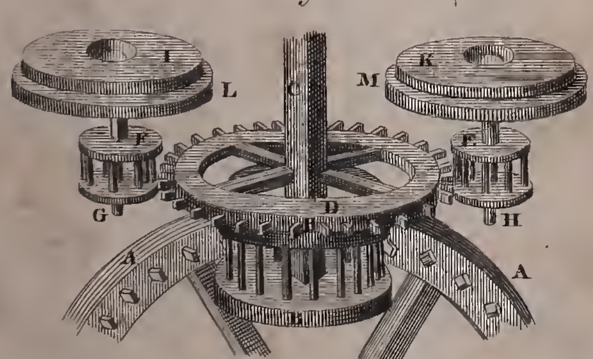

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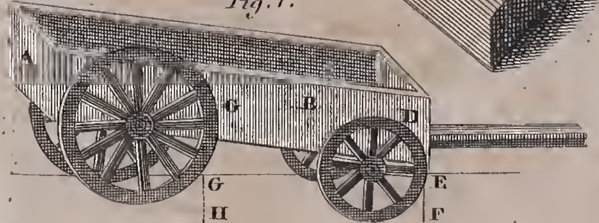

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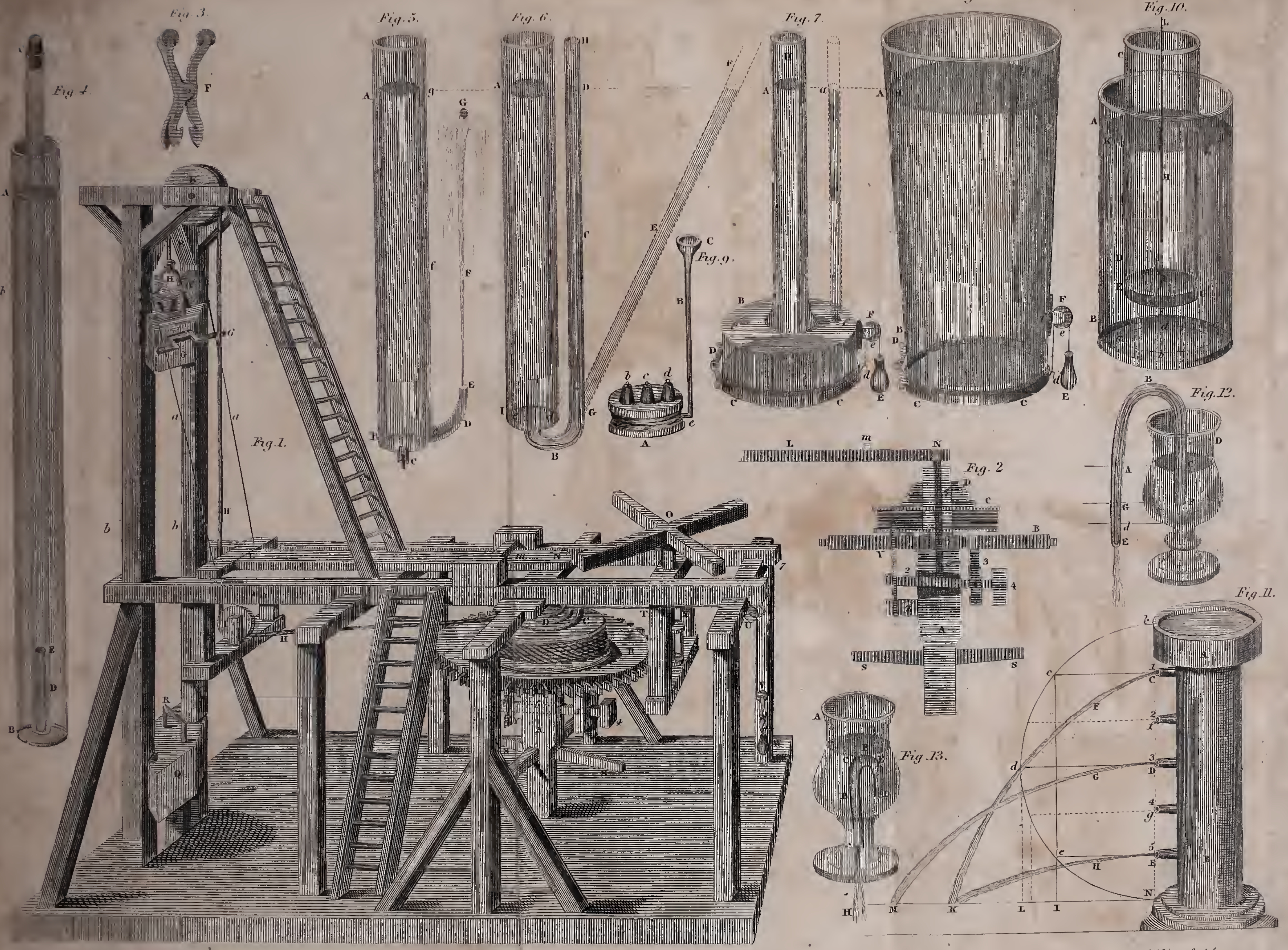

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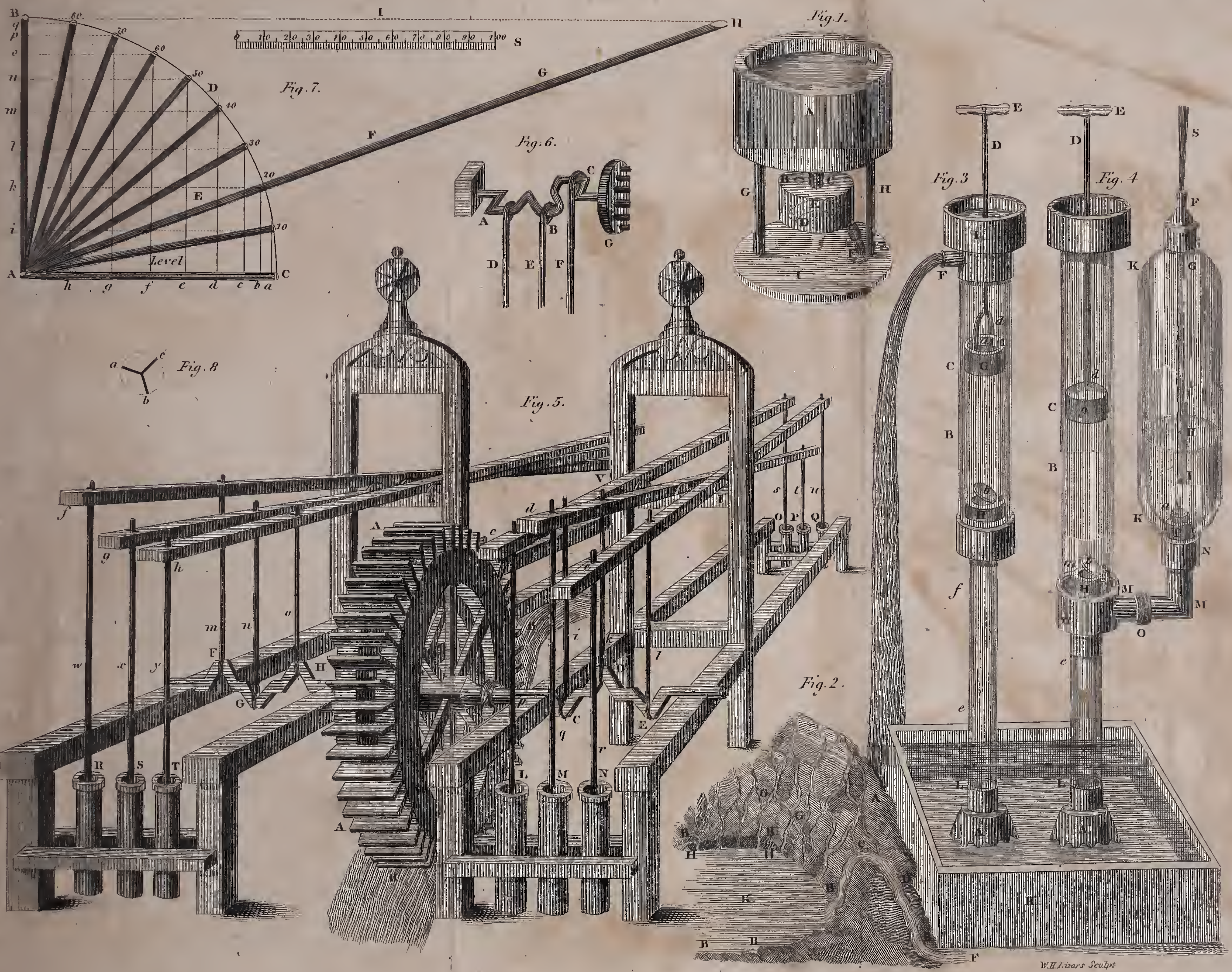
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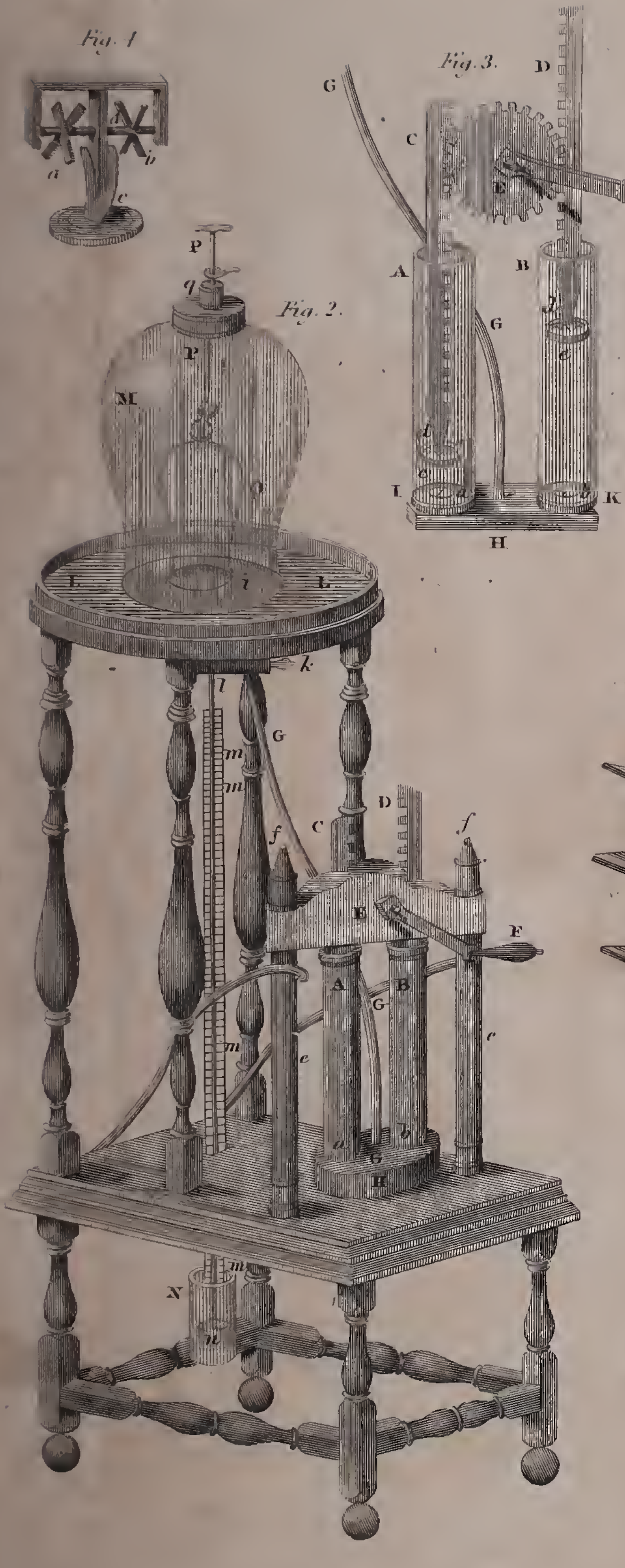

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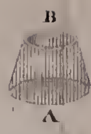

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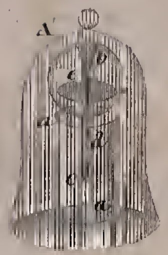

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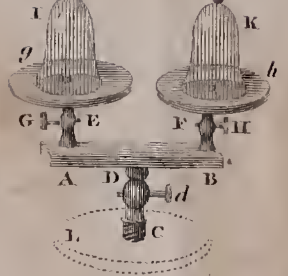

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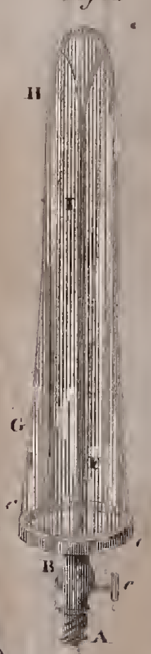

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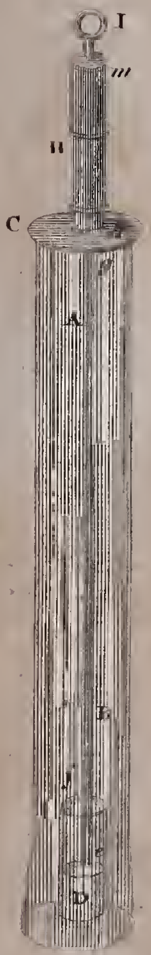

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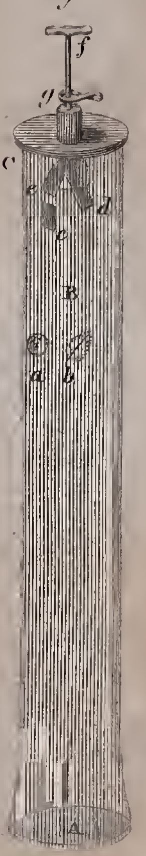

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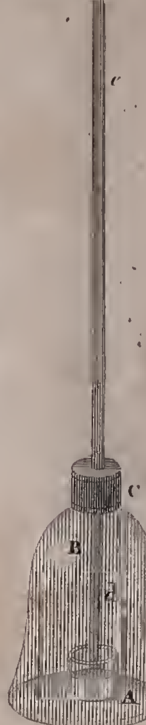

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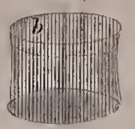

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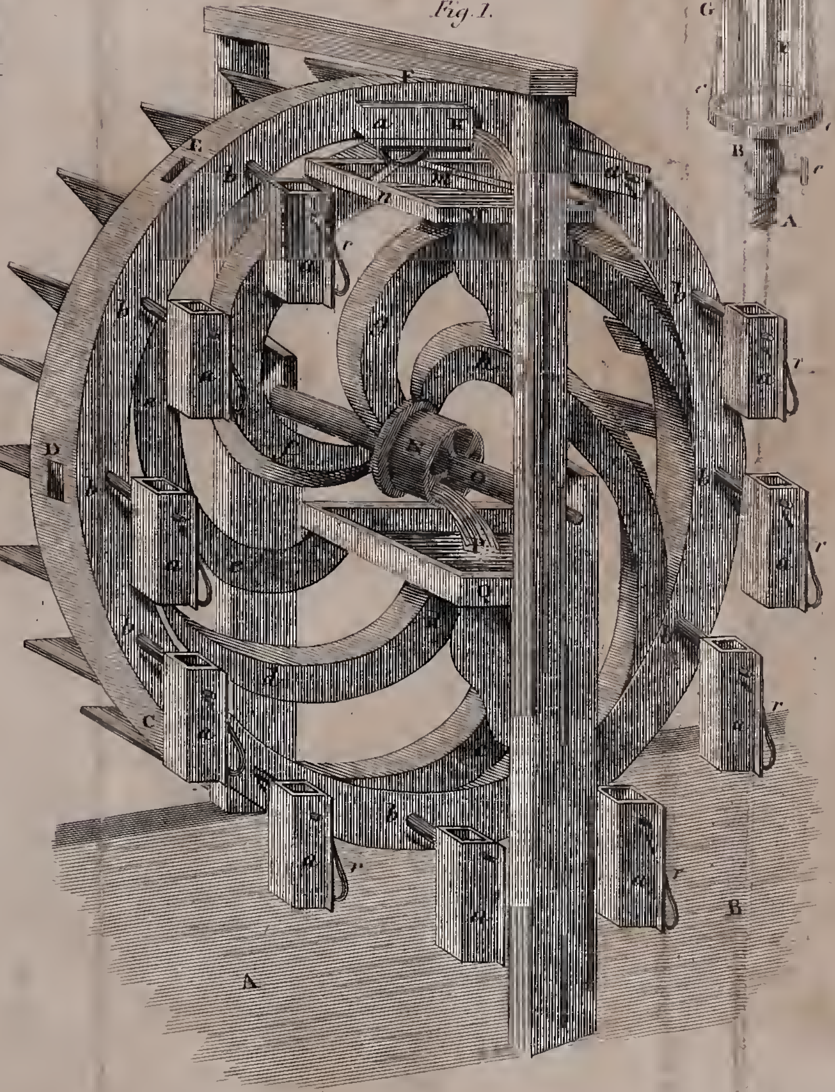

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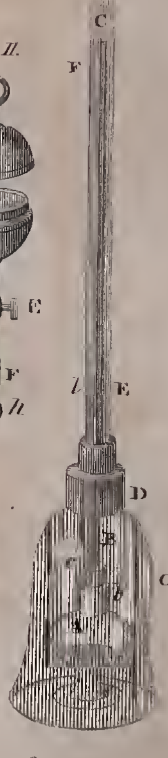

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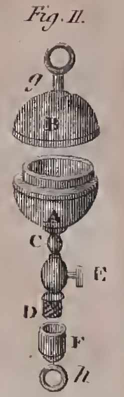

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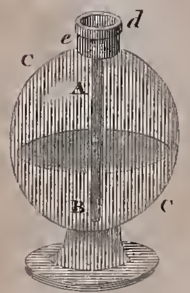

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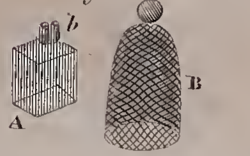

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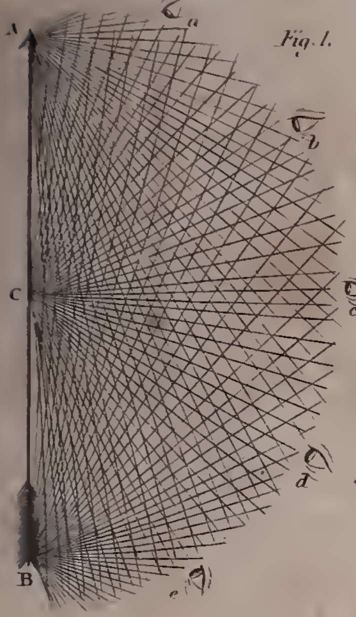

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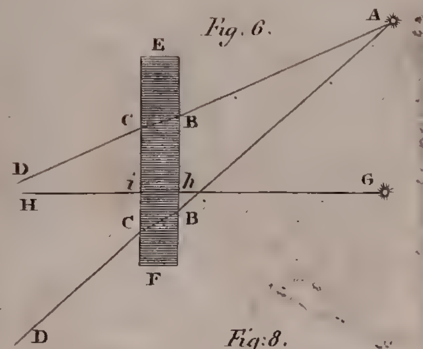

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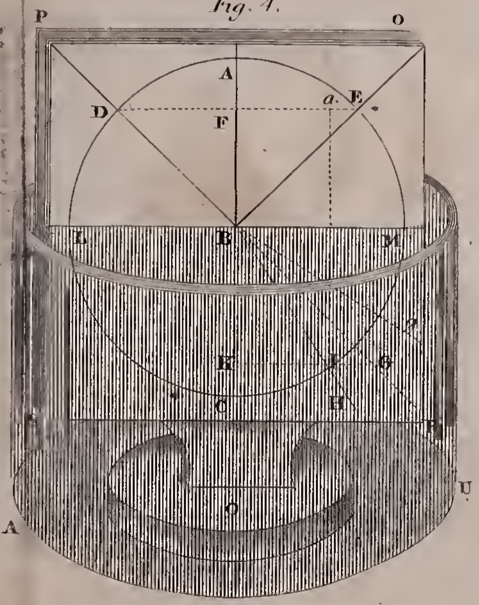

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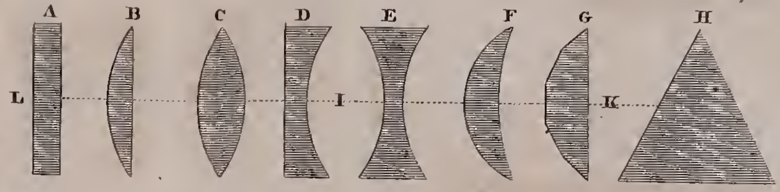

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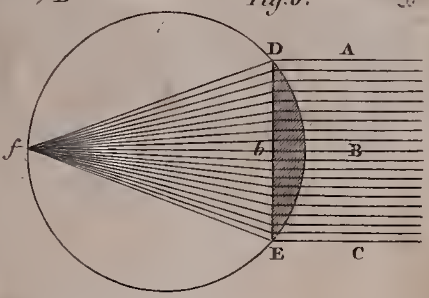

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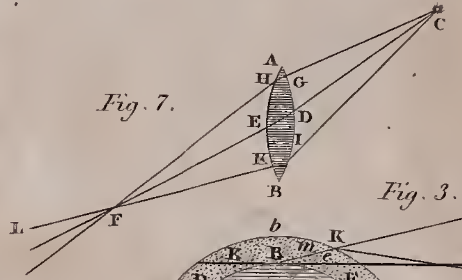

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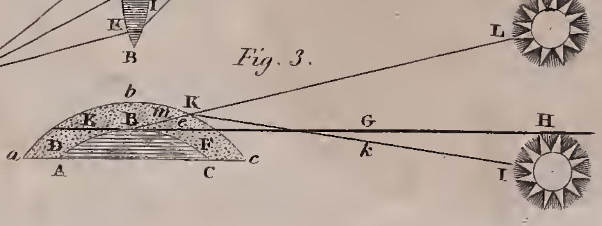

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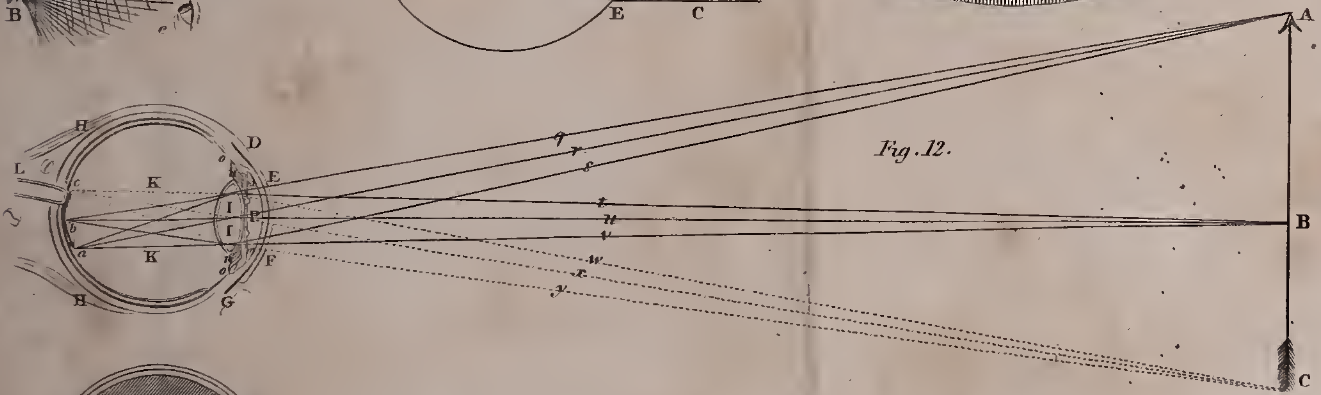

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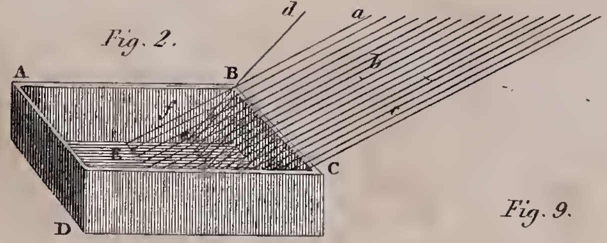

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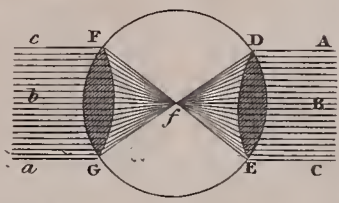

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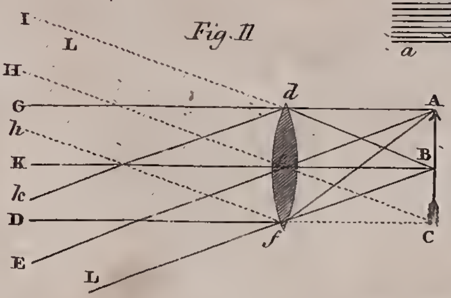

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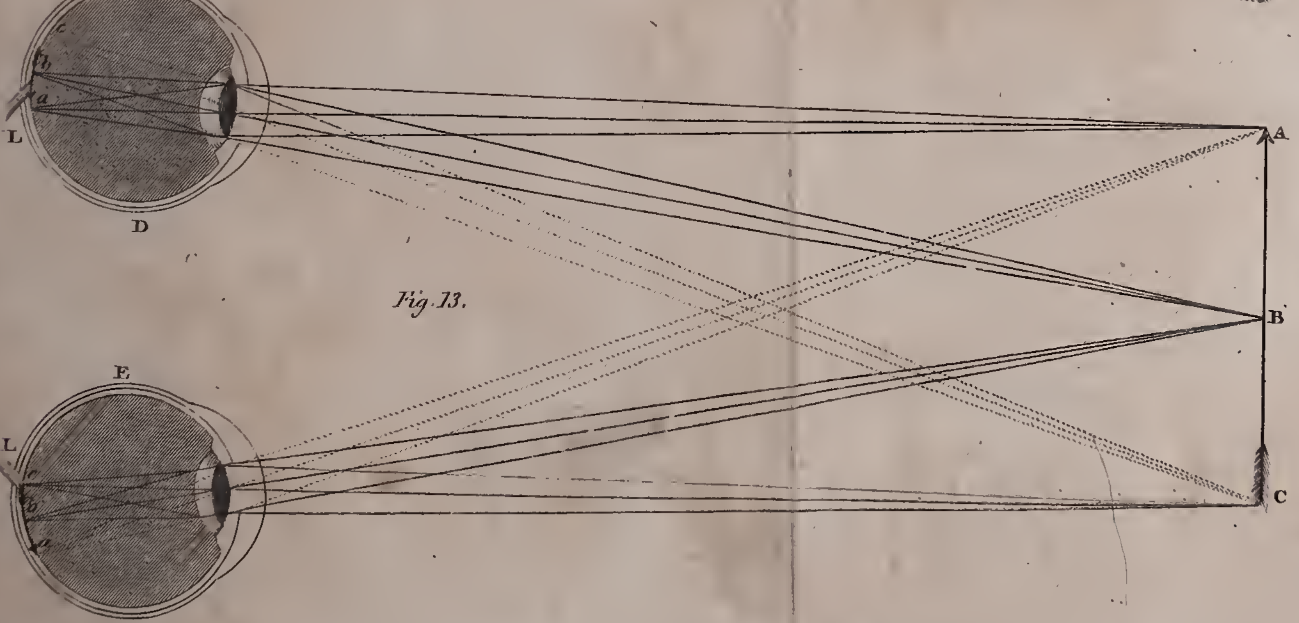

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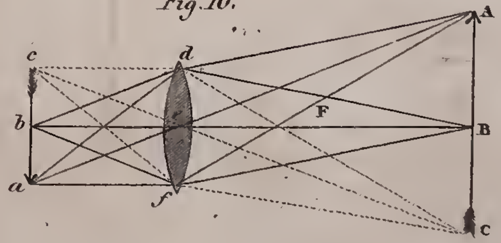

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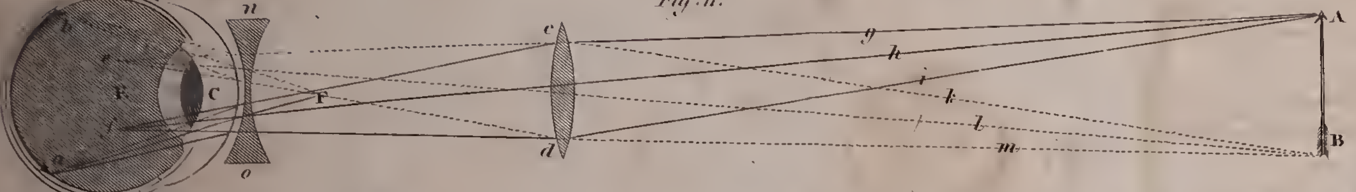
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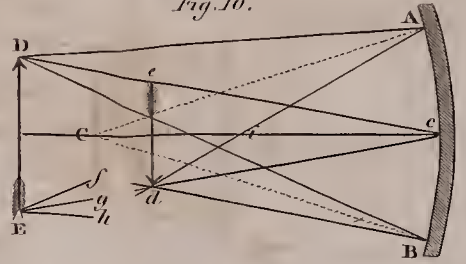
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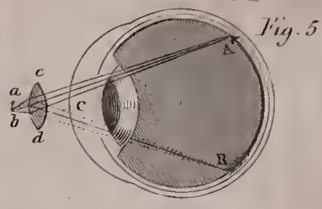
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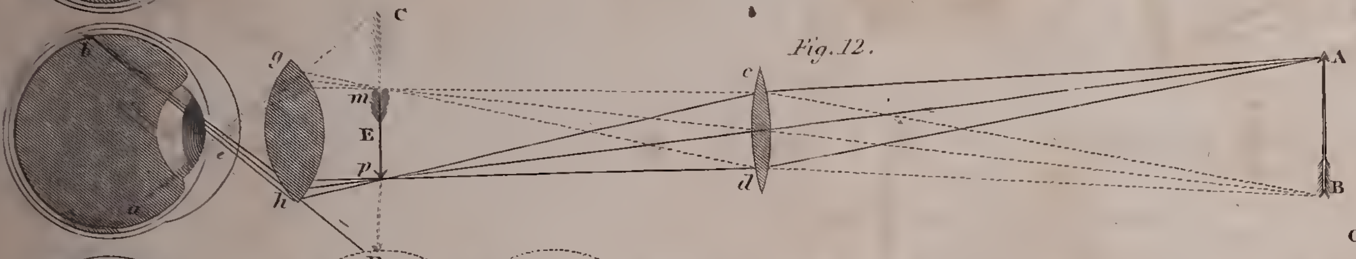
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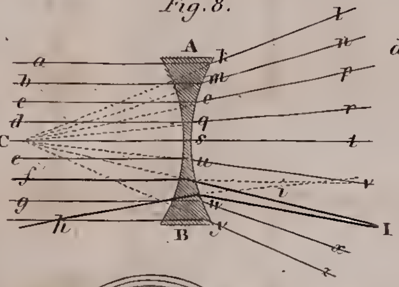
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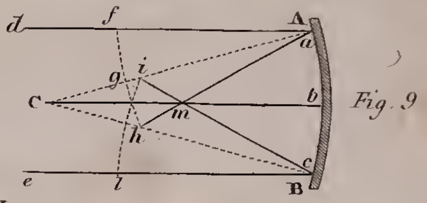
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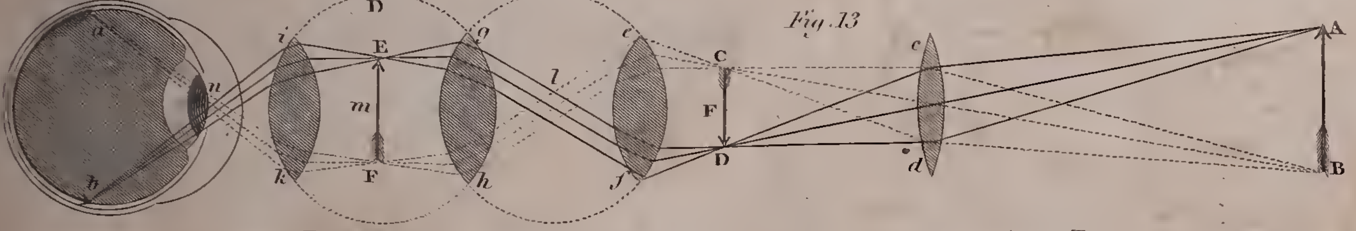
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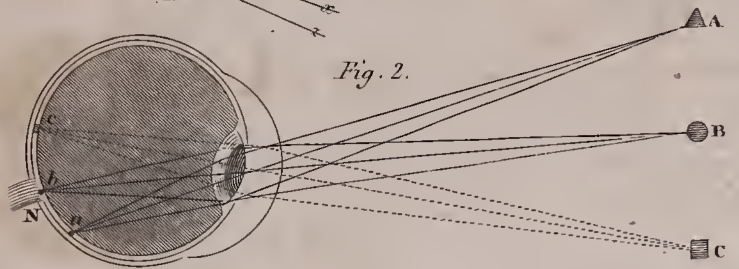
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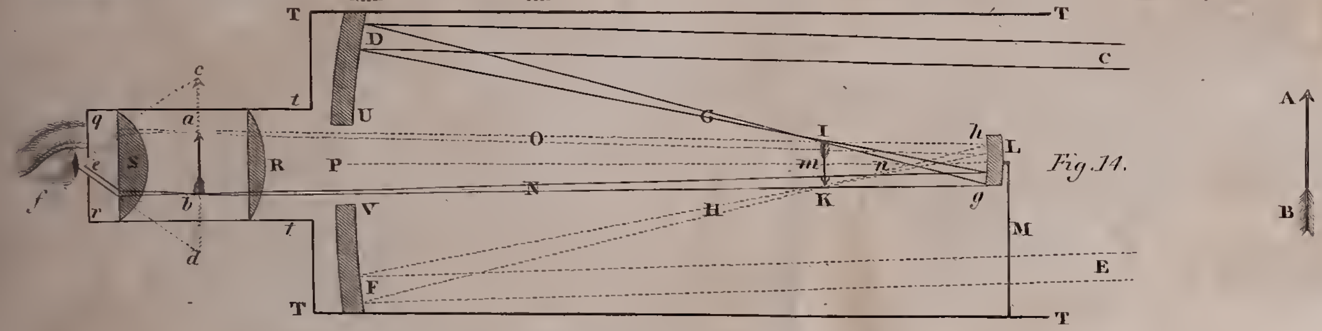
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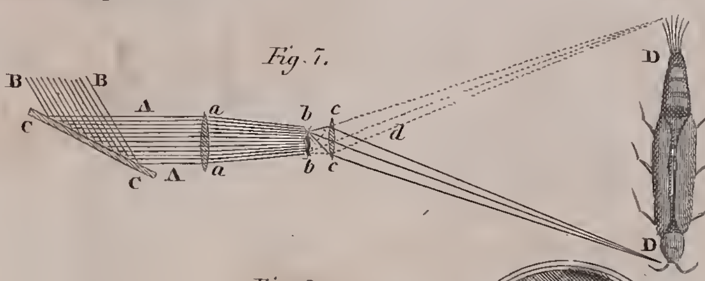
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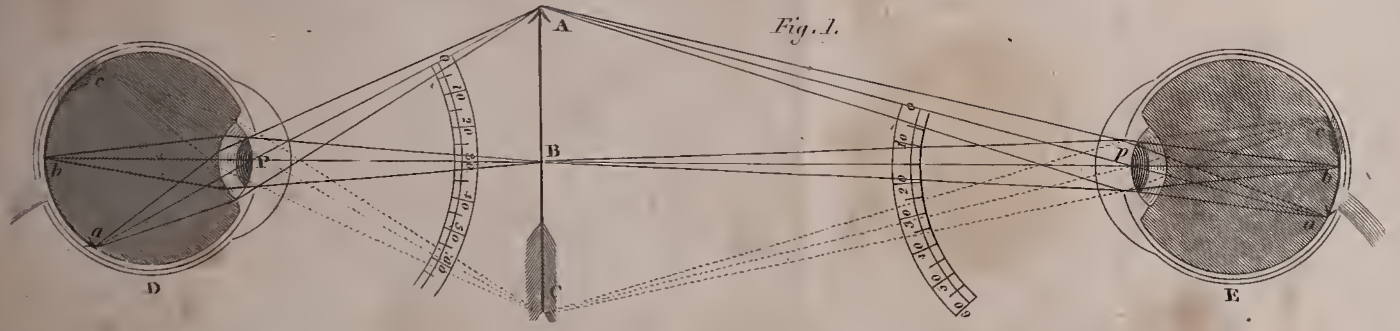
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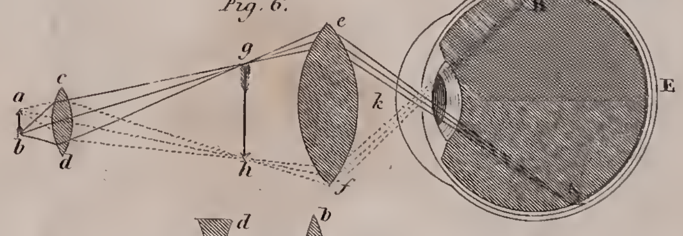
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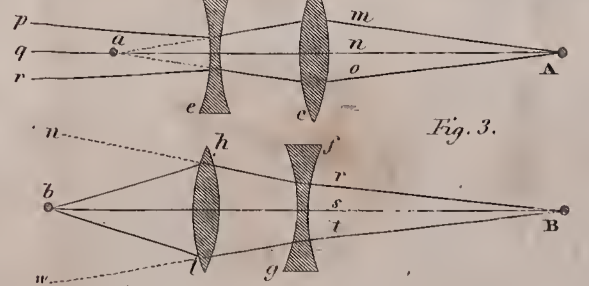
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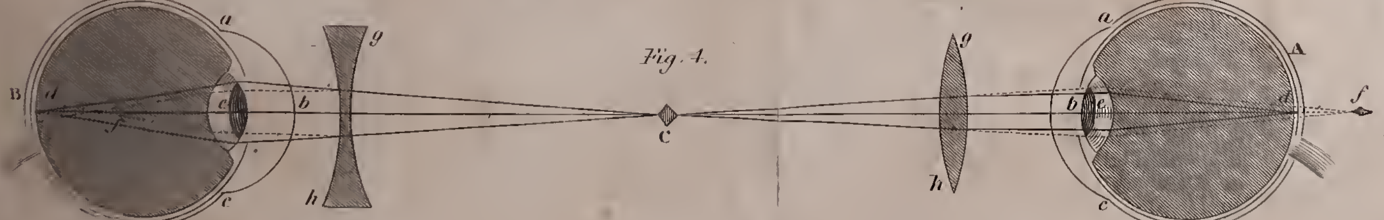
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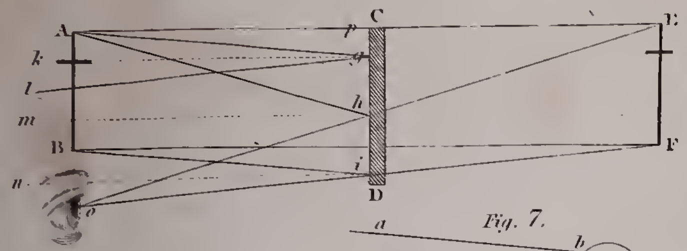

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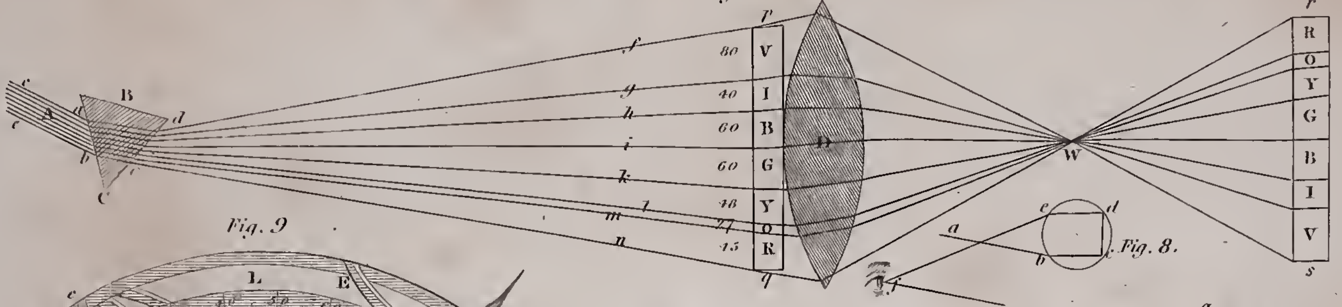

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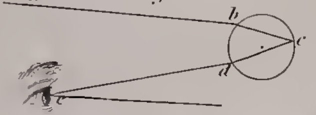

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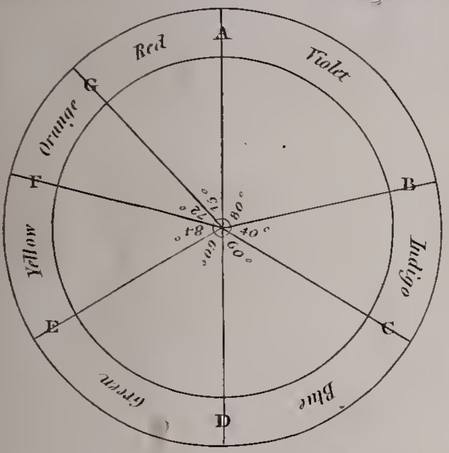

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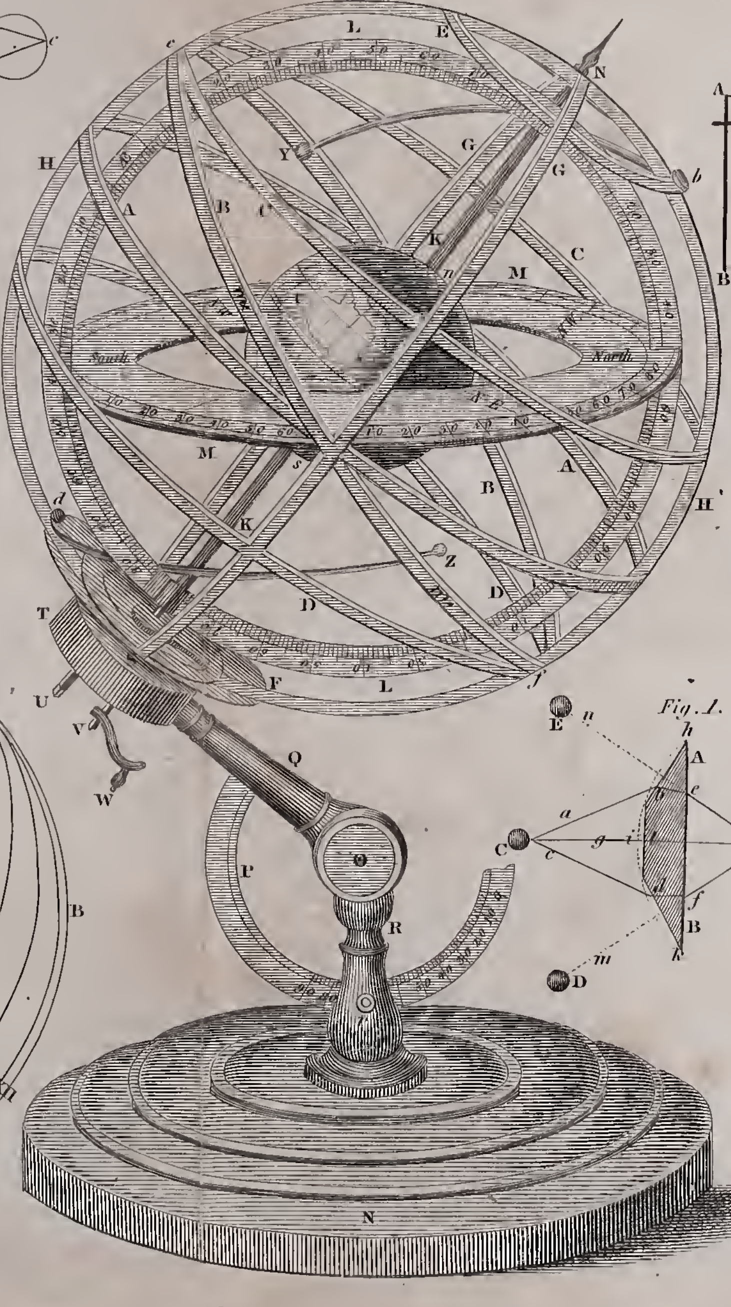

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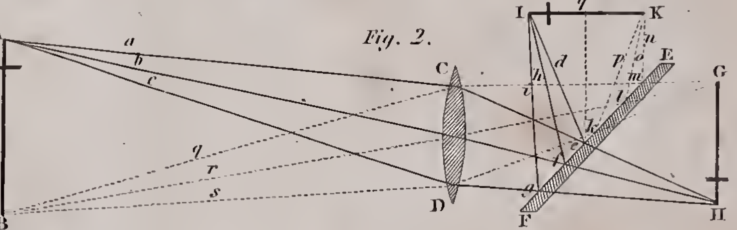

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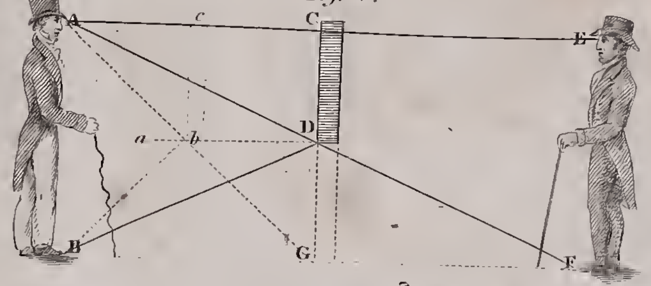

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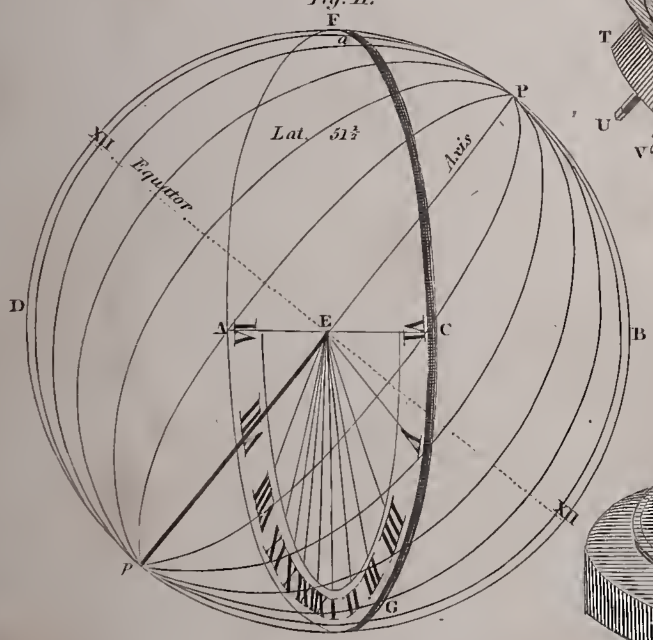

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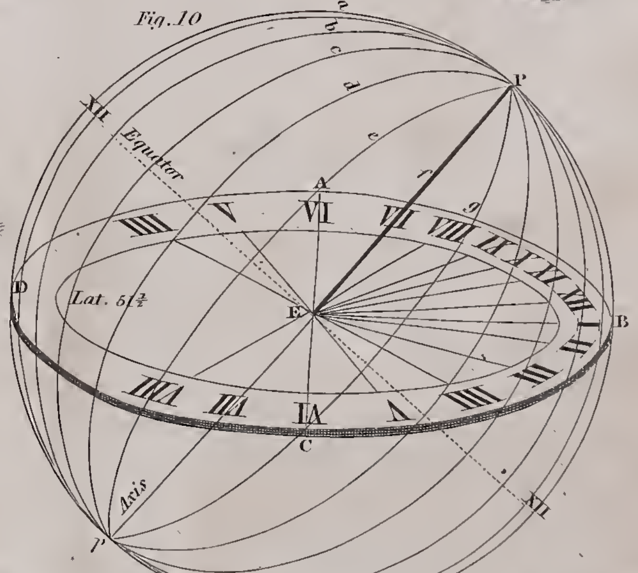

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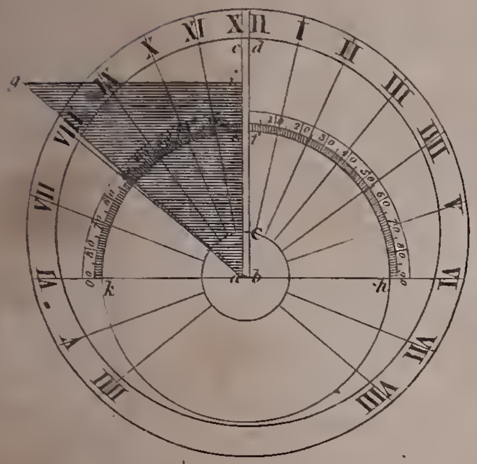
PLATE X.

Fig. 1.



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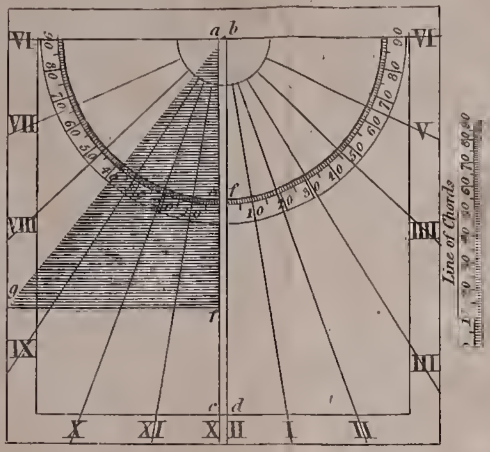

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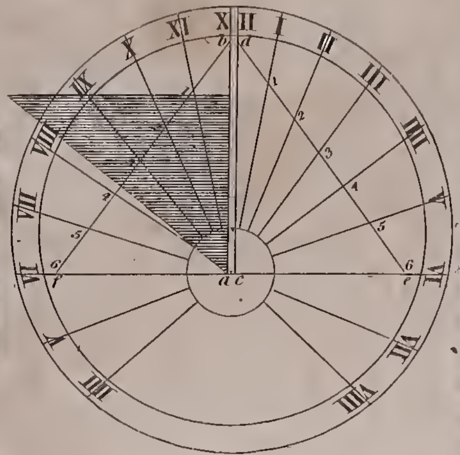

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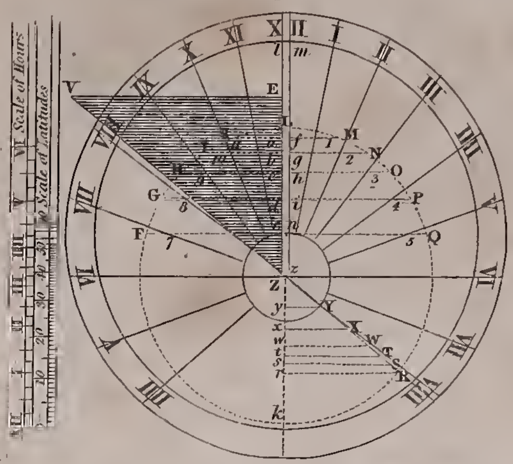

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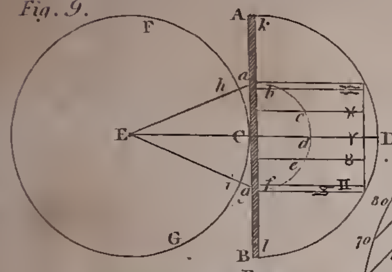

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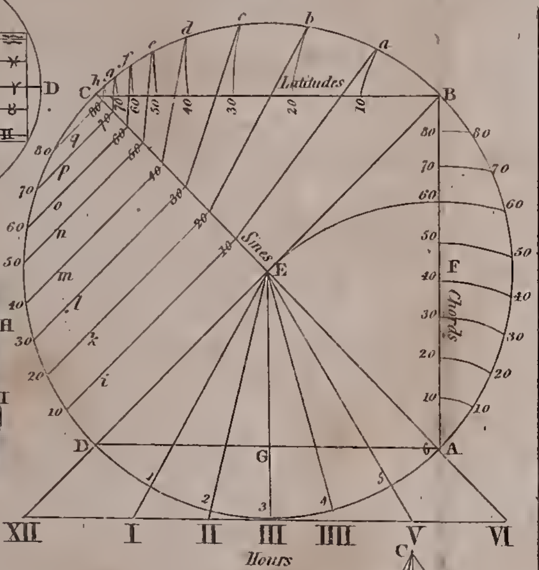

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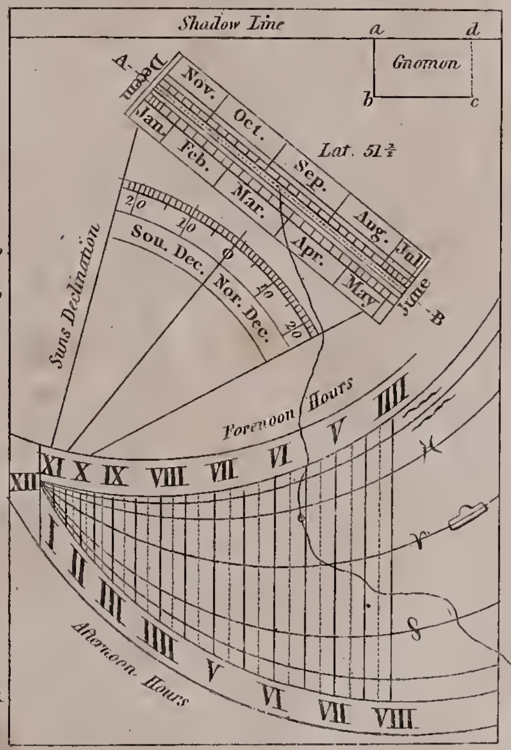

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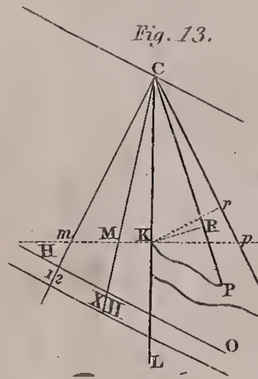

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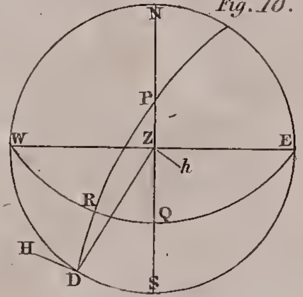

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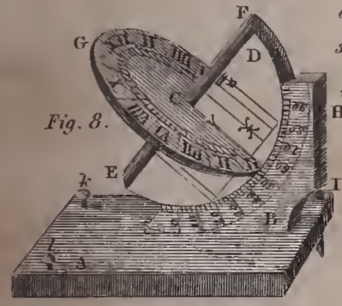

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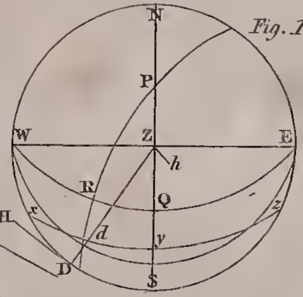

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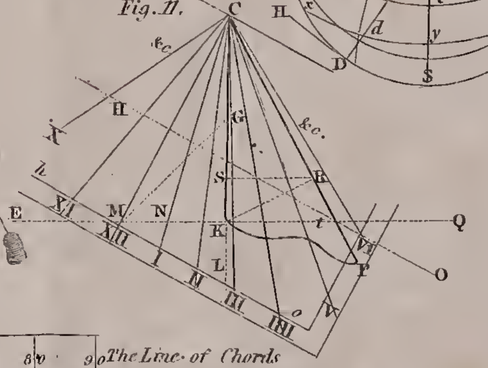

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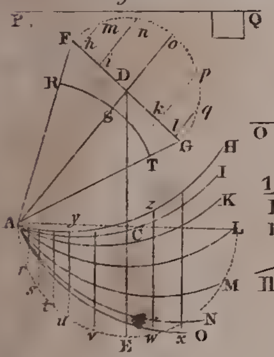

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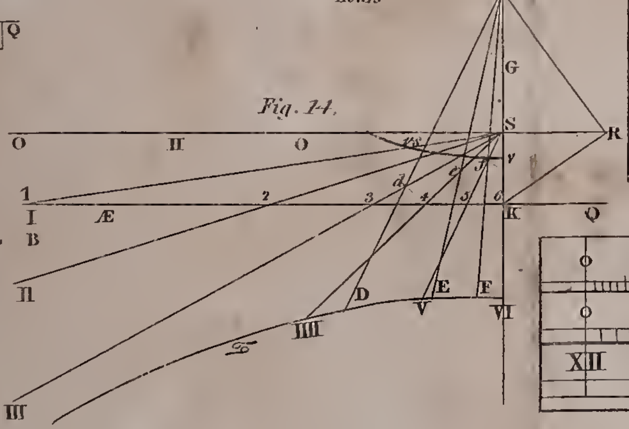

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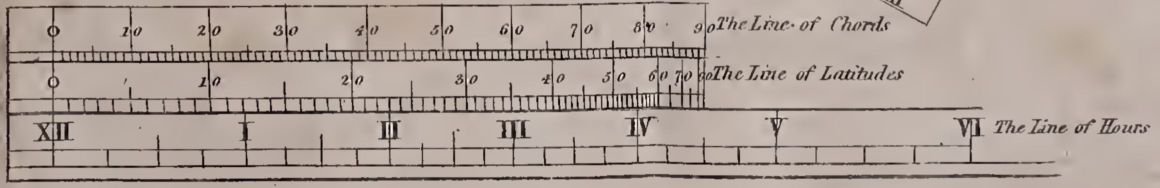

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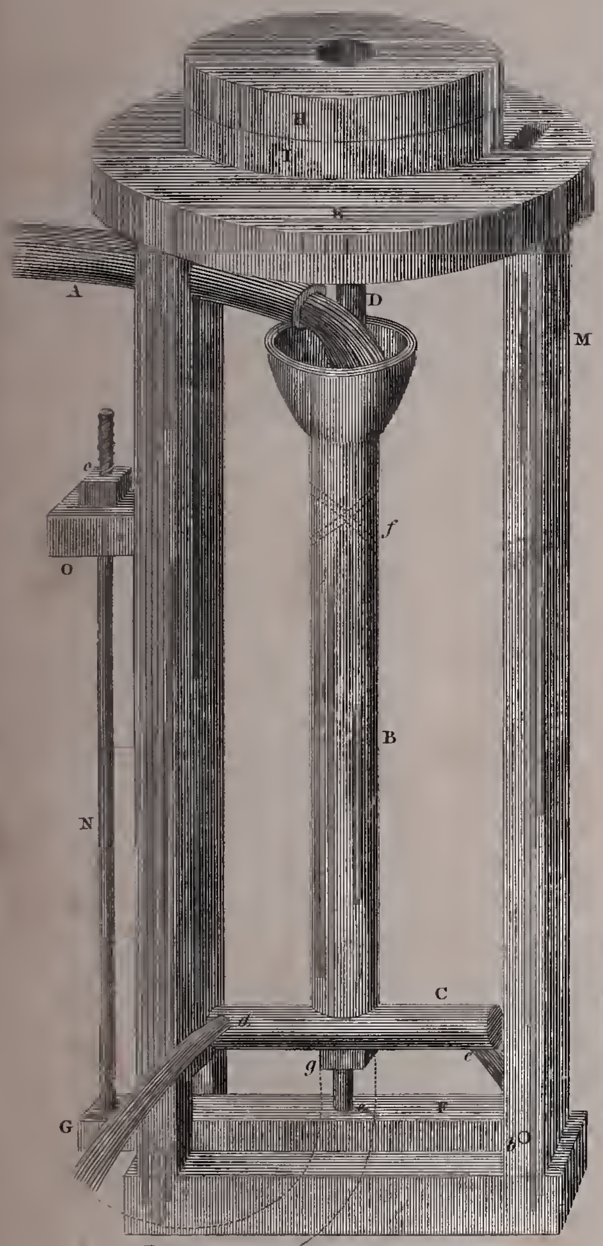
PLATE XI.

Fig. 4.



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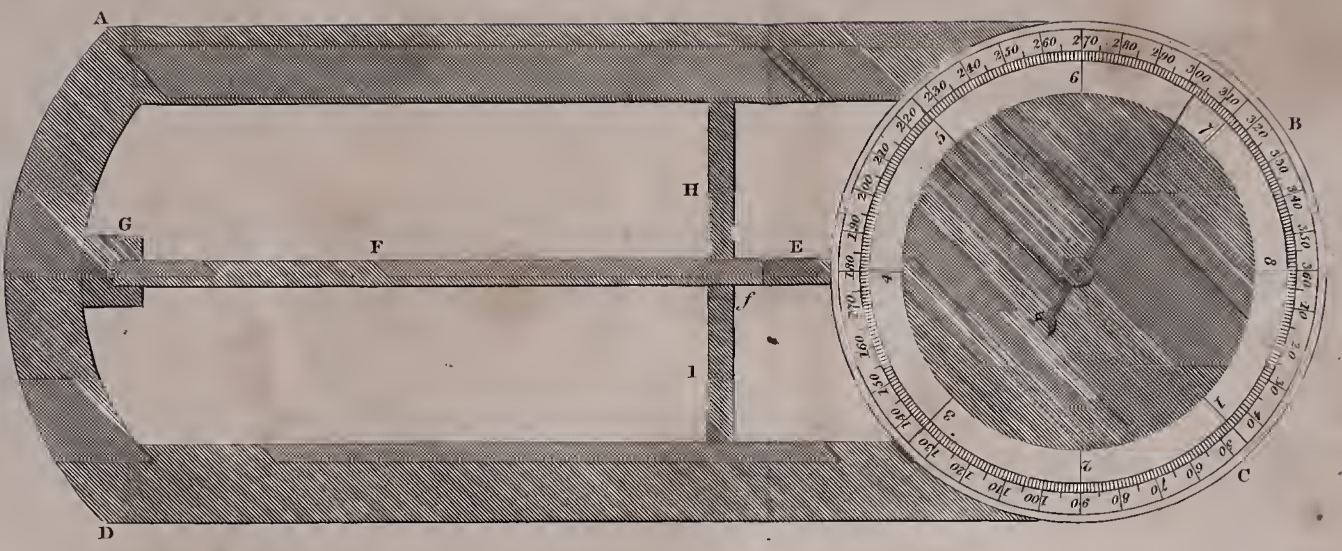

Fig. 1.

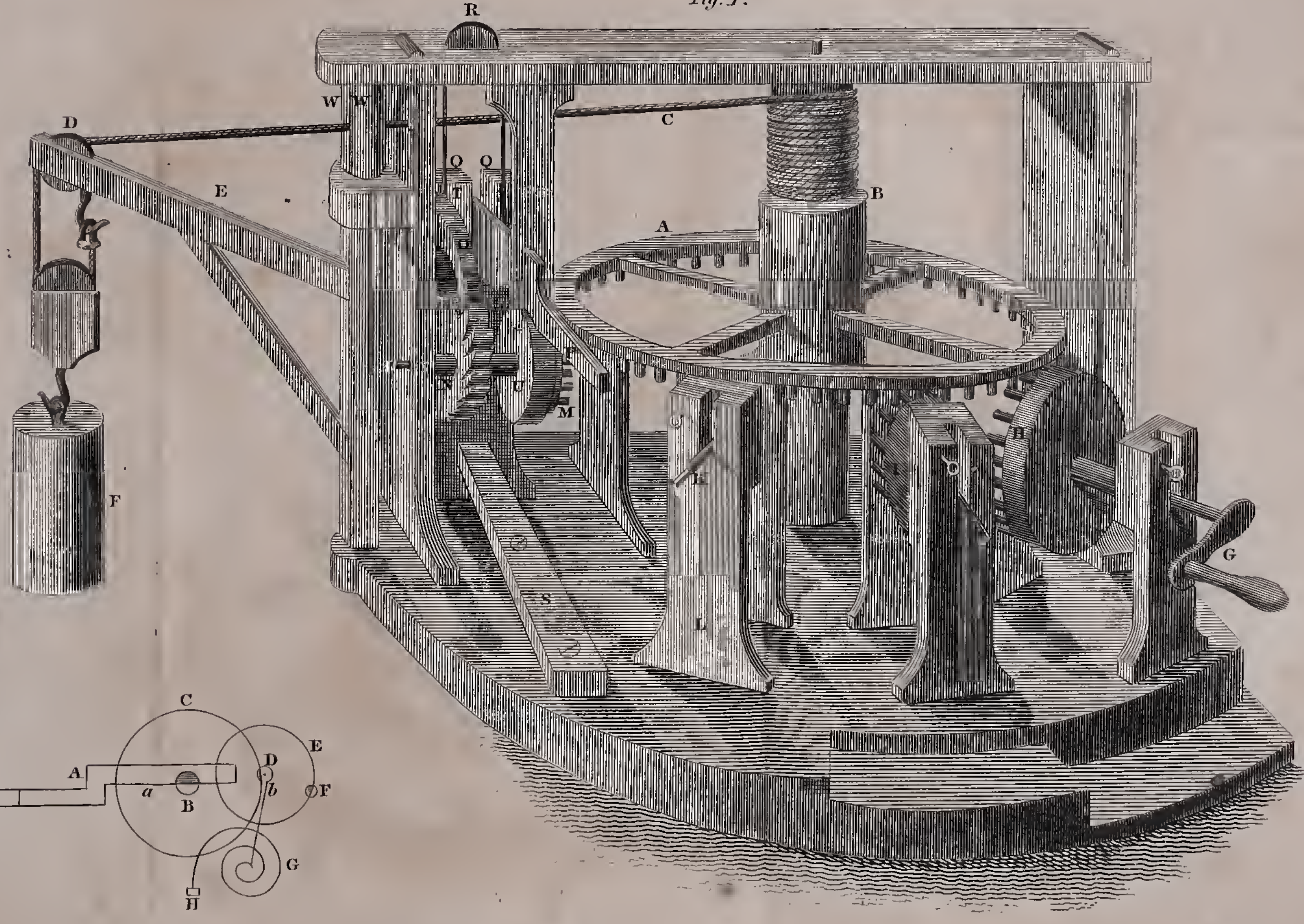

Fig. 3.

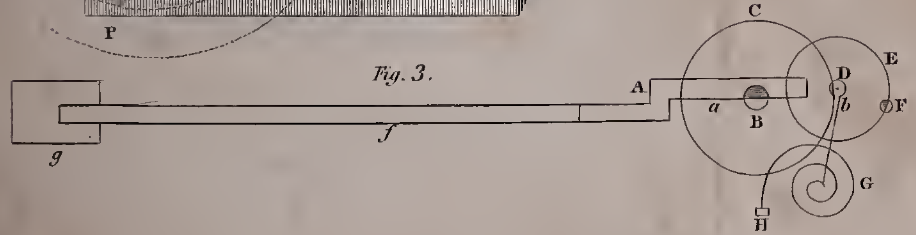

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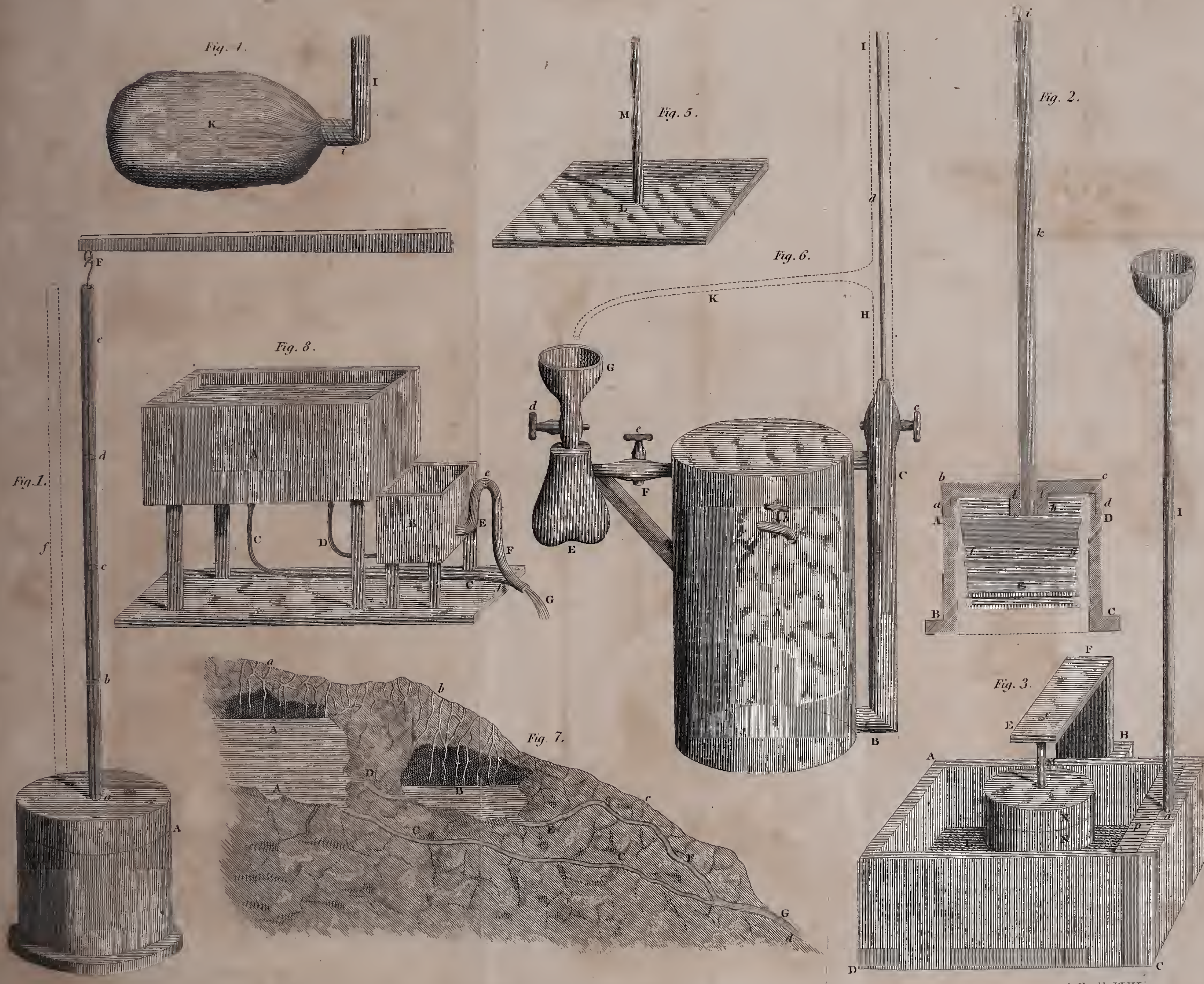
PLATE XII.



Eng^d by W. H. Lizars.

Fig. 4.

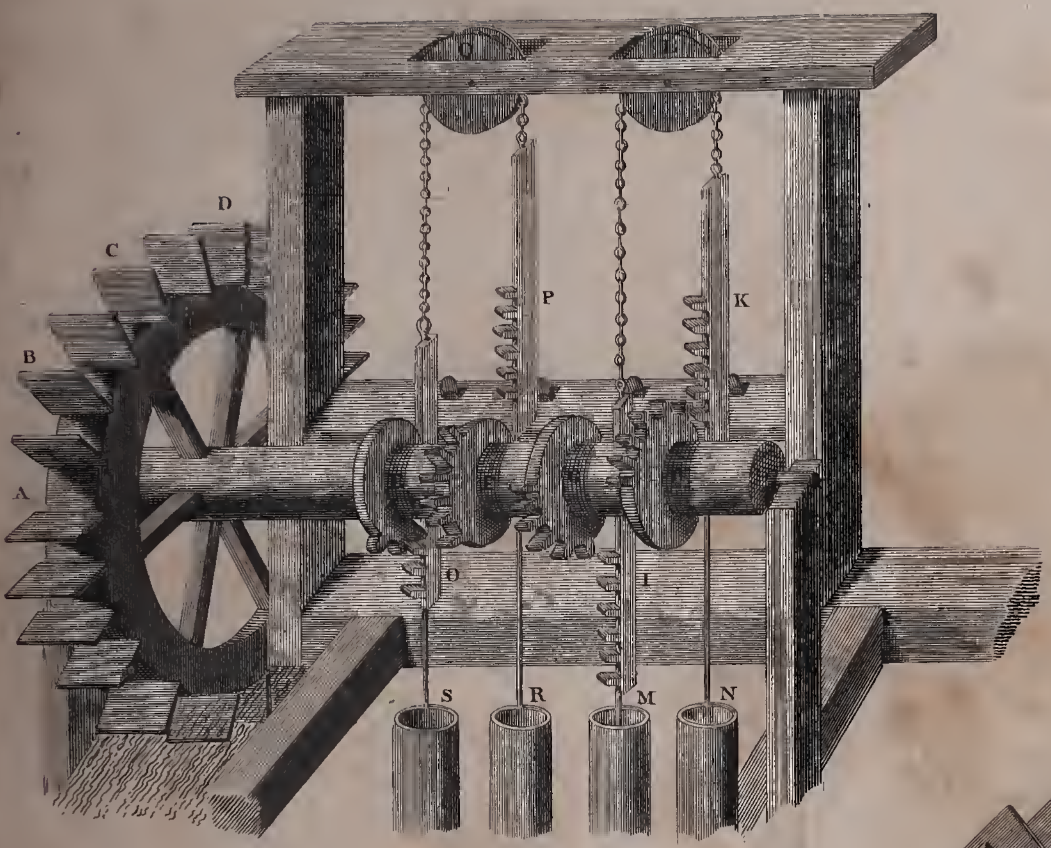


Fig. 2.

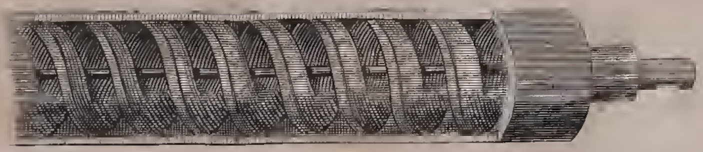


Fig. 1.

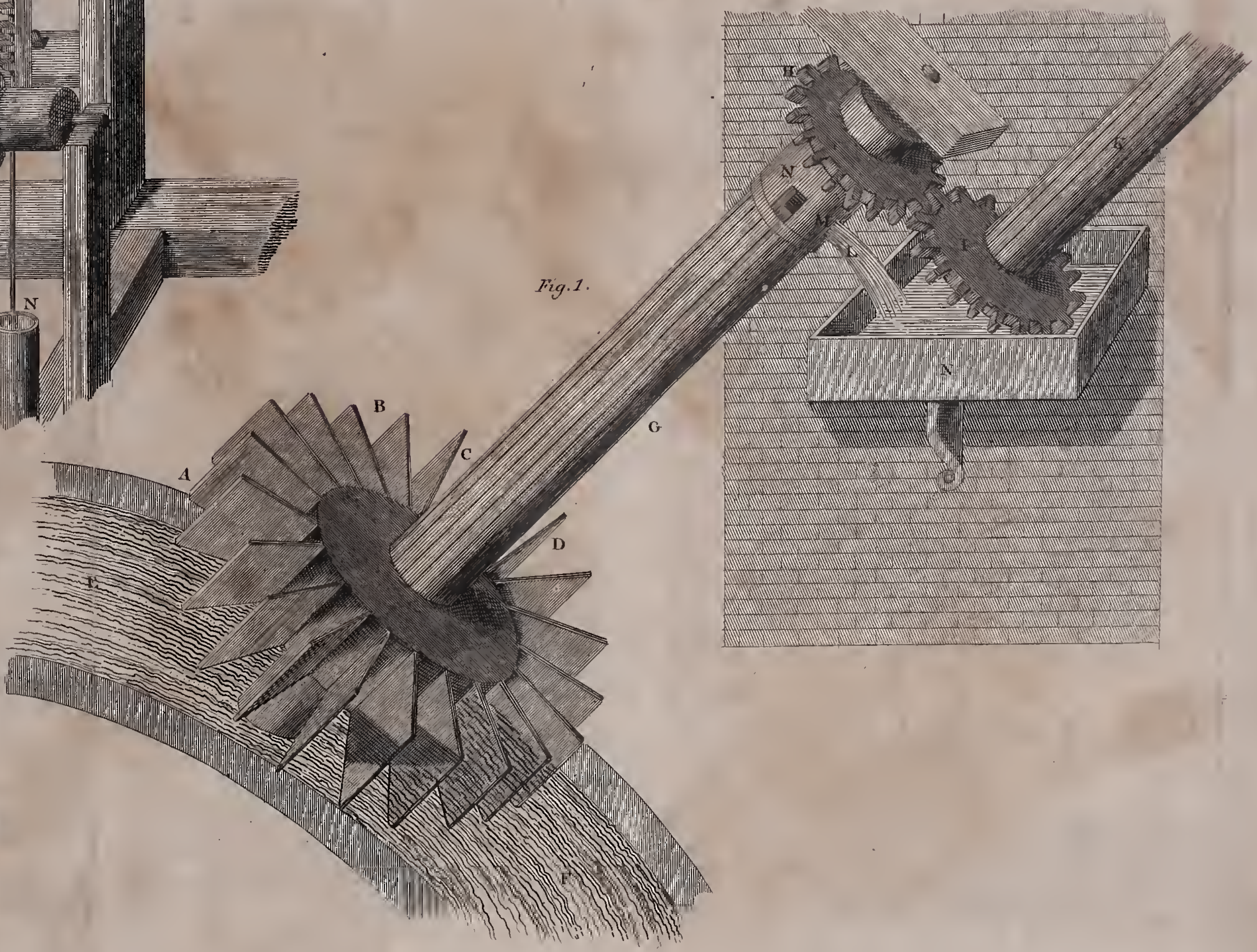


Fig. 3.

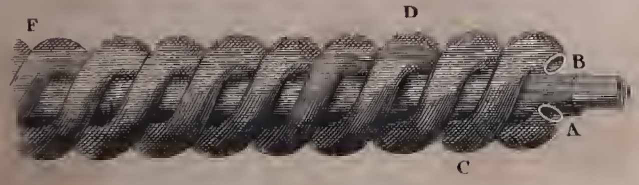


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PLATE XIV.

Fig. 4.

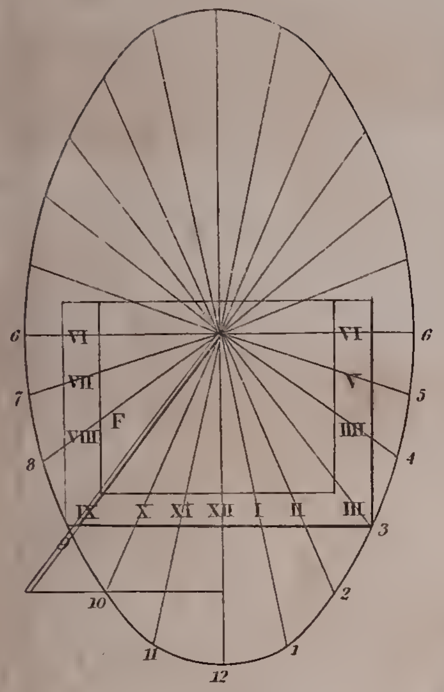

Fig. 5.

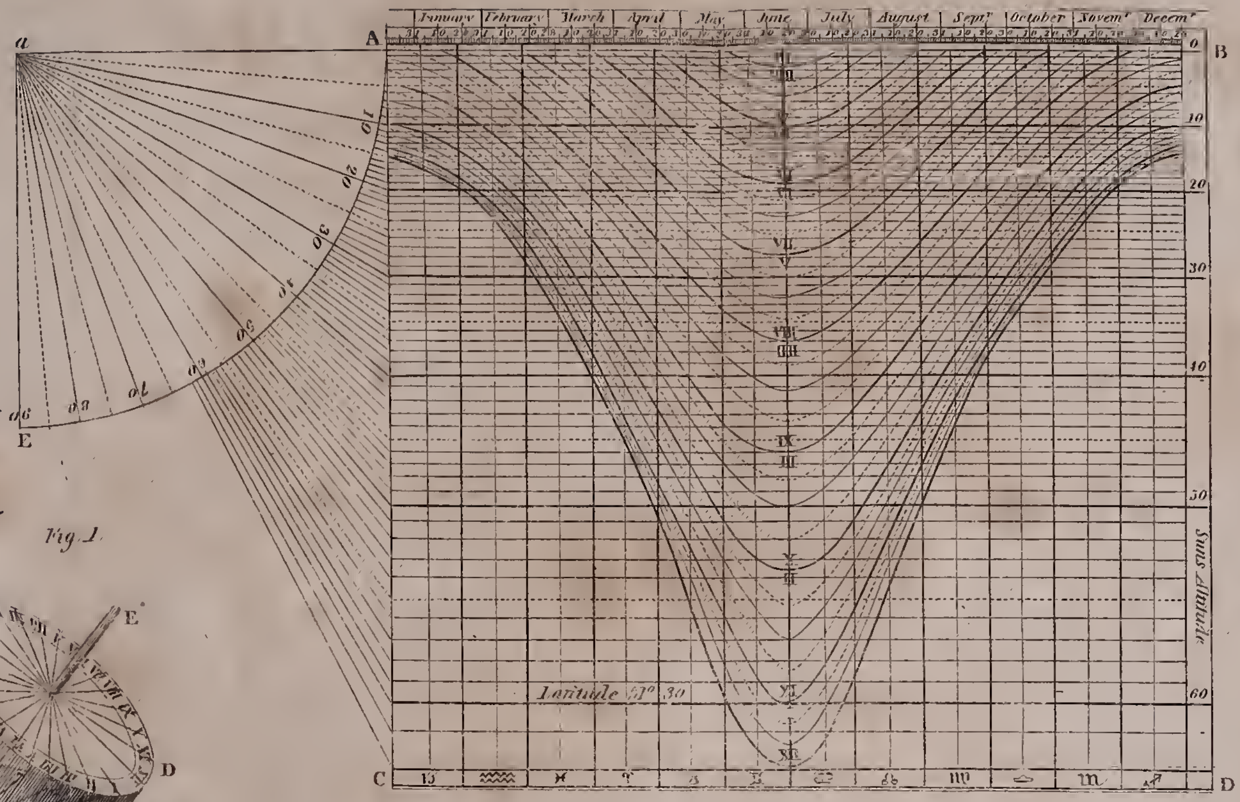

Fig. 1.

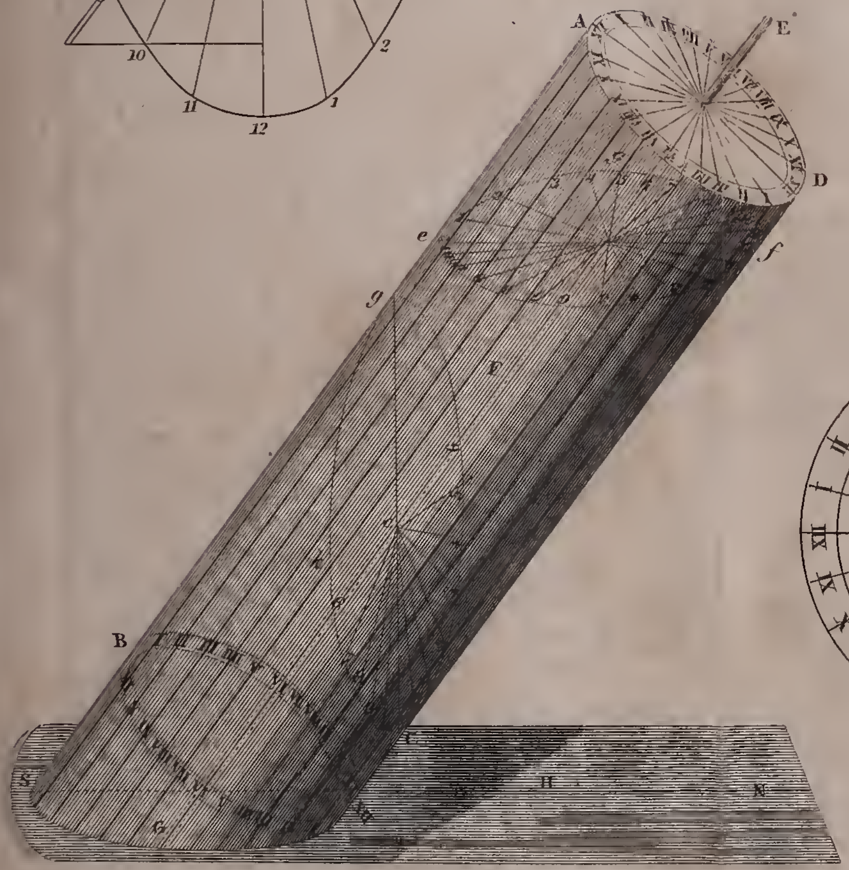

Fig. 2.

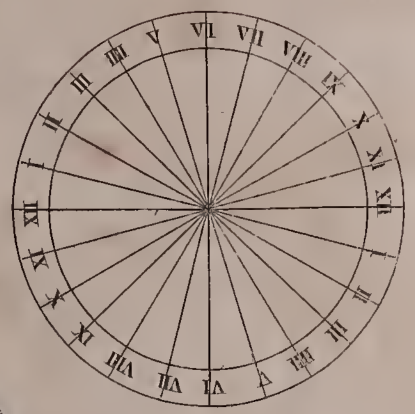

Fig. 3.

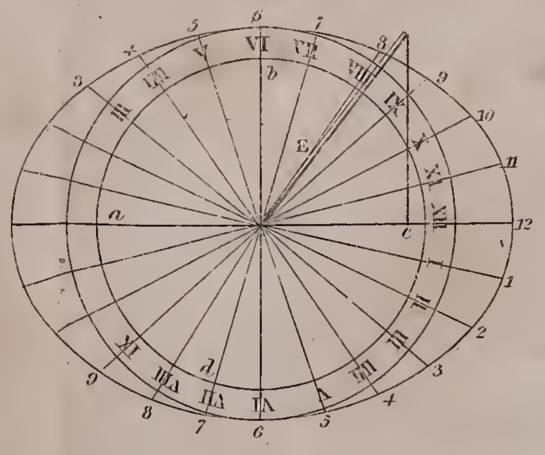

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PLATE XV.

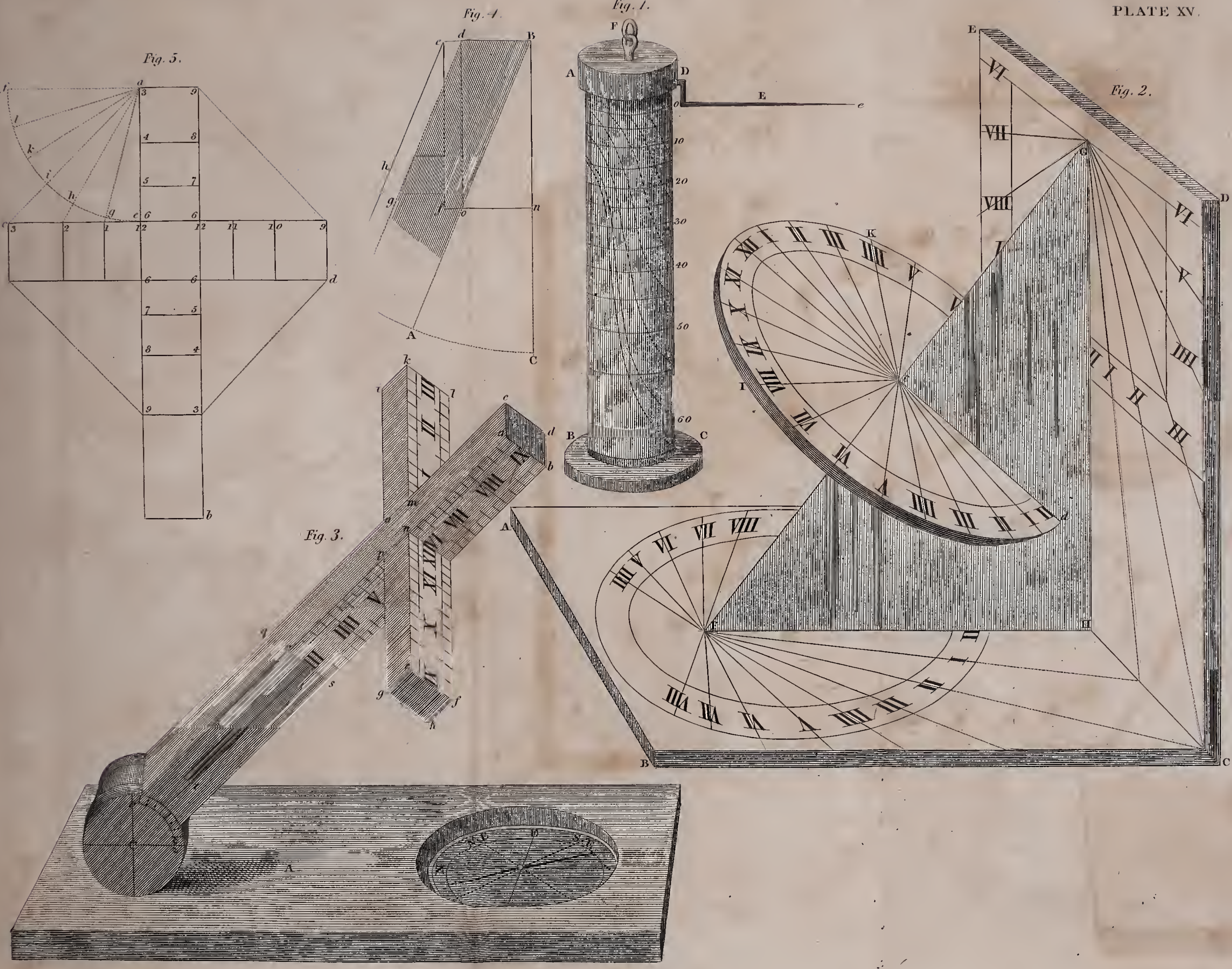

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Fig. 1.

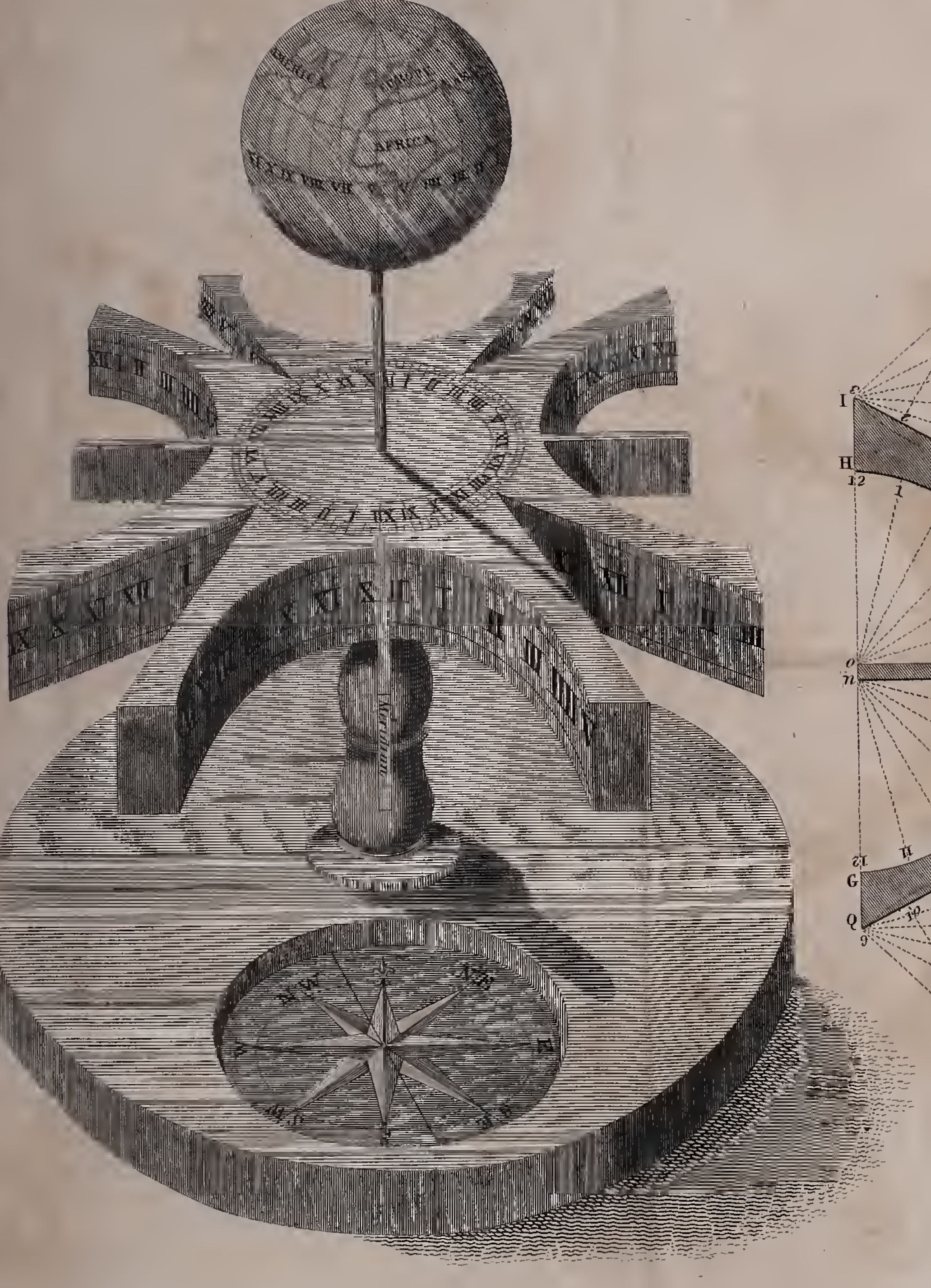
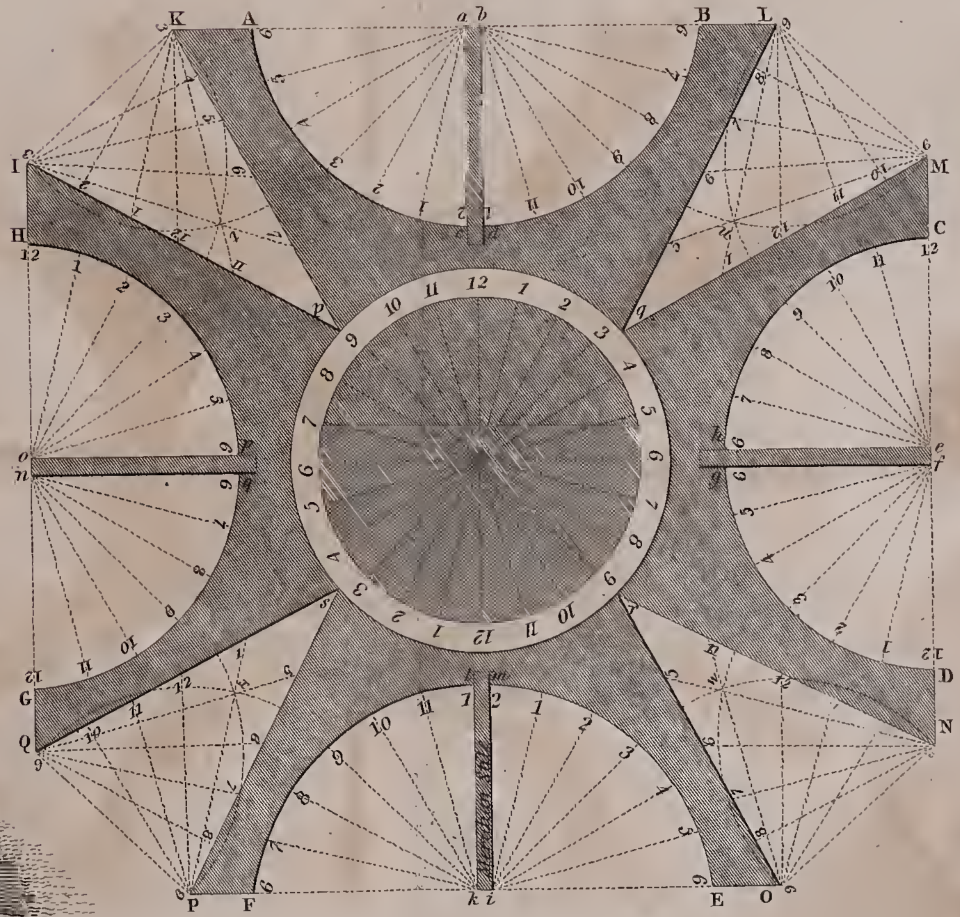


Fig. 2.



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